Broadening of cloud droplet spectra through eddy hopping: Getting it right

Wojciech W. Grabowski

with Kamal Kant Chandrakar and Hugh Morrison

NSF NCAR, Boulder, Colorado, USA

This material is based upon work supported by the National Center for Atmospheric Research, which is a major facility sponsored by the National Science Foundation under Cooperative Agreement No. 1852977.

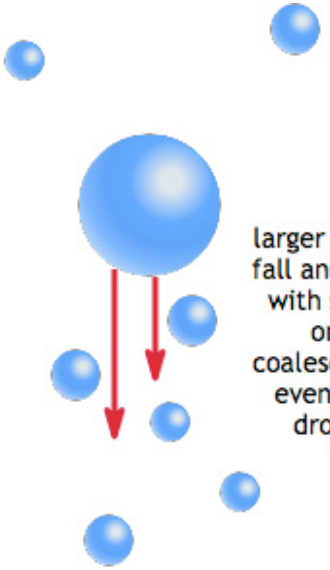
Width of the cloud droplet spectrum in warm clouds is an important parameter.

It affects transfer of solar radiation through a cloud and collision/coalescence that leads to rain formation…

Effective radius r_e:

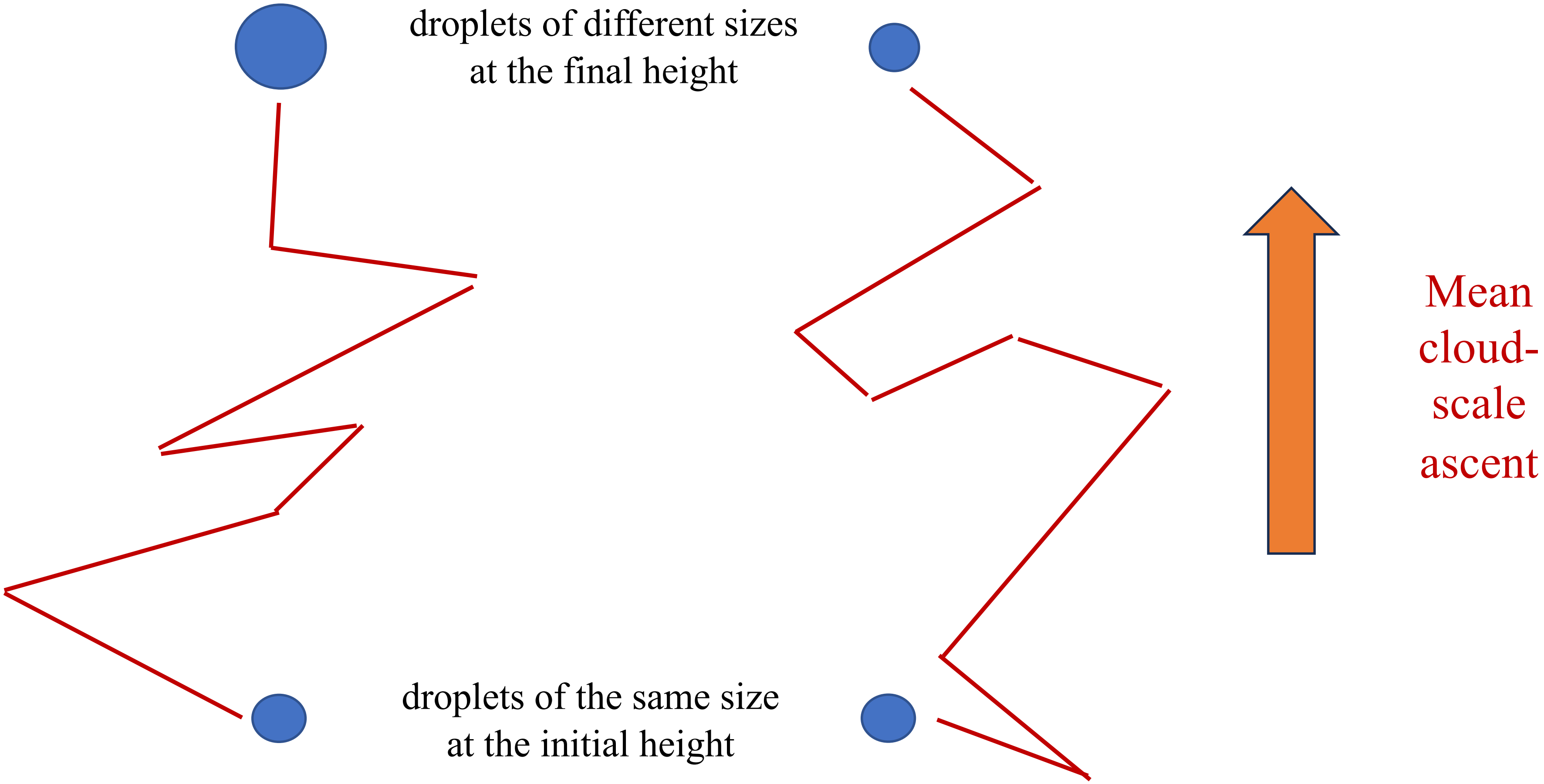
$$r_e = \frac{\int_0^\infty \pi r^3 \cdot n(r)\, dr}{\int_0^\infty \pi r^2 \cdot n(r)\, dr}$$

Gravitational droplet collisions:



larger droplets fall and collide with smaller ones, coalescing into even larger droplets

CLOUD DROPLETS HOPPING TURBULENT EDDIES

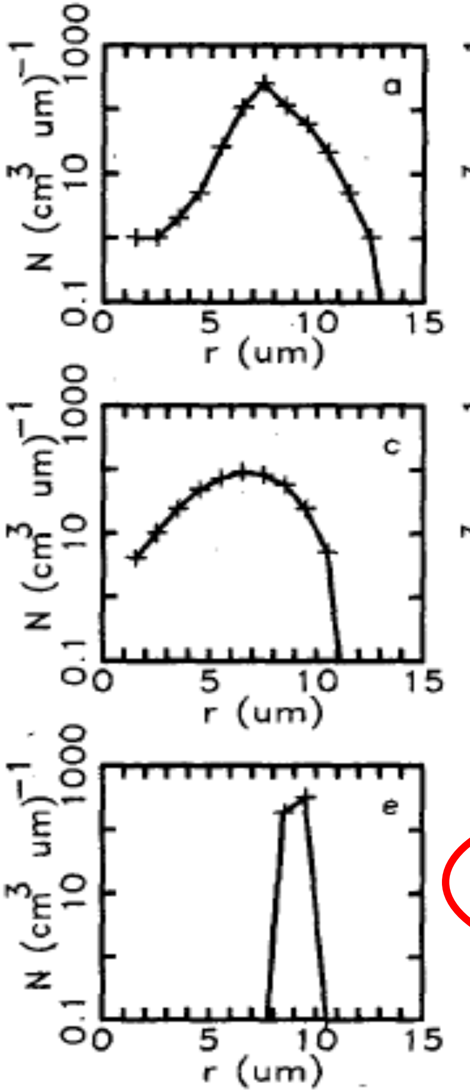


droplets of different sizes
at the final height

droplets of the same size
at the initial height

Mean
cloud-
scale
ascent

Observed cloud droplet spectra in cumulus averaged over ~100 m (1 Hz, FSSP data) around 1 km above the cloud base:



observed,
adiabatic fraction
AF ≈ 1; σ_r=1.3 μm

observed, AF ≈ 0.8;
σ_r=1.3 μm

observed, AF ≈ 0.8;
σ_r=1.8 μm

observed, AF ≈ 1;
bimodal

calculated adiabatic
spectrum; σ_r=0.1 μm

(Jensen et al. *JAS* 1985)

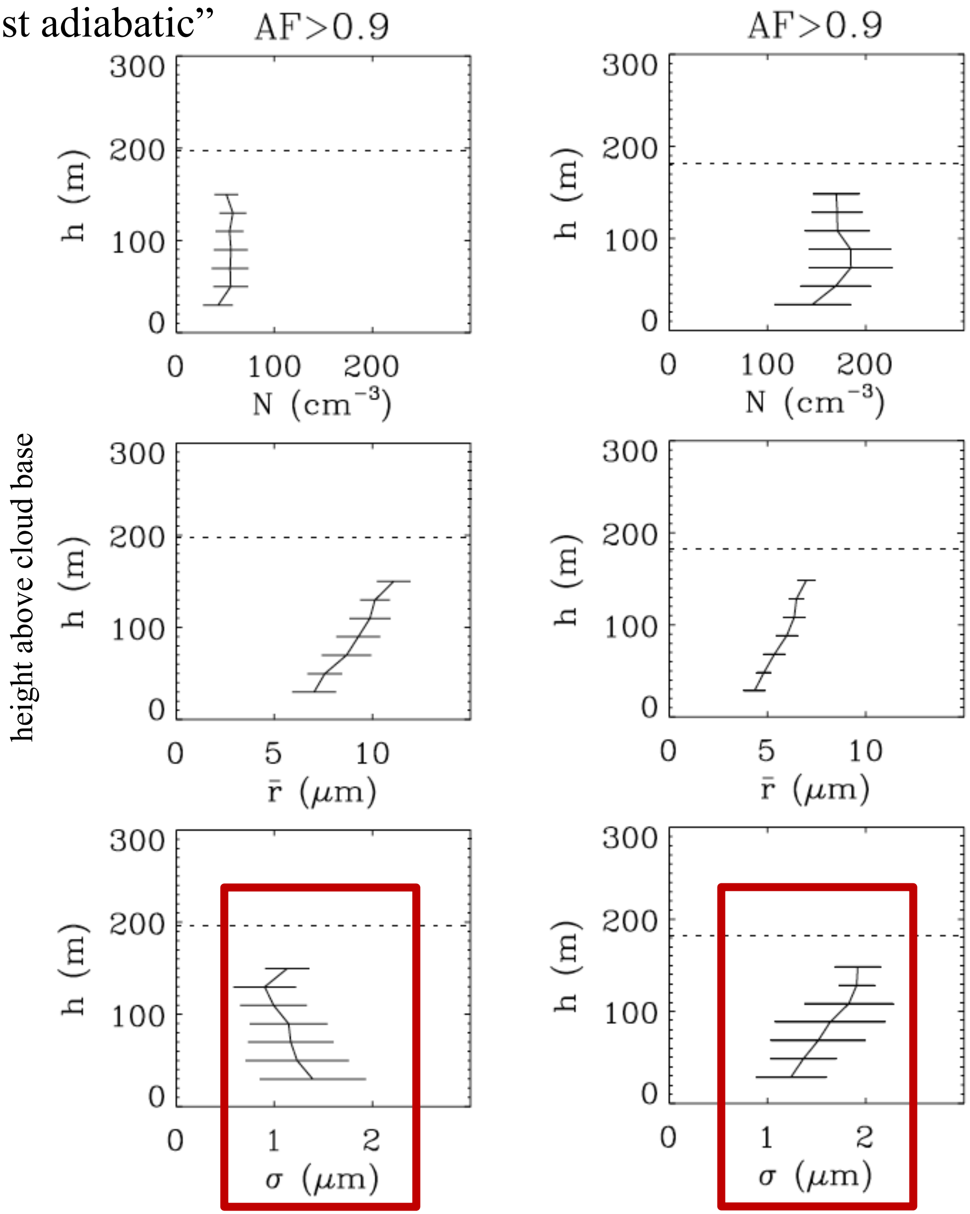
Observed cloud droplet spectra in stratocumulus averaged over ~10m (10 Hz, Fast FSSP):

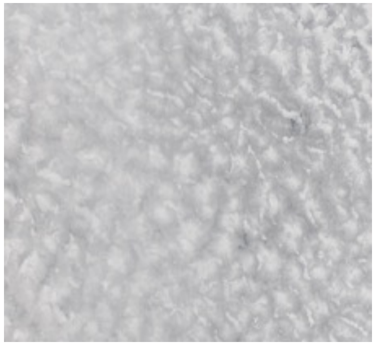
"almost adiabatic"

droplet concentration

mean radius

spectral width





(Pawlowska et al. *GRL* 2006)

Broadening of cloud droplet spectra through eddy hopping: Why did we all have it wrong?

Wojciech W. Grabowski[1] and Gustavo C. Abade[2]

[1] NSF NCAR, Boulder, Colorado, USA
[2] University of Warsaw, Warsaw, Poland

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Can small-scale turbulence explain the width of the droplet spectra in undiluted cloudy volumes?

Microscopic Approach to Cloud Droplet Growth by Condensation. Part I: Model Description and Results without Turbulence

P. A. VAILLANCOURT* AND M. K. YAU
Department of Atmospheric and Oceanic Sciences, McGill University, Montreal, Quebec, Canada

W. W. GRABOWSKI
National Center for Atmospheric Research, Boulder, Colorado

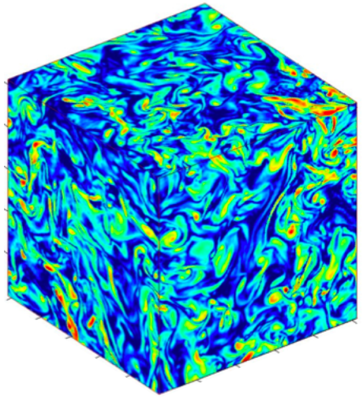
JAS 2001

Microscopic Approach to Cloud Droplet Growth by Condensation. Part II: Turbulence, Clustering, and Condensational Growth

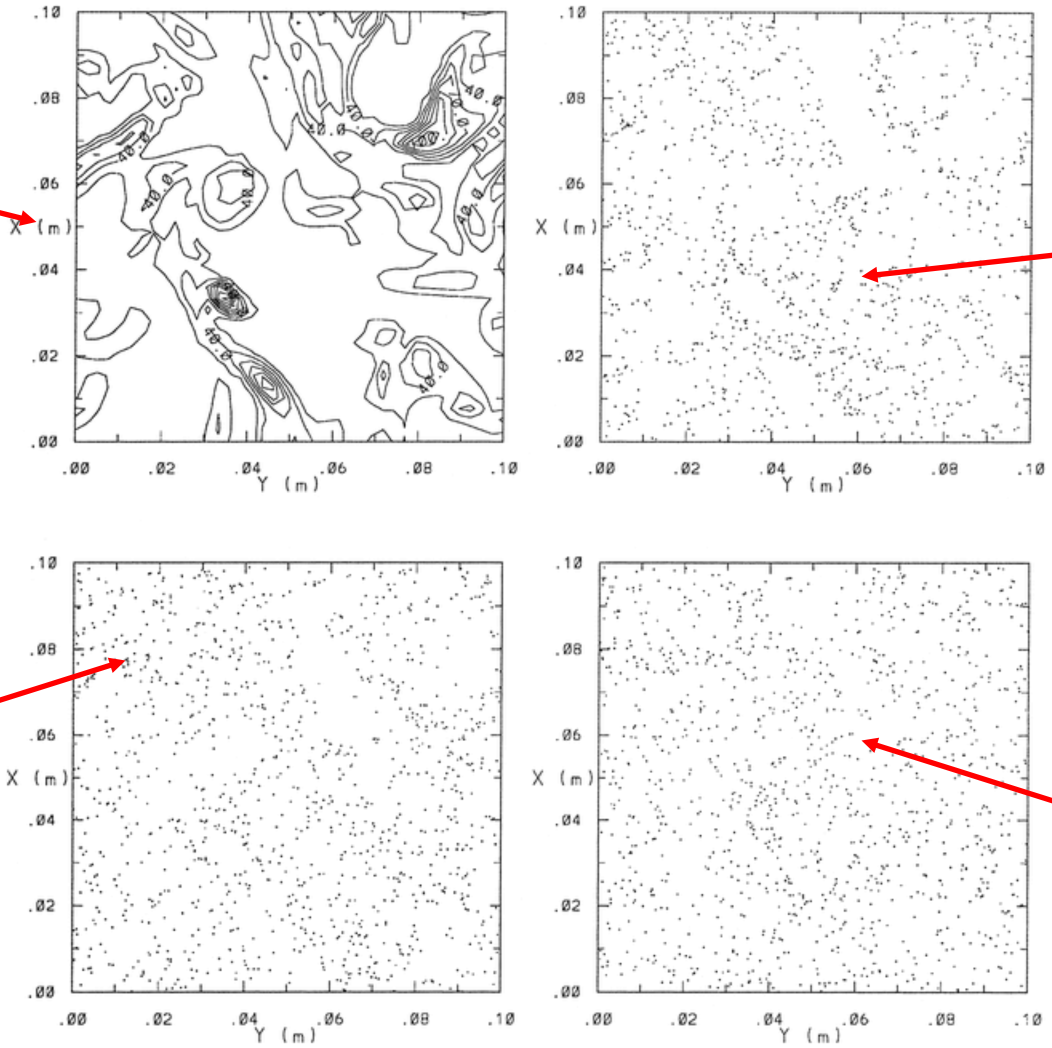
P. A. VAILLANCOURT, M. K. YAU, AND P. BARTELLO
McGill University, Montréal, Québec, Canada

W. W. GRABOWSKI
National Center for Atmospheric Research, Boulder, Colorado

JAS 2002



Direct numerical simulations (DNS) of homogeneous isotropic turbulence with cloud droplets *growing by the diffusion of water vapor* for conditions relevant to cloud physics (ε=160 cm^{2}s^{-3})



z vorticity magnitude

(contour 15 s^{-1})

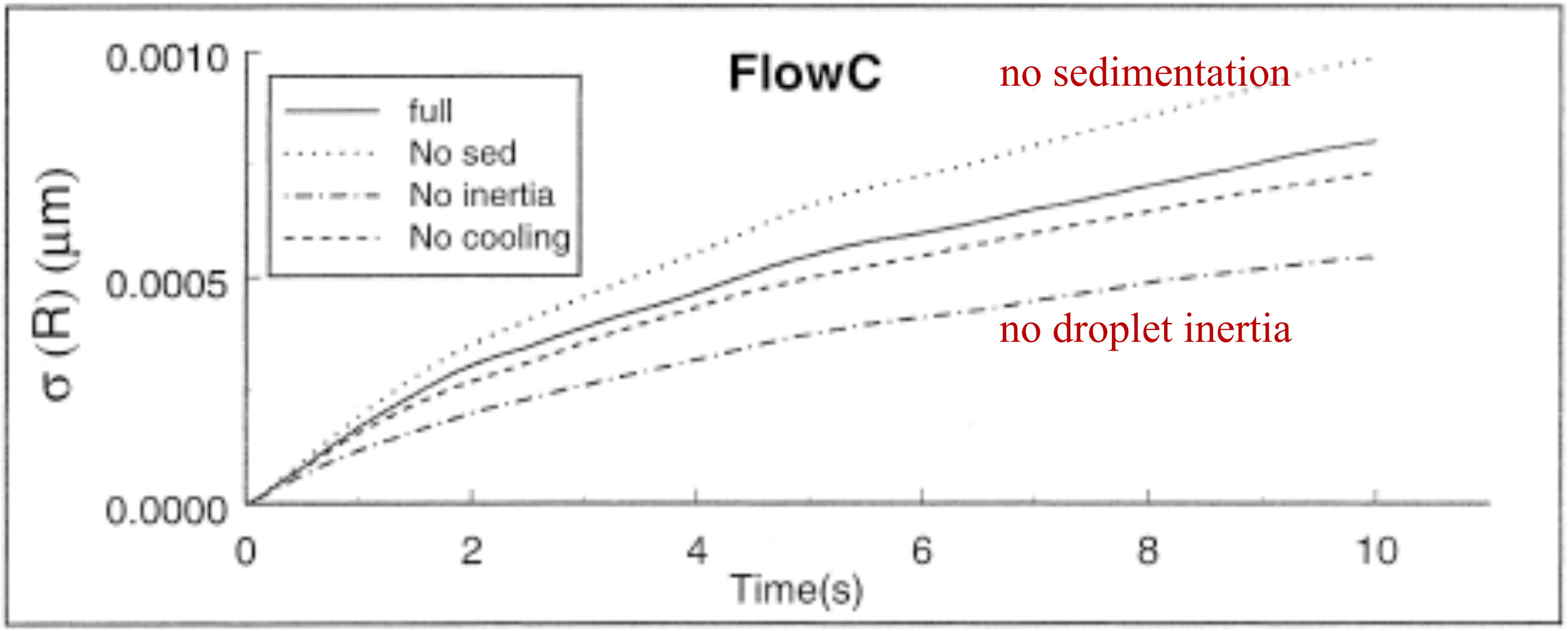
r=20 micron

r=15 micron

r=10 micron

Note the domain size: about 1 liter…

Vaillancourt et al. *JAS* 2002

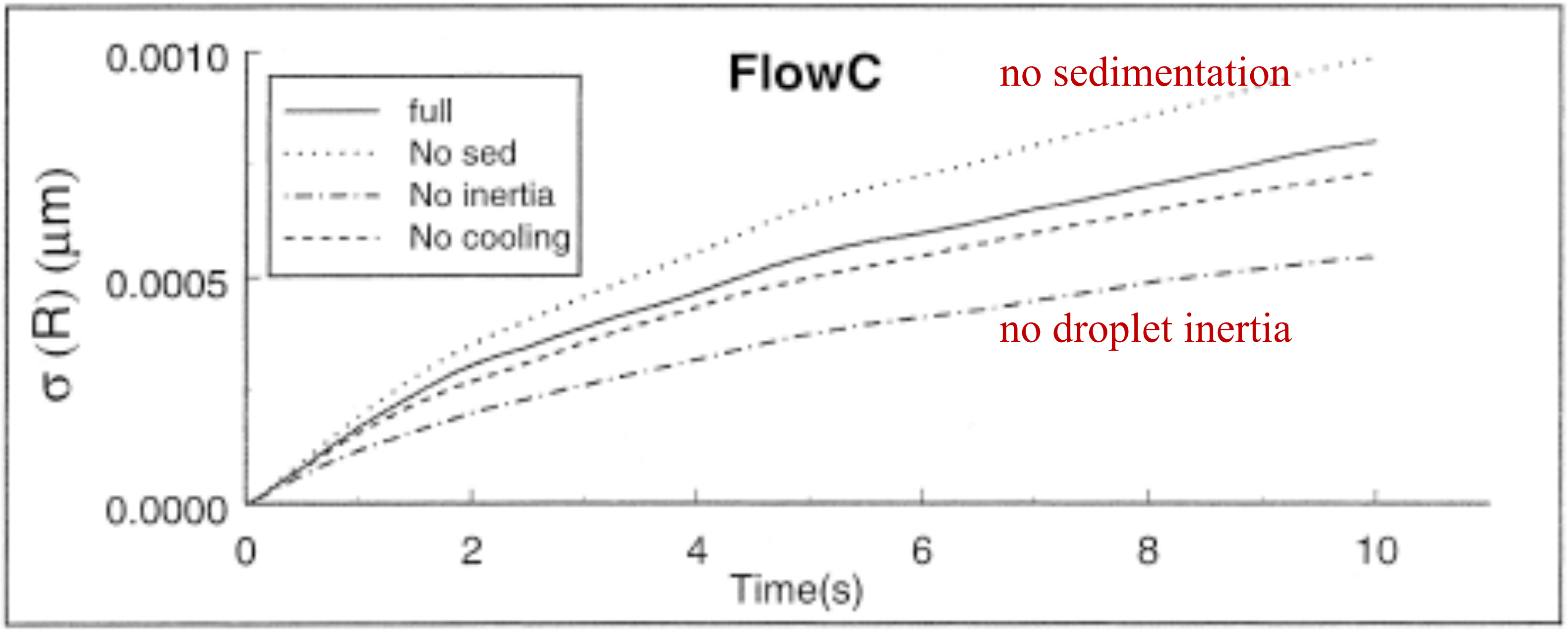


no sedimentation

no droplet inertia

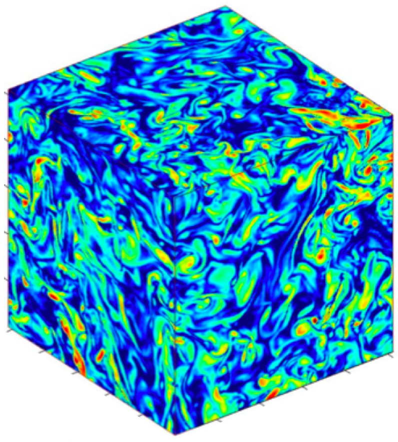
Main conclusion: centimeter-scale turbulence has a small effect for the diffusional growth…

Vaillancourt et al. *JAS* 2002



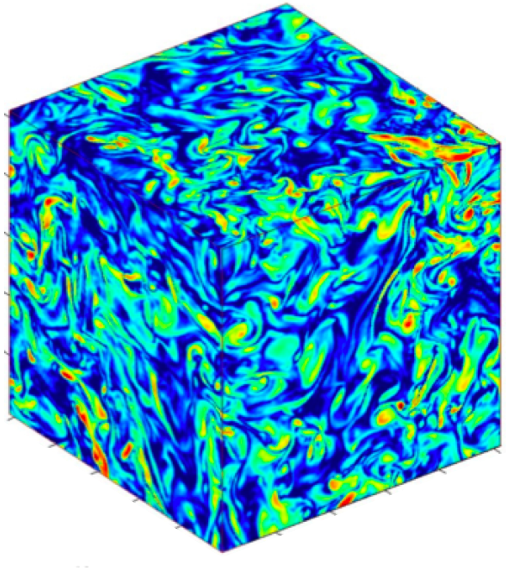
no sedimentation

no droplet inertia

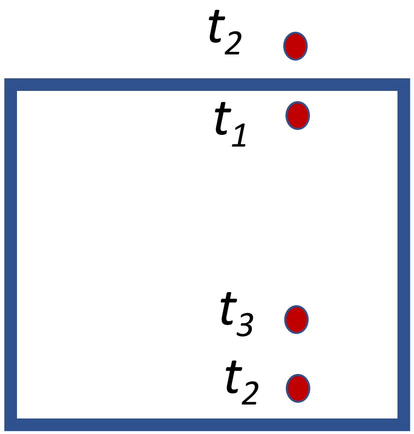


Several subsequent studies pursued this line of research…

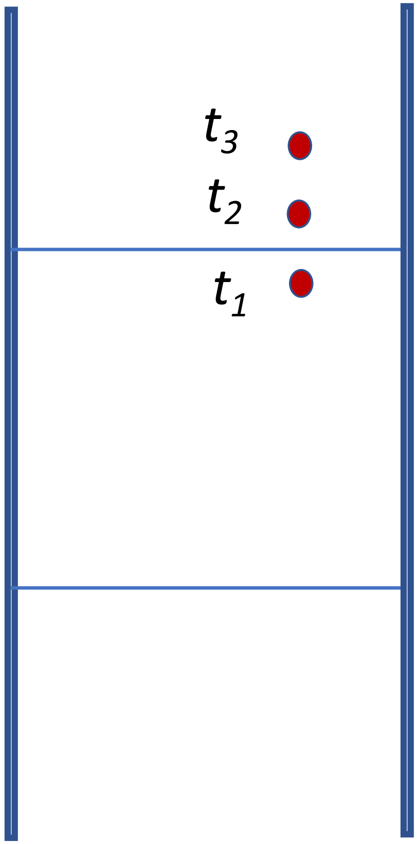
Is there anything we have not yet considered?



periodic box

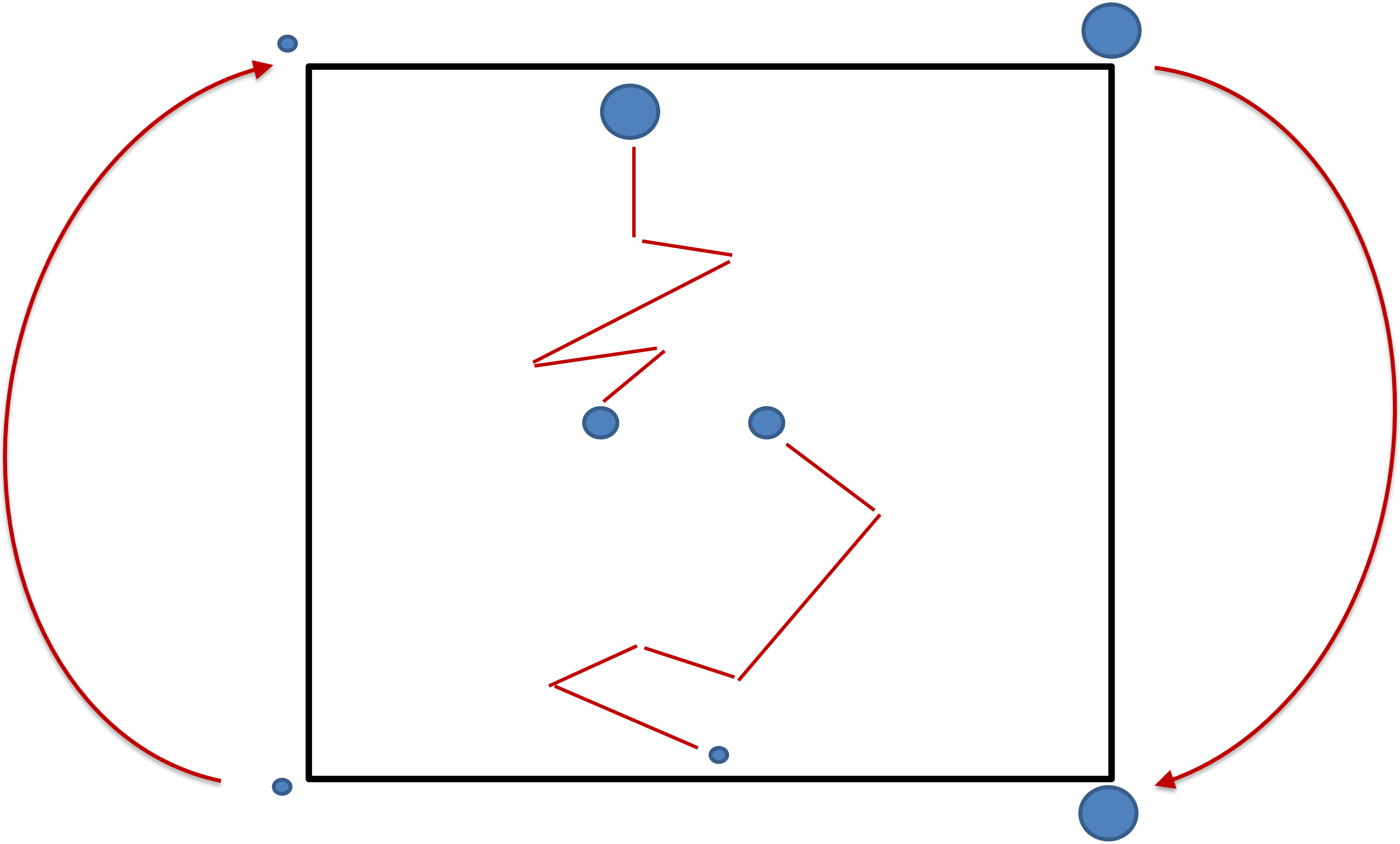


infinite box



$t_1 < t_2 < t_3$

Prabhakaran et al. *JAS* 2022
Abade *JAS* 2024
Grabowski *JAS* 2024



In a periodic domain, droplets can circulate the domain and continue growing/evaporating…

Stochastic model for droplet growth by condensation and following droplet position in the vertical:

$$\frac{dr^{2\prime}}{dt} \sim S'$$

$$\frac{dS_i'}{dt} = a_1 w' - \frac{S_i'}{\tau_{\text{relax}}}$$

$$w'(t+\delta t) = w'(t)e^{-\delta t/\tau} + \sqrt{1-e^{-2\delta t/\tau}}\,\sigma_{w'}\psi$$

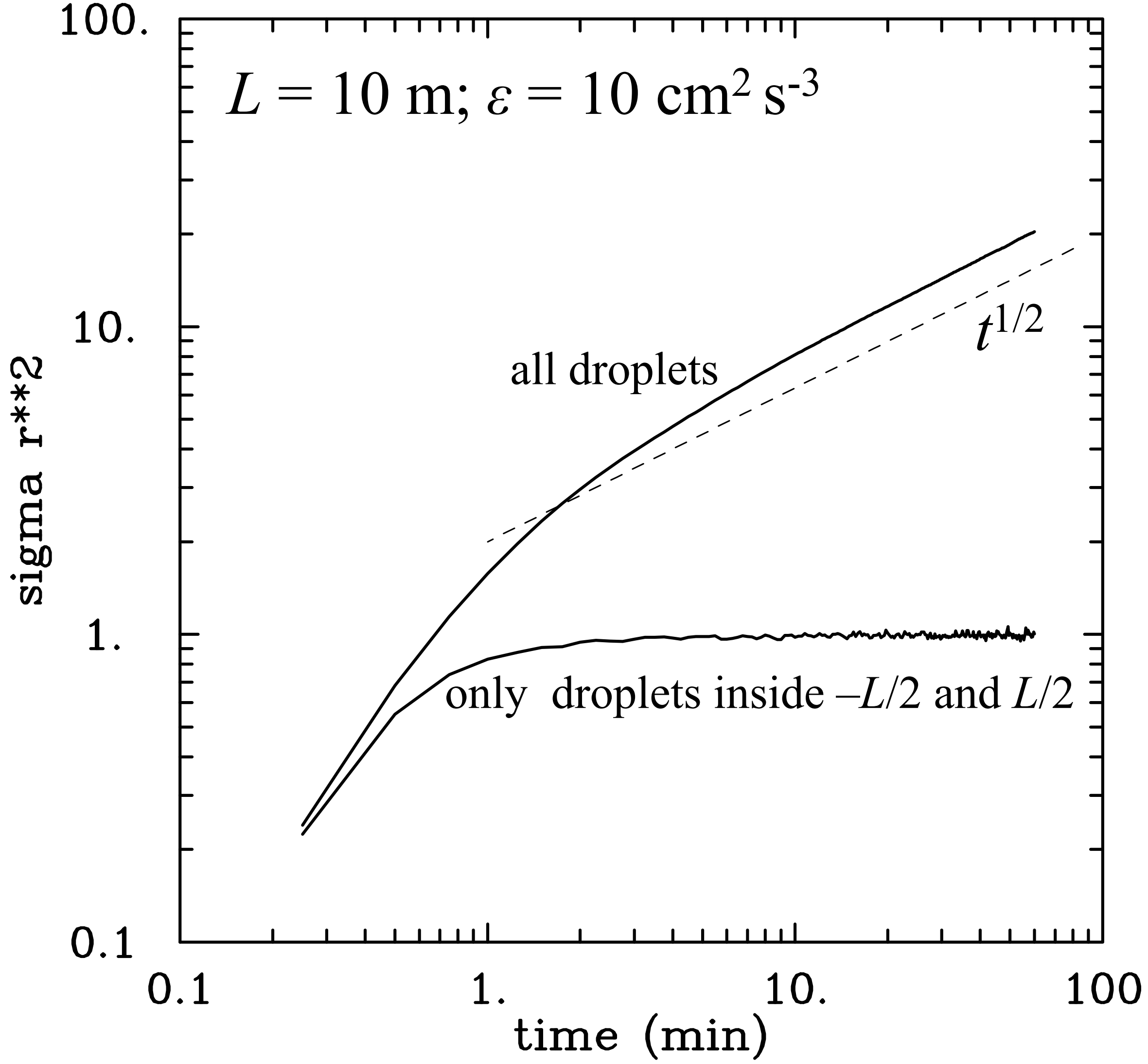
$$\sigma_{w'}^2 = \frac{2}{3}E$$

E - TKE

ψ - Gaussian random number

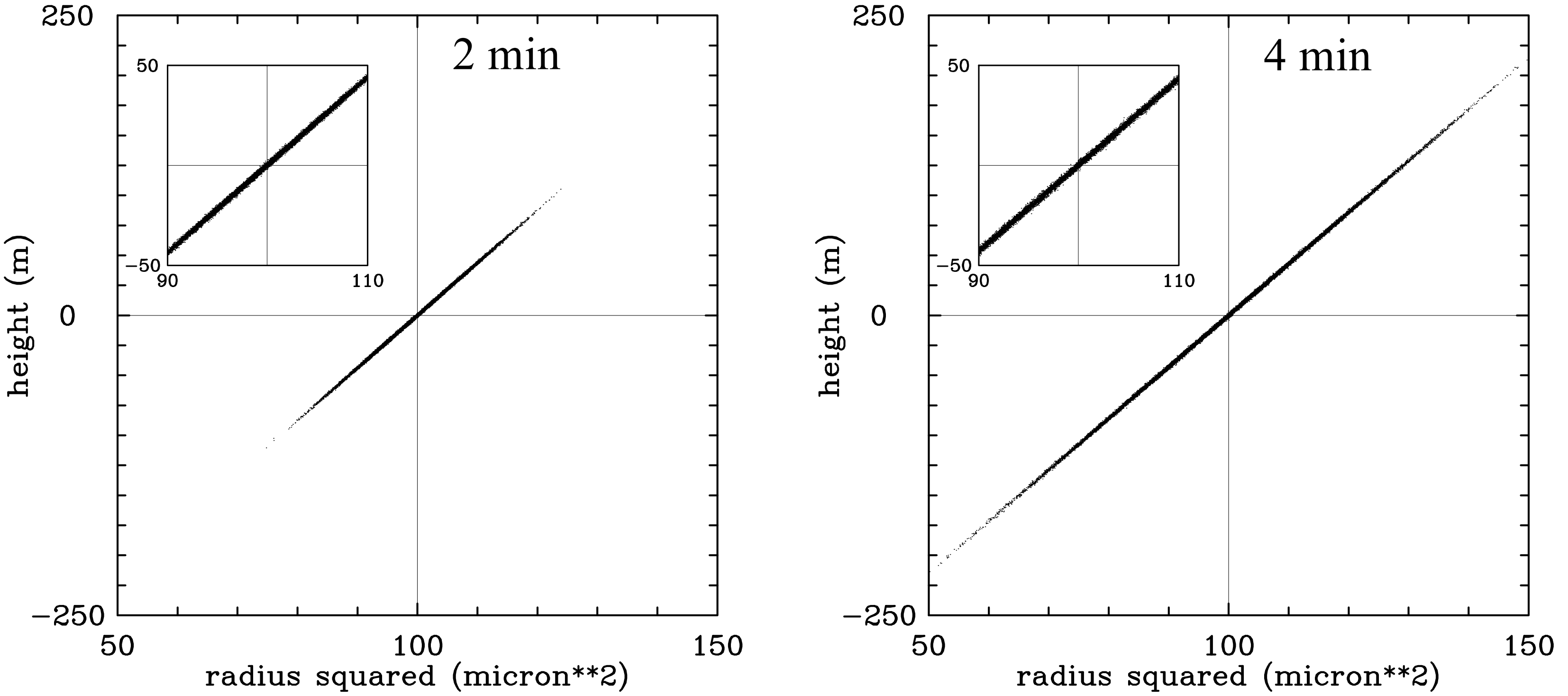
$$\tau = \frac{L}{(2\pi)^{1/3}}\left(\frac{C_\tau}{E}\right)^{1/2} \qquad E = \left(\frac{L\varepsilon}{C_E}\right)^{2/3}$$

keeping track of height changes: $z(t+\delta t) = z(t) + w'\,\delta t$

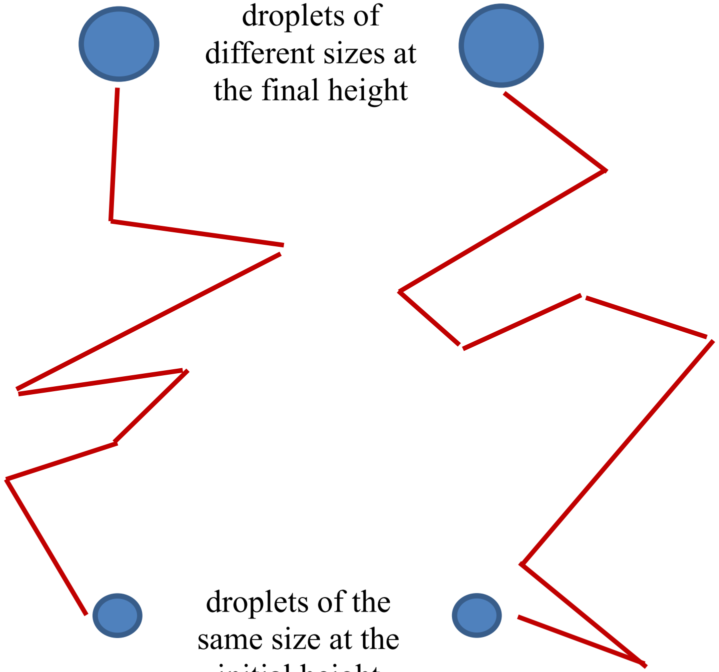

$L = 10$ m; $\varepsilon = 10$ cm^2 s^{-3}
all droplets
$t^{1/2}$
only droplets inside $-L/2$ and $L/2$
sigma r**2
time (min)
100.
10.
1.
0.1
0.1
1.
10.
100

One can derive the true impact of eddy hoping by removing the mean vertical gradient of the droplet radius

TKE dissipation: 10 cm^2 s^{-3} *L*: 100 m

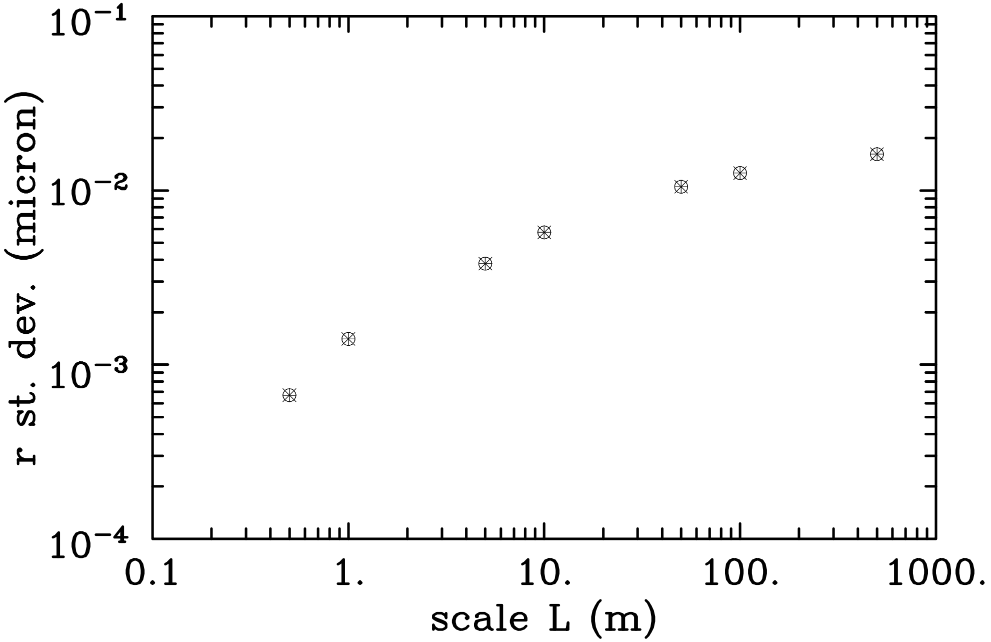


Bartlett and Jonas *QJRMS* 1972
Grabowski *JAS* 2025



droplets of different sizes at the final height

droplets of the same size at the initial height



$$r_i^2(t+\delta t) - r_i^2(t) = A\,[z_i(t+\delta t) - z_i(t)] \;+\; B\,[S_i(t) - S_{qe}(t)]$$

$$S_{qe}(t) \sim w'(t)$$

Grabowski *JAS* 2025

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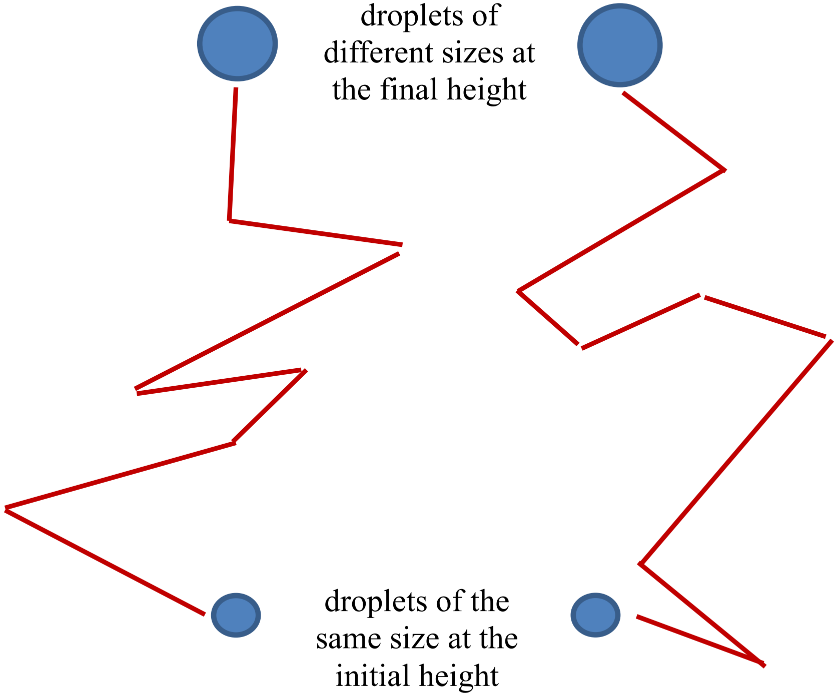
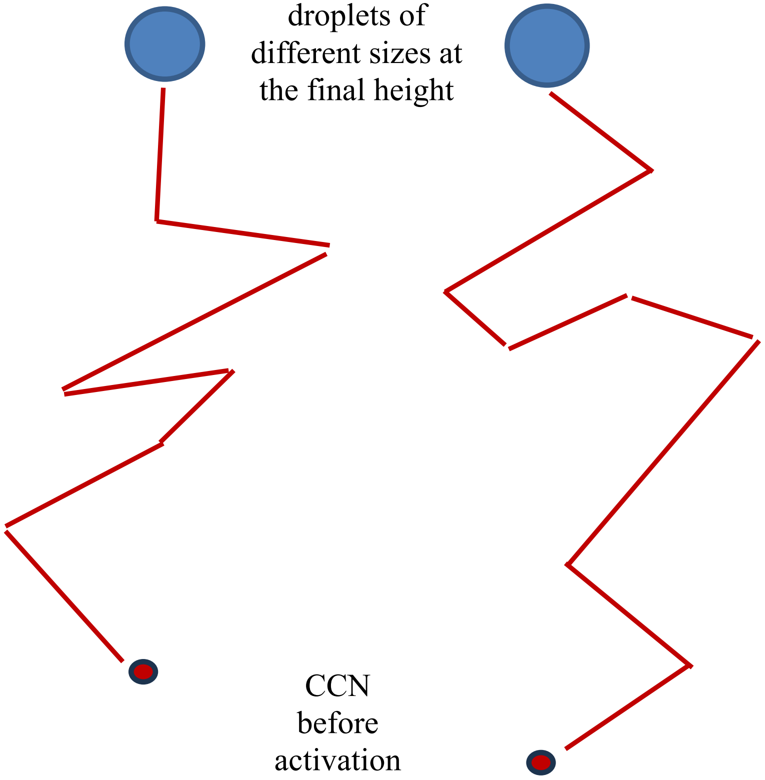
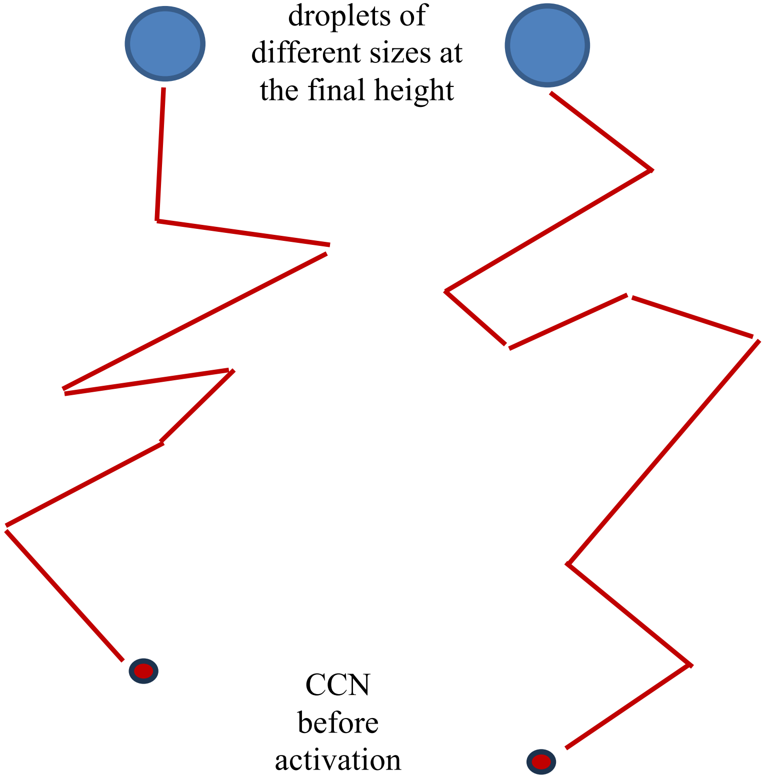
Bartlett and Jonas *QJRMS* 1972
Grabowski *JAS* 2025



Lasher-Trapp et al. *QJRMS* 2005
Grabowski et al. (this study)

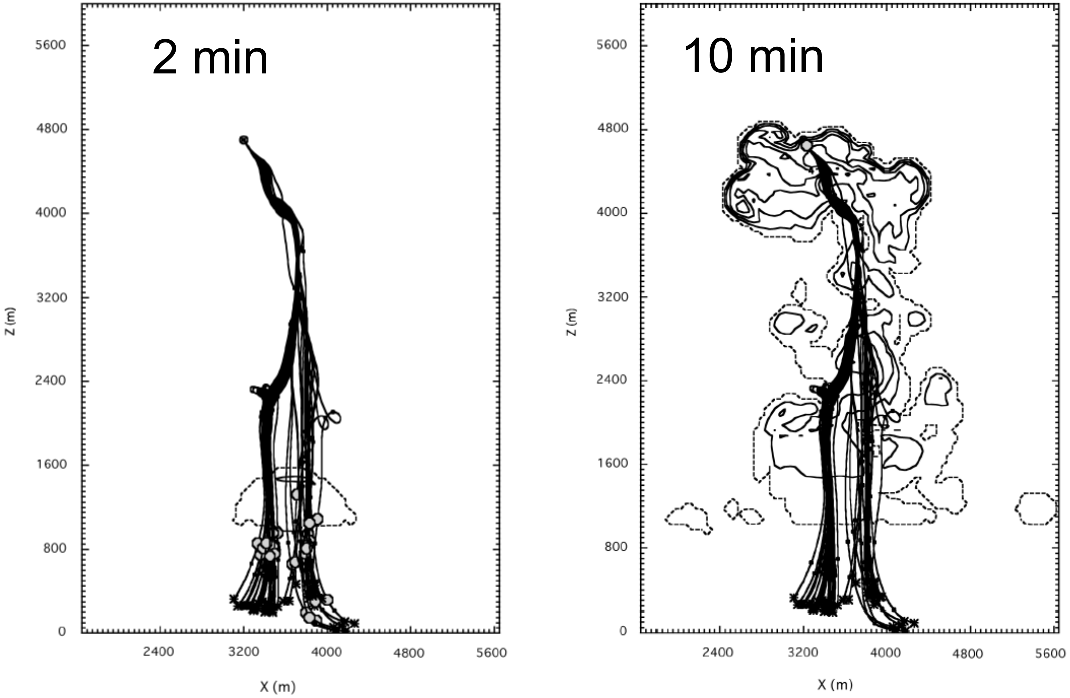


Lasher-Trapp et al. *QJRMS* 2005



droplets of
different sizes at
the final height

CCN
before
activation

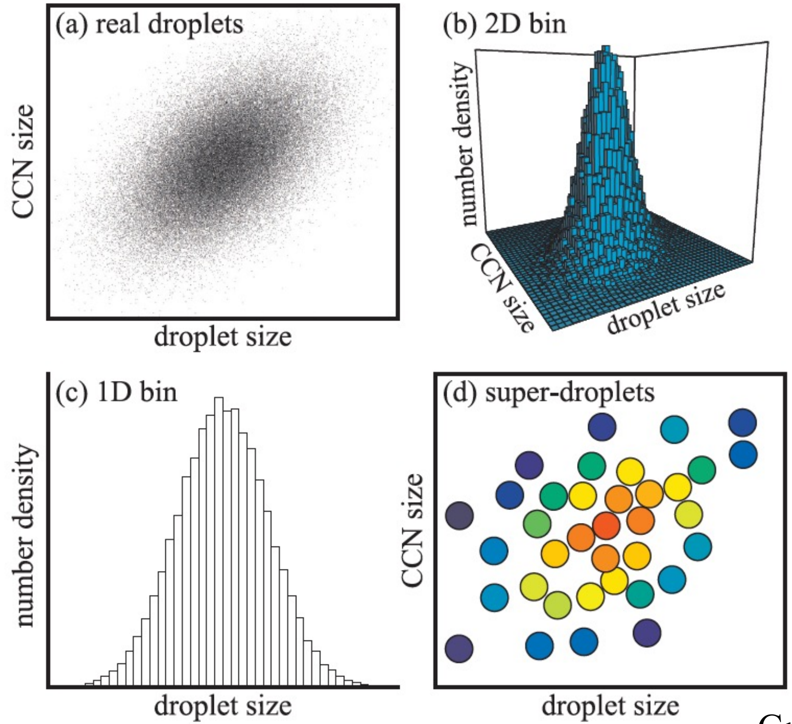


Lasher-Trapp et al. *QJRMS* 2005

Lasher-Trapp et al. (2005) used an elaborate scheme to calculate individual droplet growth along their trajectories through a turbulent cloud.

This is not needed when cloud model features Lagrangian particle-based microphysics as each superdroplet follows its own trajectory.

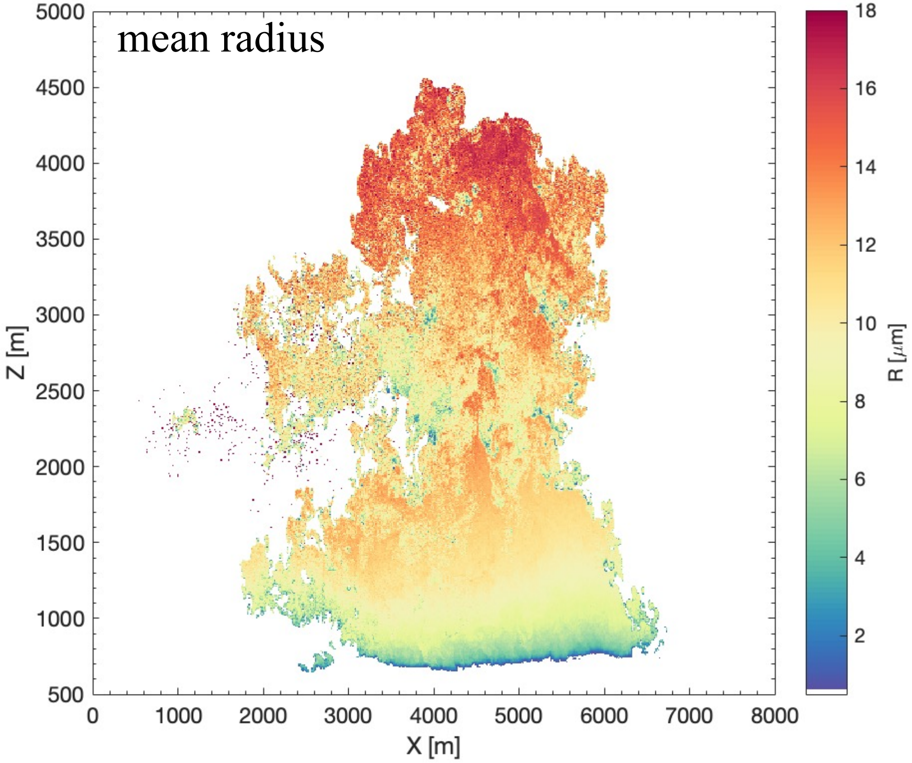
However, high spatial resolution is needed to appropriately simulate the impact of cloud turbulence on the droplet growth.



blue - low multiplicity
red - high multiplicity

Grabowski et al. *BAMS* 2019

Recent ASD project lead by MMM's Kamal Kant Chandrakar: Simulations of an isolated cumulus congestus from CAMP2Ex



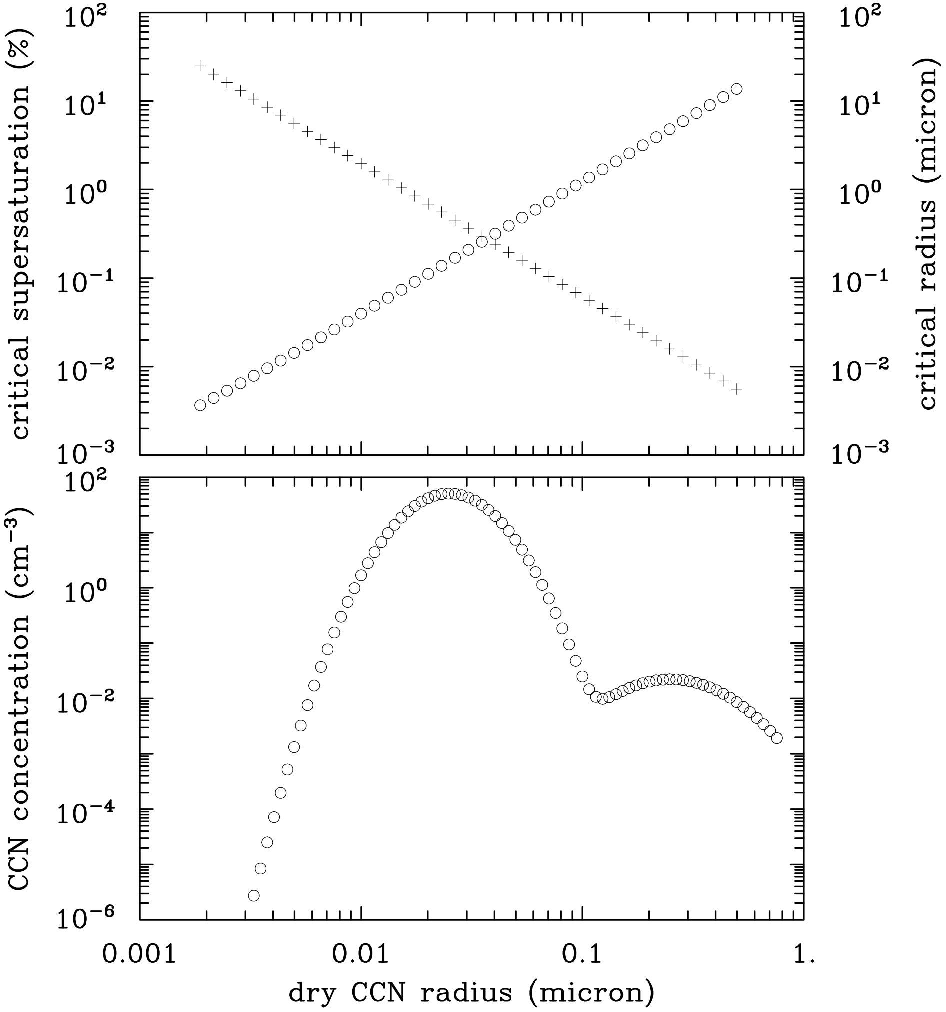
Domain size: 12 km x 12 km x 11 km (grid: 1600 x 1600 x 1465)

Grid spacings: dx = dy = dz = 7.5 m

Super-particles: 64 per grid box (non precipitating) and 62 (precipitating)

Aerosols: bimodal lognormal distribution of $(NH_4)_2SO_4$; 639.7 cm^{-3}; vertically varying

CCN spectrum based on observations in CAMP2Ex



total concentration: 640 cm^{-3}

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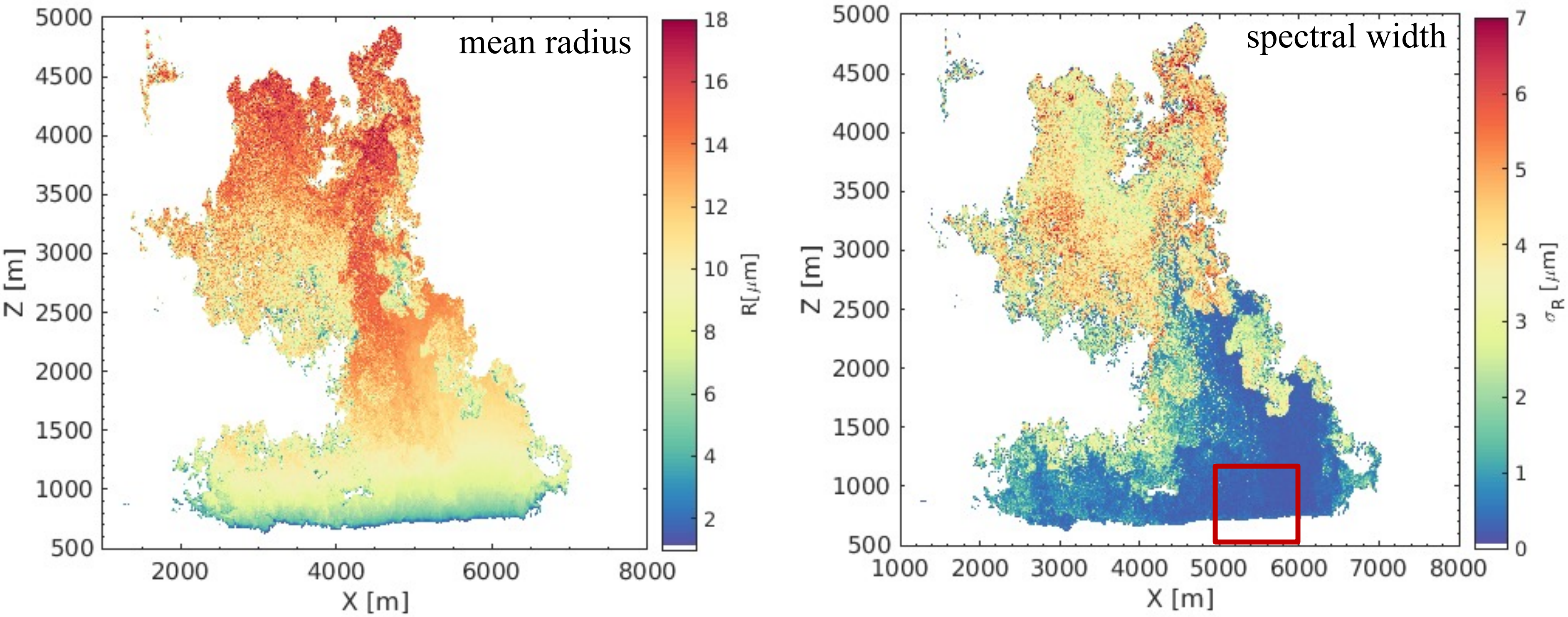
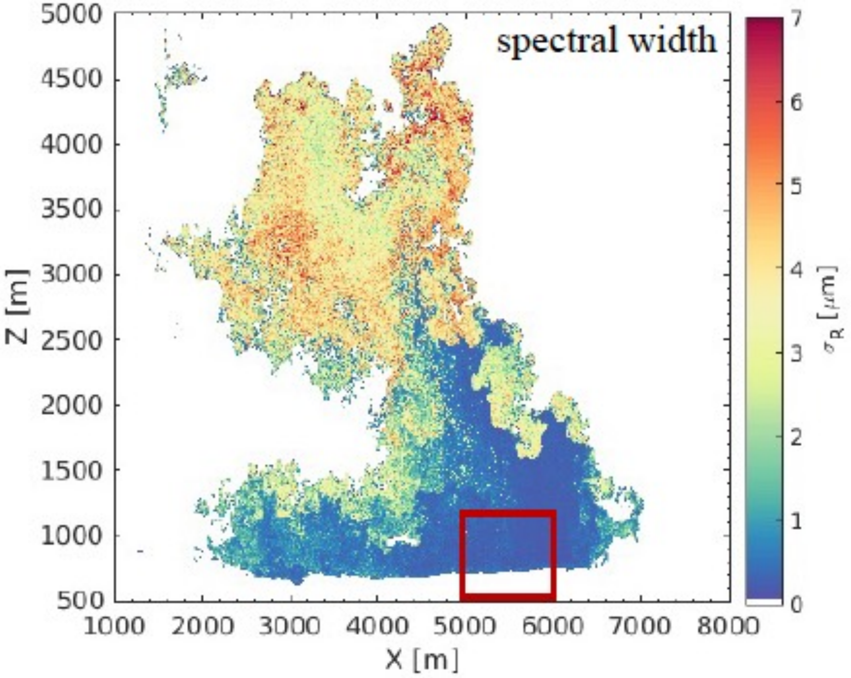
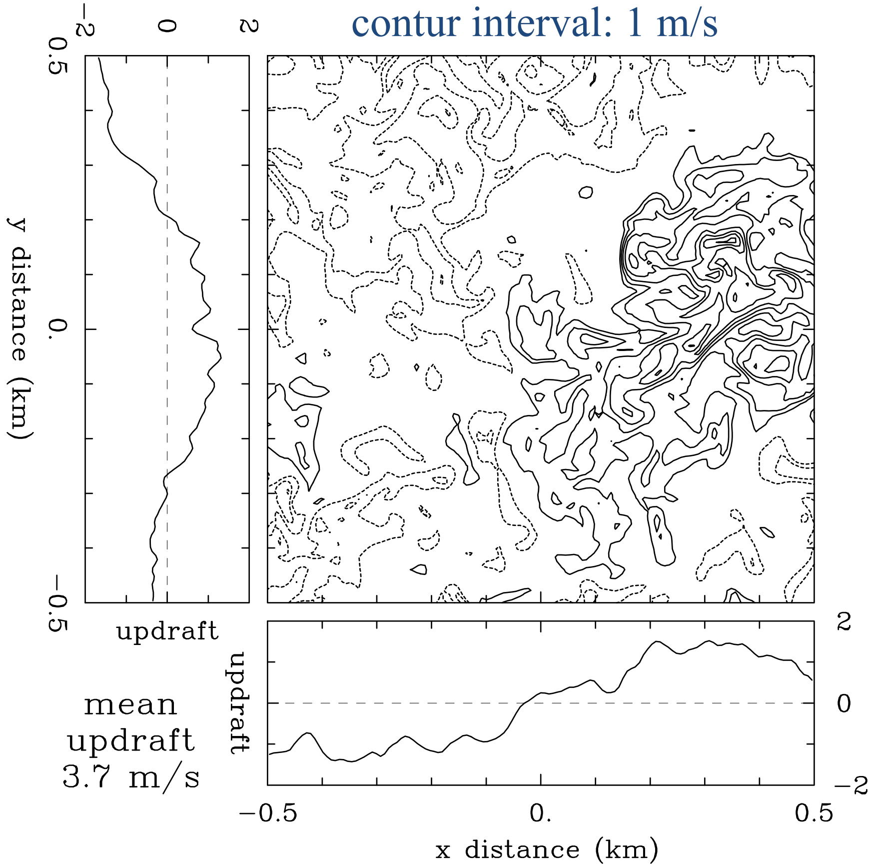
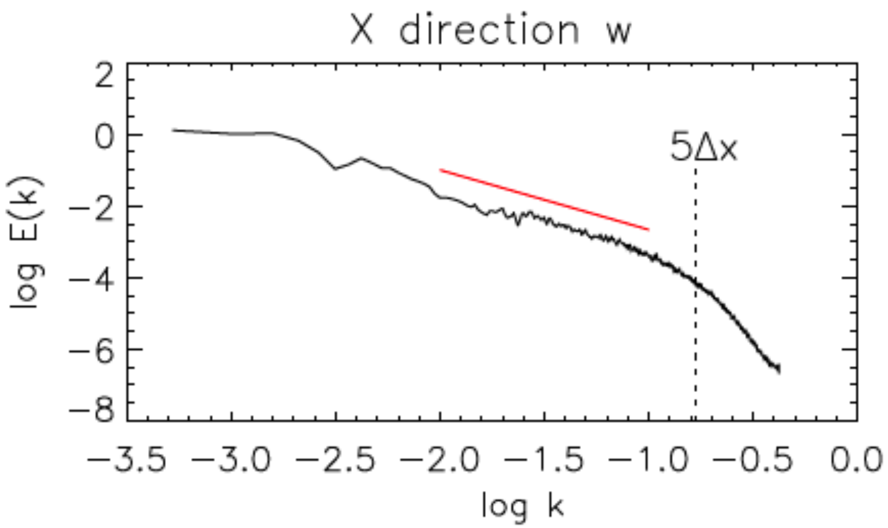
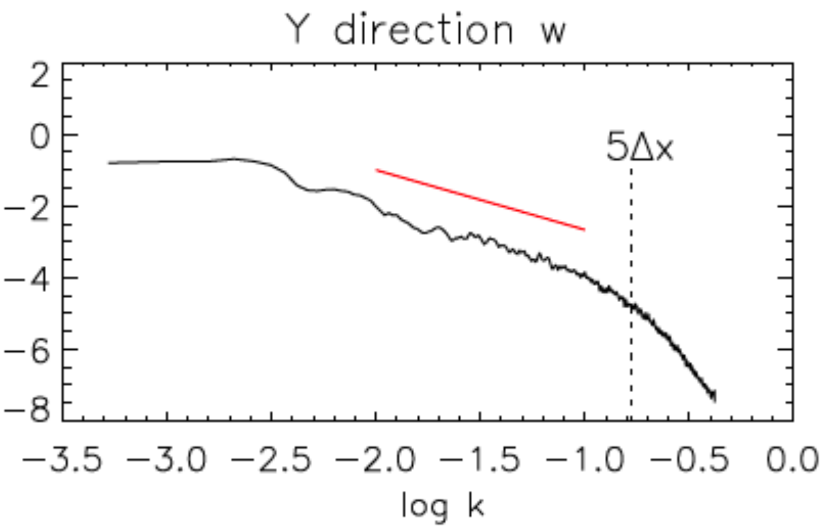




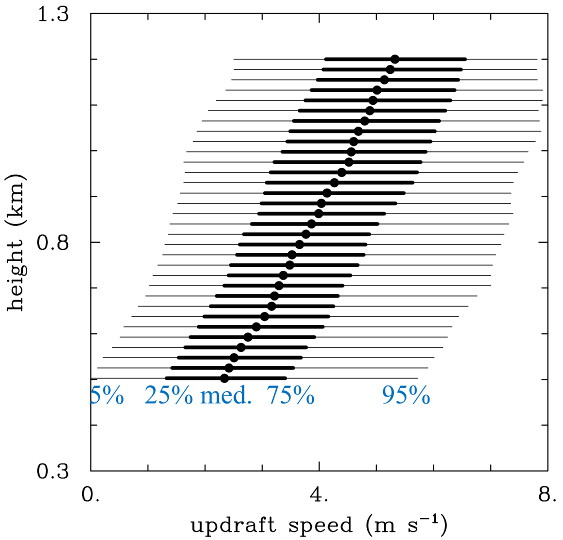
perturbation updraft
distribution at the cloud
base (~0.7 km)
contur interval: 1 m/s



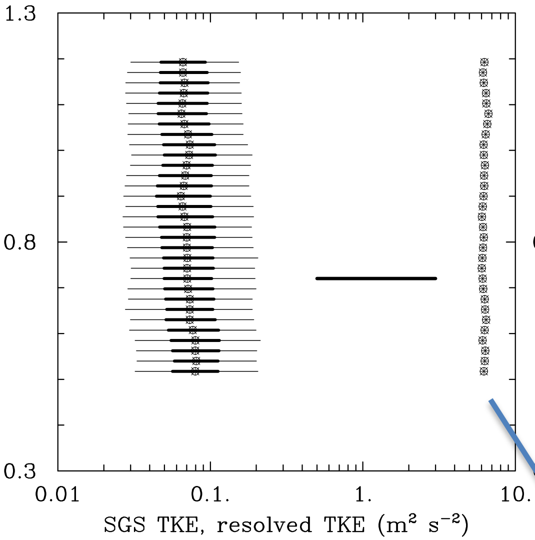

X direction w
log E(k)
2
0
−2
−4
−6
−8
5Δx
−3.5
−3.0
−2.5
−2.0
−1.5
−1.0
−0.5
0.0
log k


Y direction w
2
0
−2
−4
−6
−8
5Δx
−3.5
−3.0
−2.5
−2.0
−1.5
−1.0
−0.5
0.0
log k

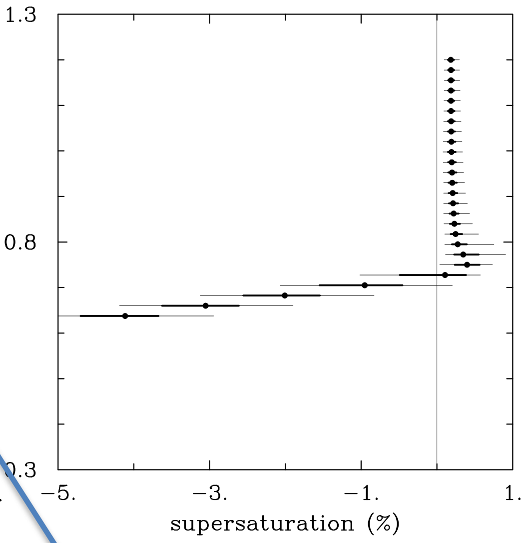
updraft speed (median, percentiles)

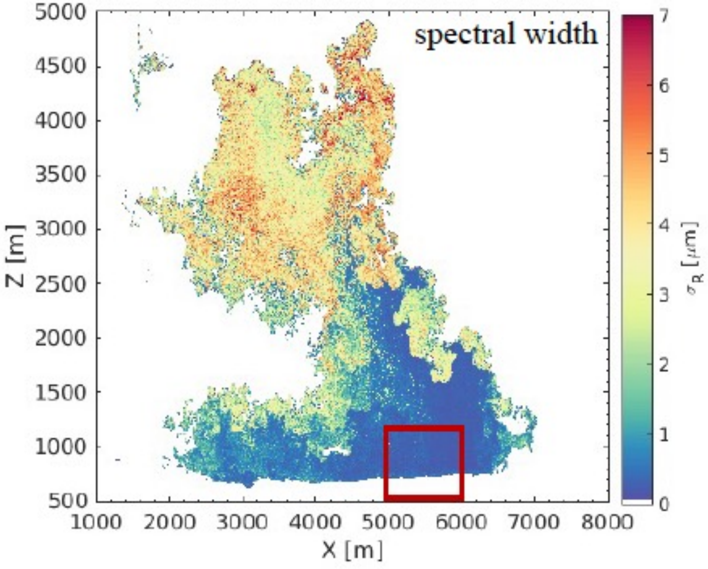


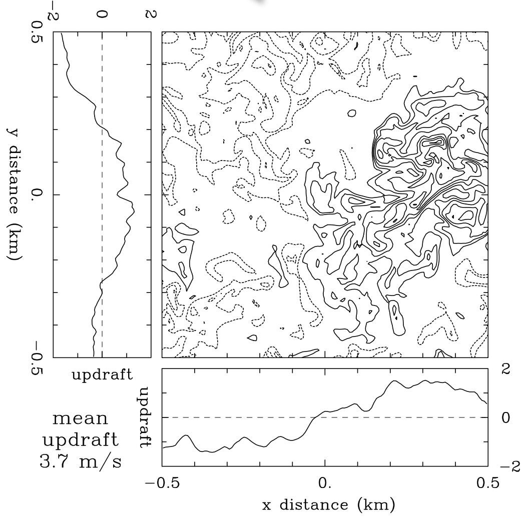
TKE (SGS, resolved)



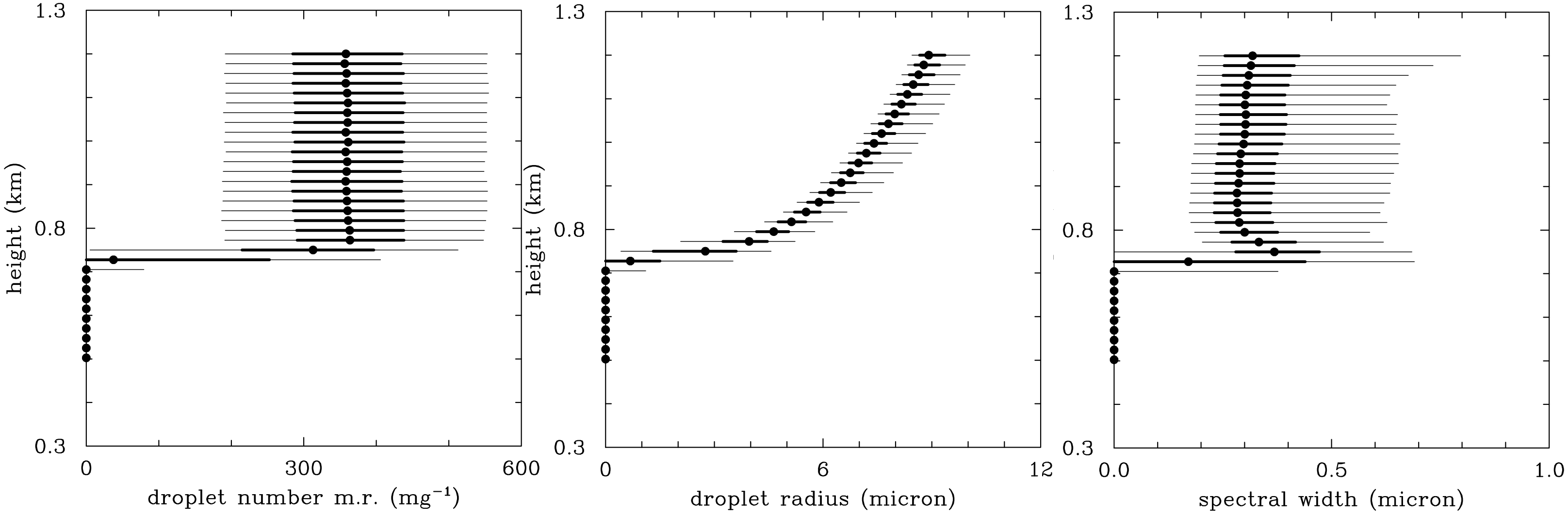
supersaturation (median, percentiles)



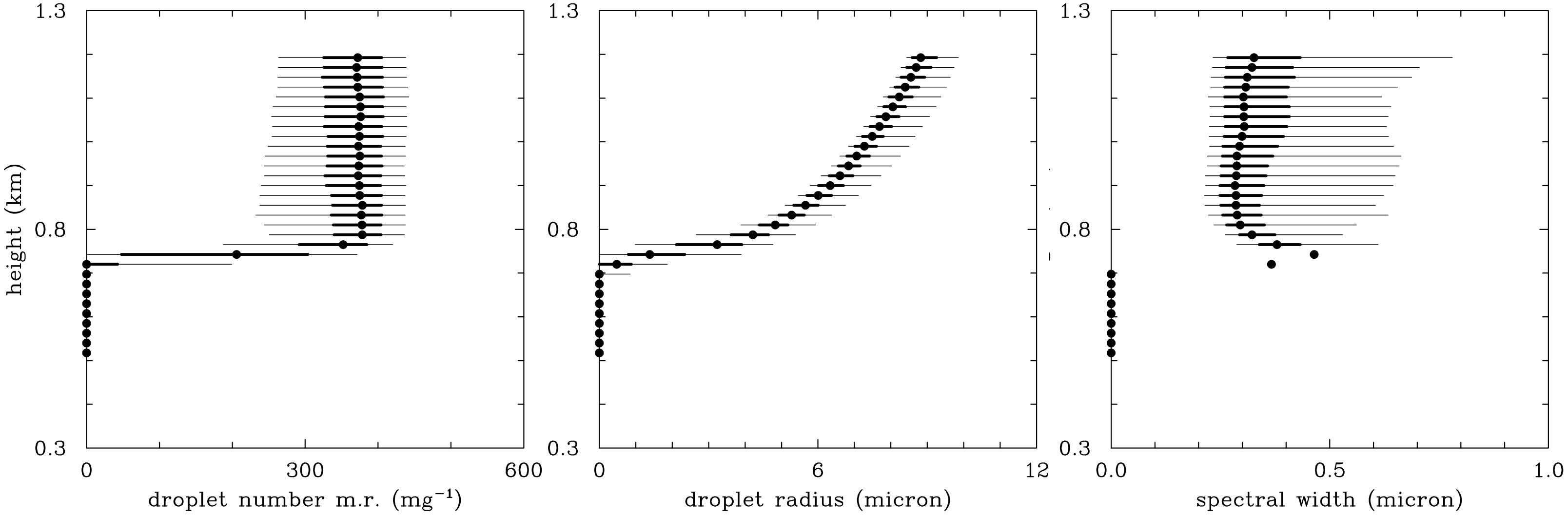




7.5 m data



22.5 m data



droplet number mixing ratio

mean radius

spectral width

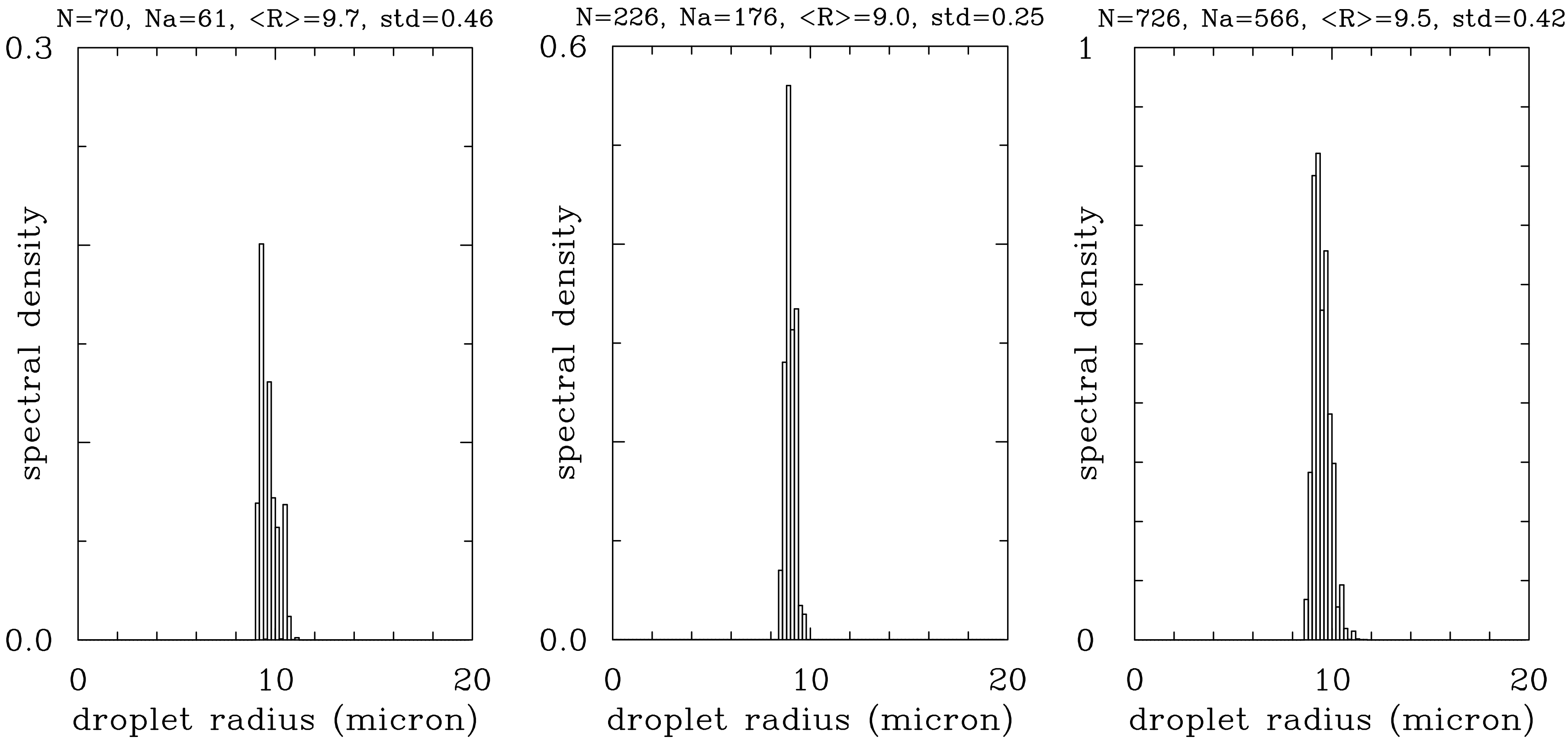
Examples of droplet spectra at about 1200 m height:

N – number of SDs in the statistics (~ 64 per 7.5^3 m^3 volume)

Na – number of activated SDs

<R> – mean radius (micron)

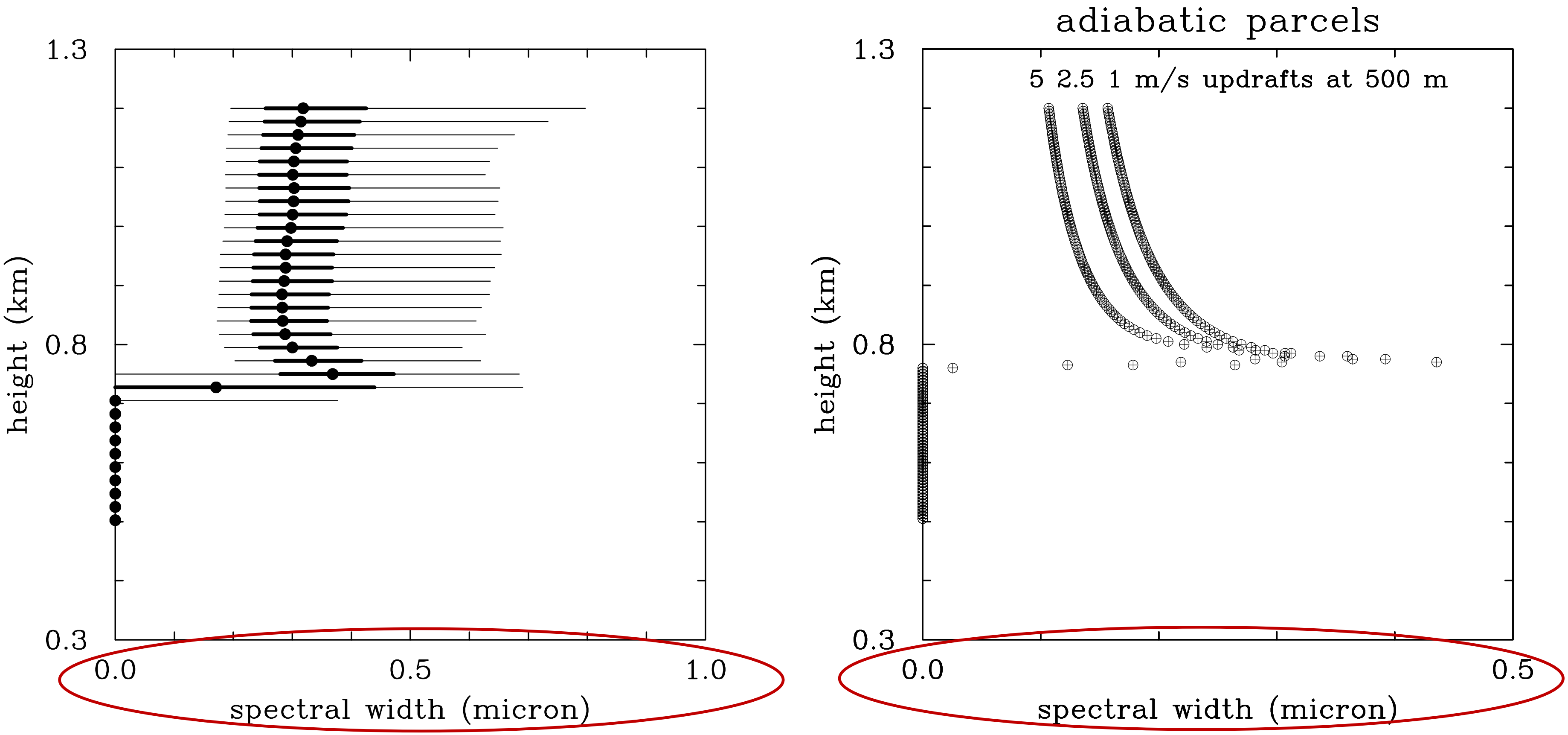
std – spectral width (micron)



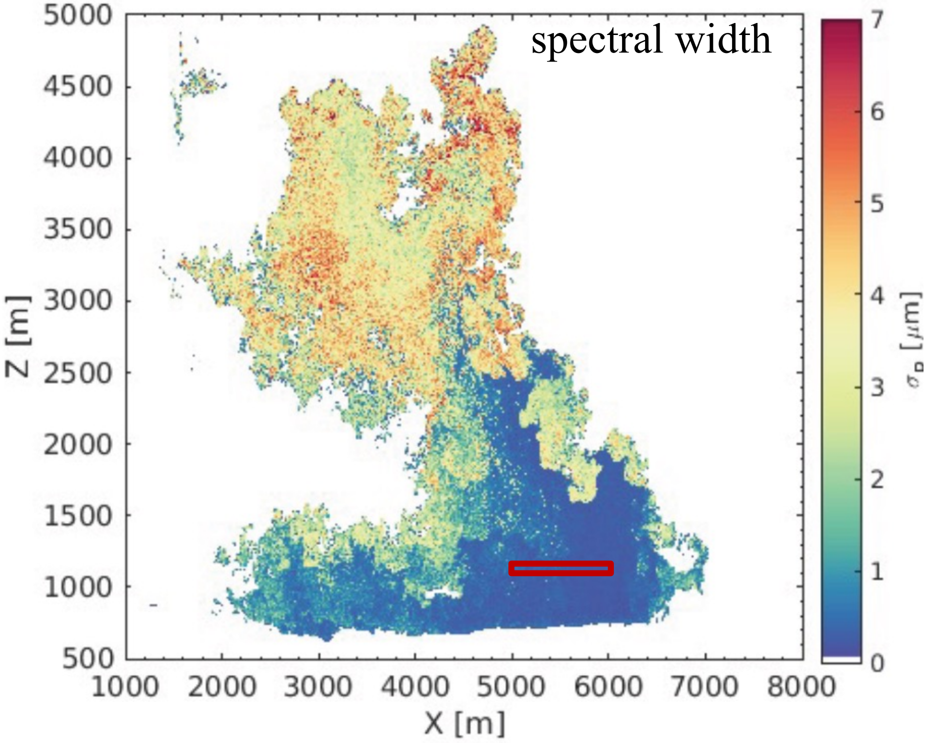
N=70, Na=61, <R>=9.7, std=0.46

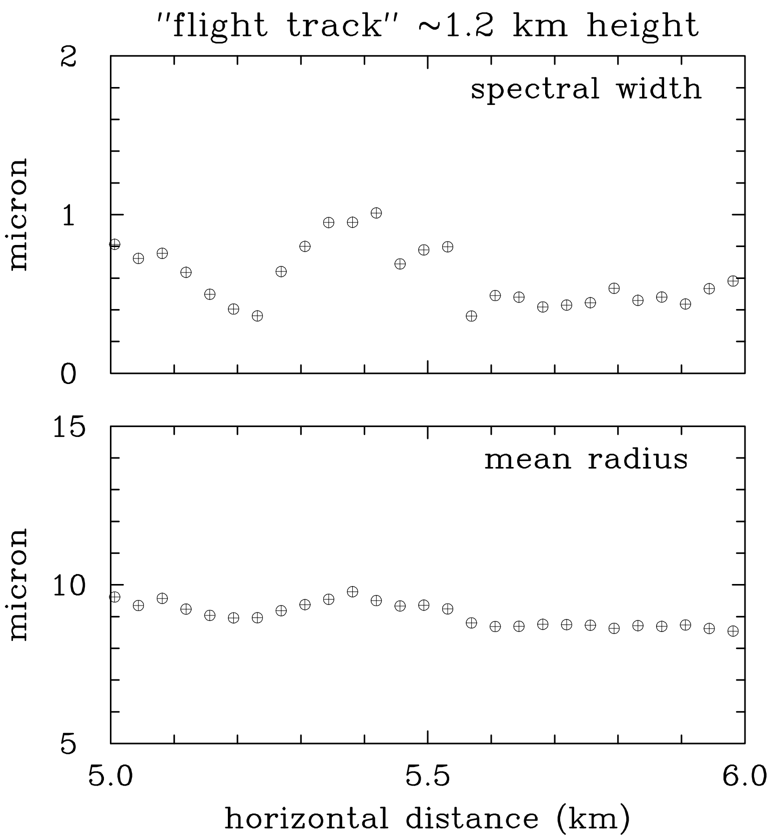
N=226, Na=176, <R>=9.0, std=0.25

N=726, Na=566, <R>=9.5, std=0.42


1.3
0.8
0.3
height (km)
0.0
0.5
1.0
spectral width (micron)
adiabatic parcels
5 2.5 1 m/s updrafts at 500 m
1.3
0.8
0.3
height (km)
0.0
0.5
spectral width (micron)

Recent ASD project lead by MMM’s Kamal Kant Chandrakar: Simulations of an isolated cumulus congestus from CAMP2Ex





SDs data averaged over 5^3 grid boxes (37.5^3 m^3)

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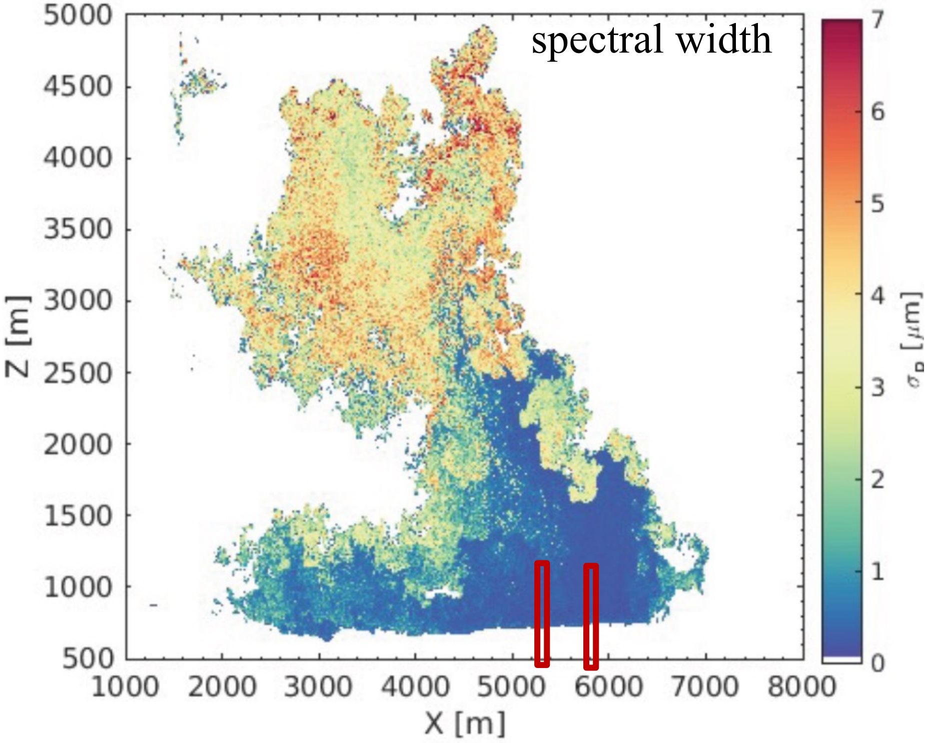
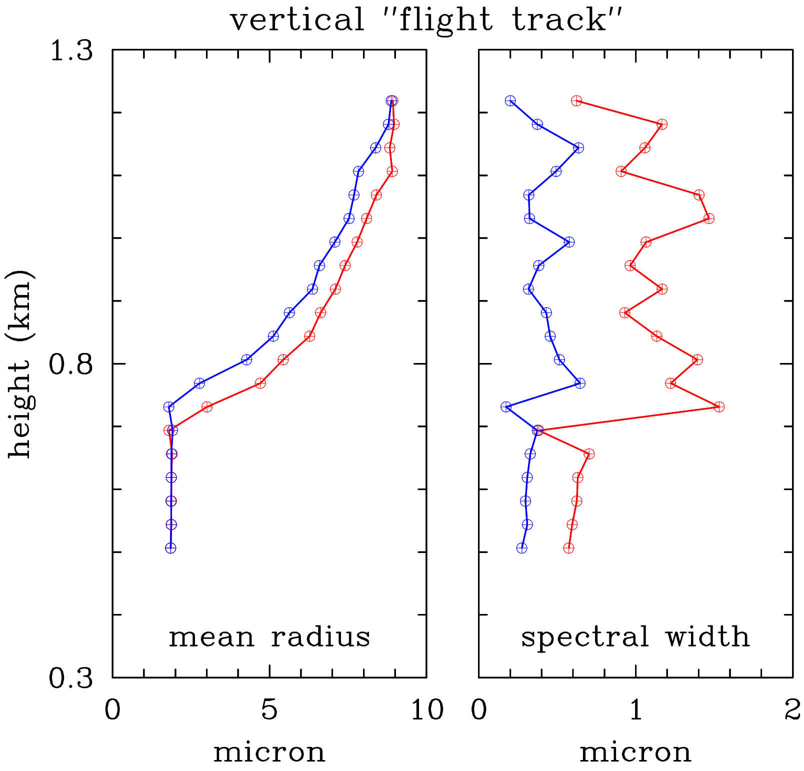




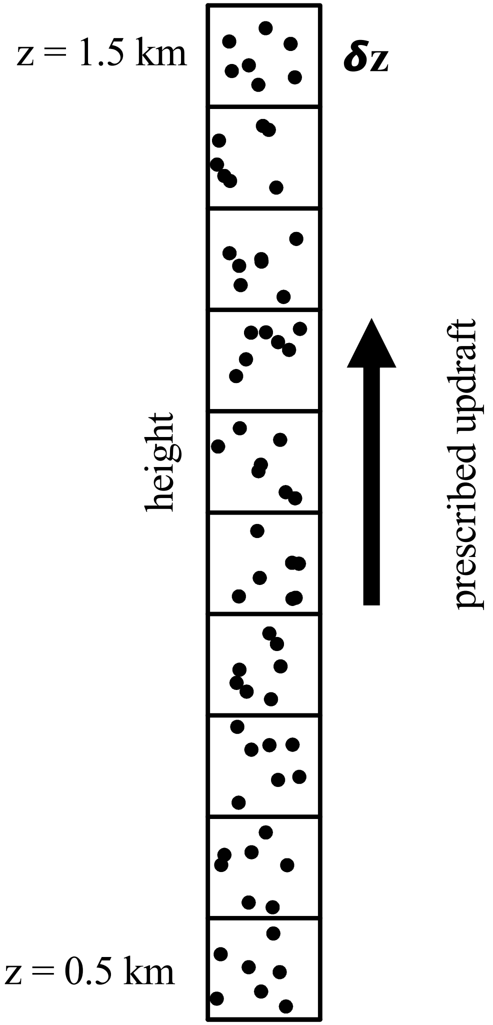
SDs data averaged over 5^3 grid boxes (37.5^3 m^3)

Can we understand the impact of turbulence seen in 3D simulations?

We use a simplified system – 1D Eulerian (flow) and Lagrangian (superdroplets) – to understand the broadening seen in the 3D simulation

No turbulence:

1D prescribed-flow Eulerian model with superdroplets



$$\frac{\partial \rho q_v}{\partial t} + \frac{\partial}{\partial z}(\rho w q_v) = -\rho C_d$$

$$\frac{\partial \rho \theta}{\partial t} + \frac{\partial}{\partial z}(\rho w \theta) = \frac{L\theta_e}{c_p T_e}\rho C_d$$

1D thermodynamics

$$C_d = \frac{d}{dt}\left(\Sigma_i \frac{4}{3}\pi r_i^3 N_i \frac{\rho_w}{\rho}\right)$$

$$\frac{dr_i}{dt} = \frac{1}{r_i} A(S - S_{eq})$$

superdroplets

$$\frac{dz_i}{dt} = w_i$$

$w_i = w$

No turbulence:

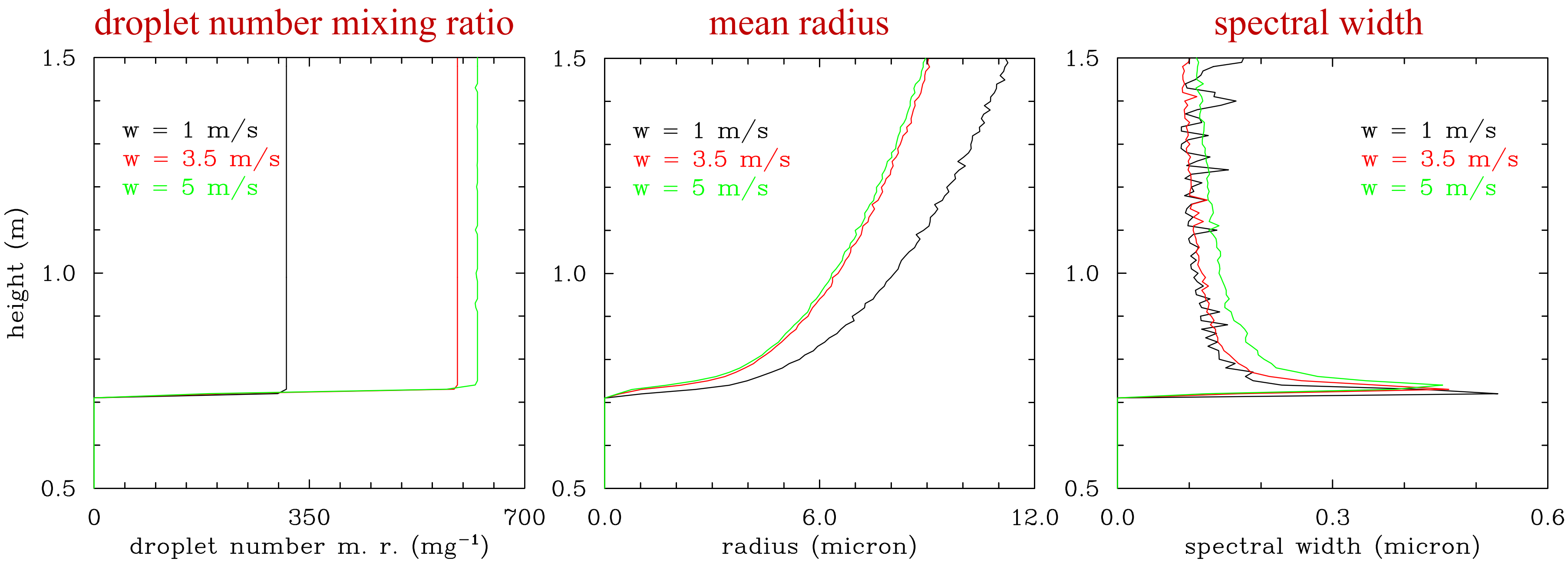
dz = 10 m, dt = 0.1 sec (sub-stepping for droplet growth)

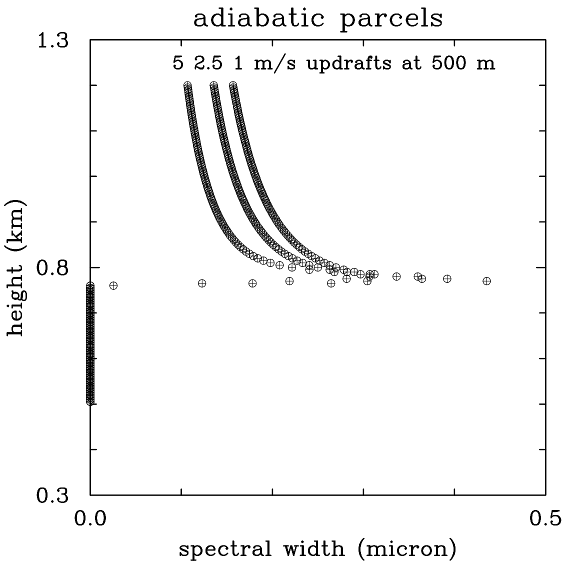
1D advection: MPDATA scheme (Smolarkiewicz et al.)

- inflow – constant in time
- outflow – unimportant (whatever…)

Superdroplets:

- 100 CCN bins, 64 per grid volume (all 640,000 SDs)
- randomly positioned at the onset with initial radii at equilibrium with local supersaturation
- advected with the mean flow
- moved to the bottom grid volume when leaving top of the domain with radius reset to the initial radius
- all superdroplets within a given grid volume grow
- in the same supersaturation $S(q_v, T)$


droplet number mixing ratio
mean radius
spectral width
1.5
1.0
0.5
height (m)
w = 1 m/s
w = 3.5 m/s
w = 5 m/s
0
350
700
droplet number m. r. (mg^{-1})
0.0
6.0
12.0
radius (micron)
0.0
0.3
0.6
spectral width (micron)


adiabatic parcels
5 2.5 1 m/s updrafts at 500 m
1.3
0.8
0.3
height (km)
0.0
0.5
spectral width (micron)

Adding turbulence:

Each superdroplet grows in the supersaturation
that comes from the combination of the mean $S(q_v, T)$
and fluctuations S' driven by a stochastic model

Stochastic model parameters derived from 3D cloud simulation:

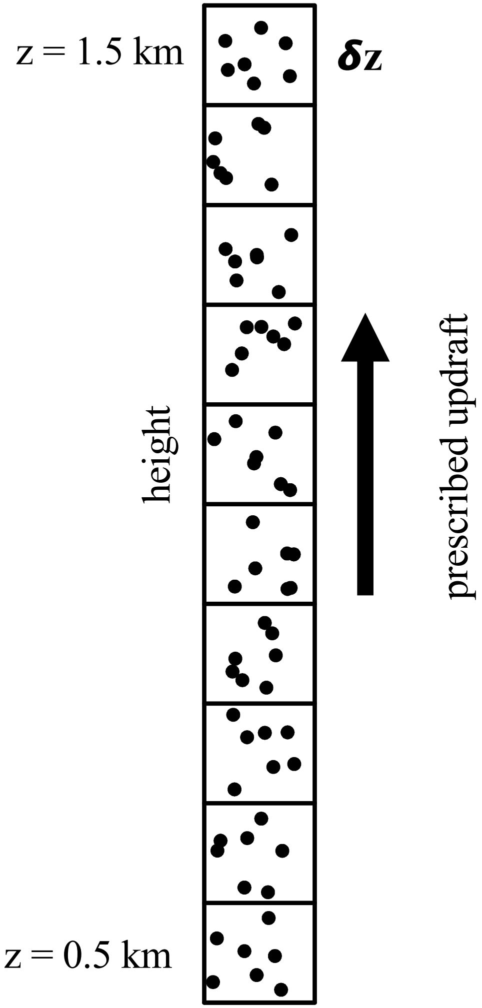
TKE ~ 6 m^2 s^2

L (integral length scale) ~ 300 m

phase relaxation time the same for all superdroplets
(derived from turbulence-free simulation)

With turbulence:

1D prescribed-flow Eulerian model with superdroplets and turbulence



$$\frac{\partial \rho q_v}{\partial t} + \frac{\partial}{\partial z}(\rho w q_v) = -\rho C_d$$

$$\frac{\partial \rho \theta}{\partial t} + \frac{\partial}{\partial z}(\rho w \theta) = \frac{L\theta_e}{c_p T_e}\rho C_d$$

1D thermodynamics

$$C_d = \frac{d}{dt}\ (\Sigma_i \frac{4}{3}\pi r_i^3 N_i \frac{\rho_w}{\rho})$$

$$\frac{dr_i}{dt} = \frac{1}{r}A(S + S'_i - S_{eq})$$

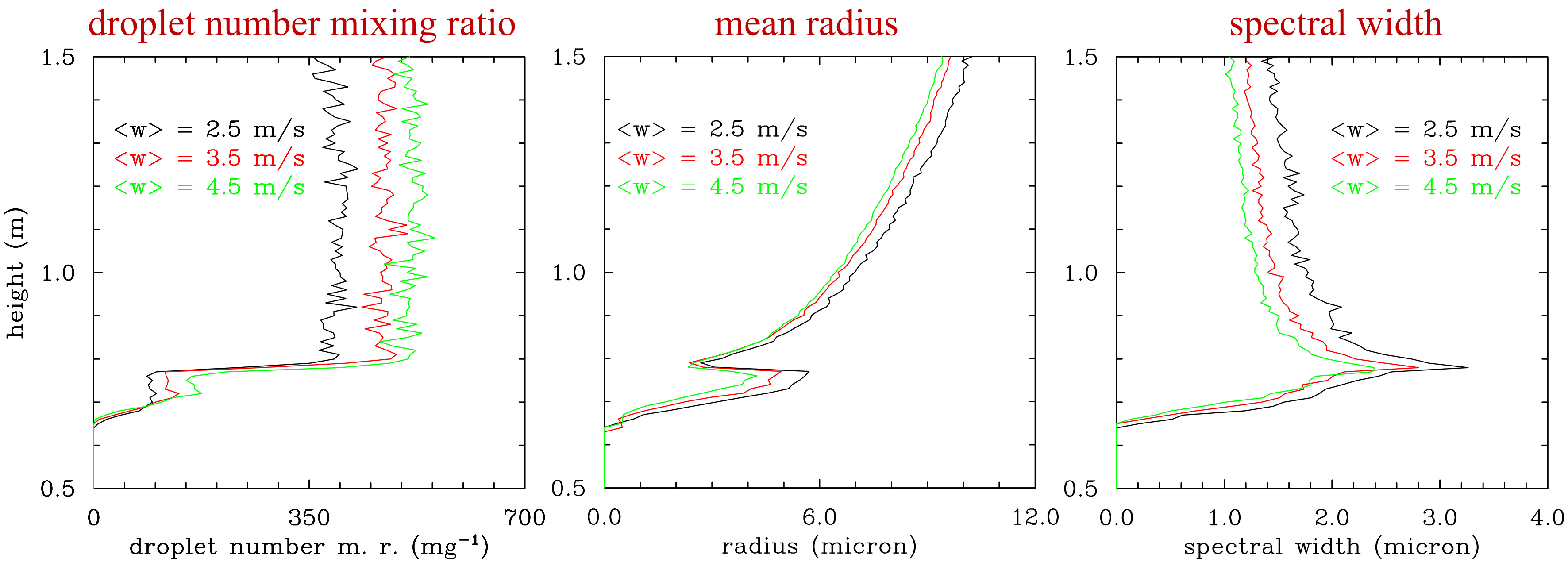
$$\frac{dz_i}{dt} = w_i + w'_i$$

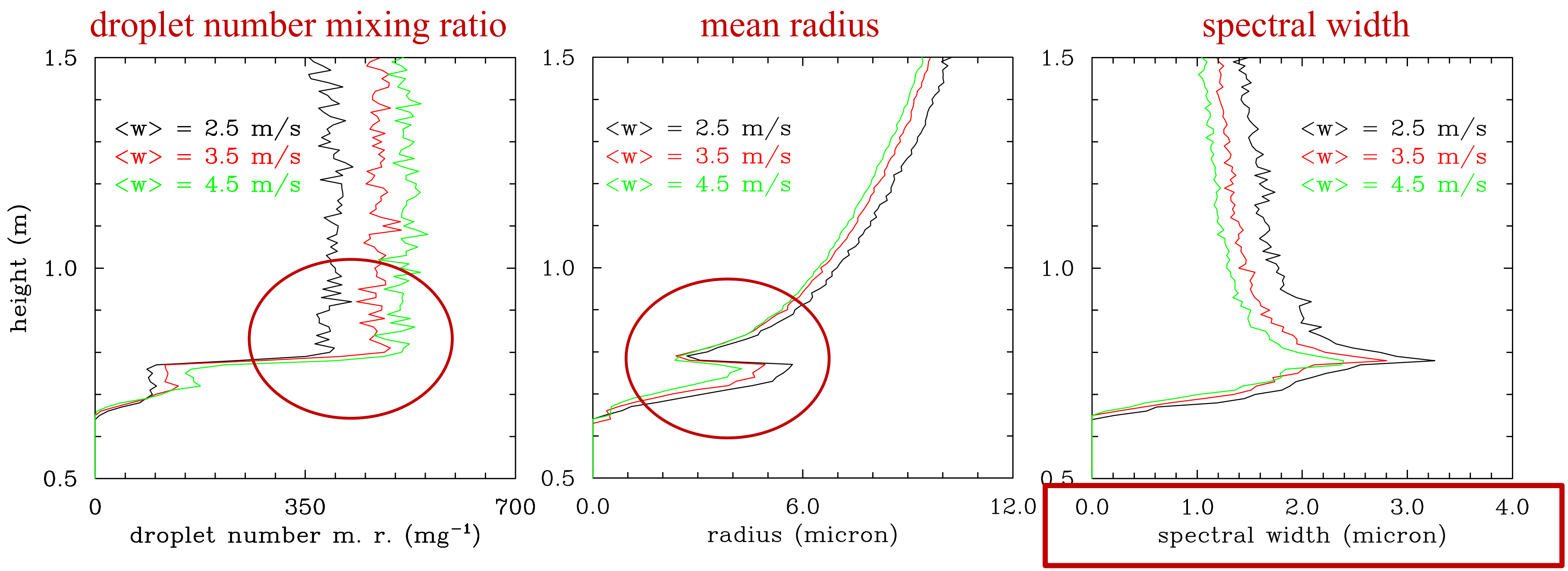
$$\frac{dS'_i}{dt} = a_1 w'_i - \frac{S'_i}{\tau_1}$$

$$w'(t+\delta t) = w'(t)e^{-\delta t/\tau_2} + \sqrt{1 - e^{-2\delta t/\tau_2}}\ \sigma_{w'}\psi$$

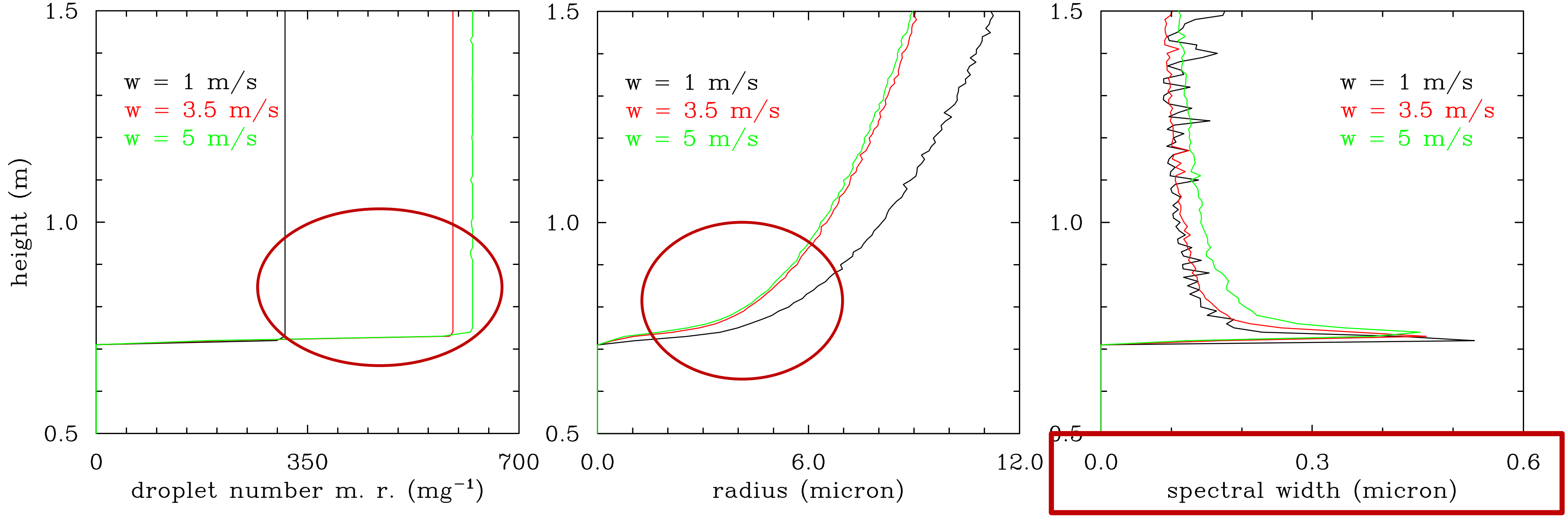
superdroplets

w', S' – turbulence!

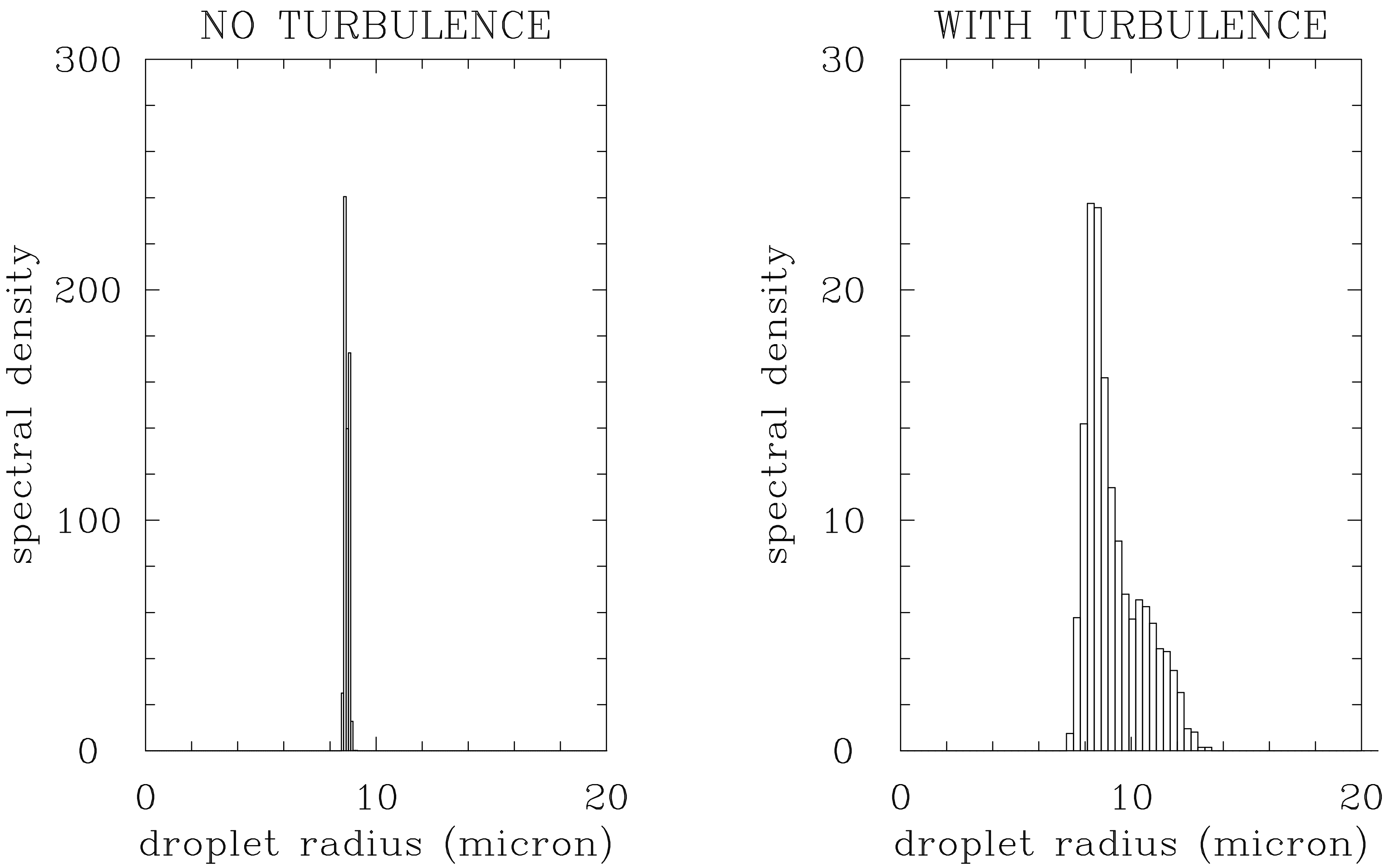

droplet number mixing ratio
mean radius
spectral width
height (m)
1.5
1.0
0.5
<w> = 2.5 m/s
<w> = 3.5 m/s
<w> = 4.5 m/s
0
350
700
droplet number m. r. (mg^{-1})
0.0
6.0
12.0
radius (micron)
0.0
1.0
2.0
3.0
4.0
spectral width (micron)



NO TURBULENCE



Spectra at 1.5 km height:

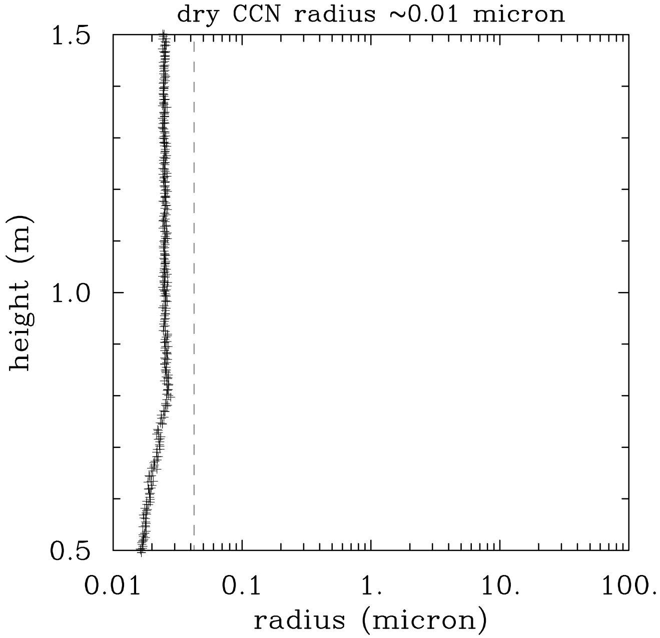


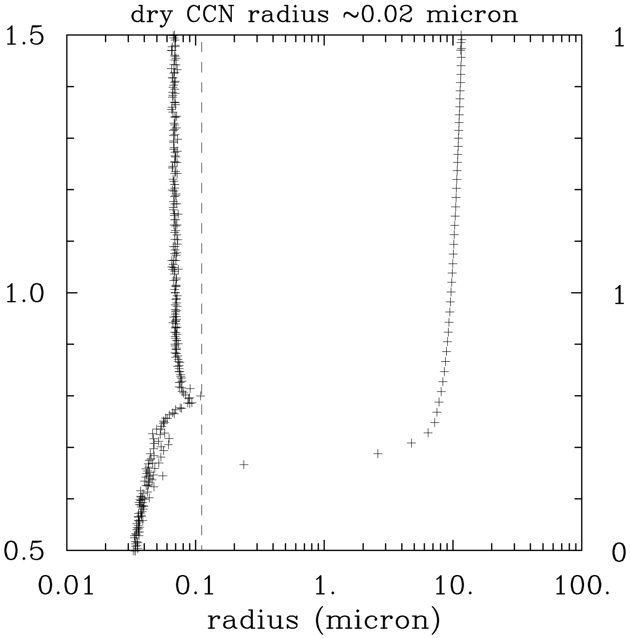
Where does the difference come from?

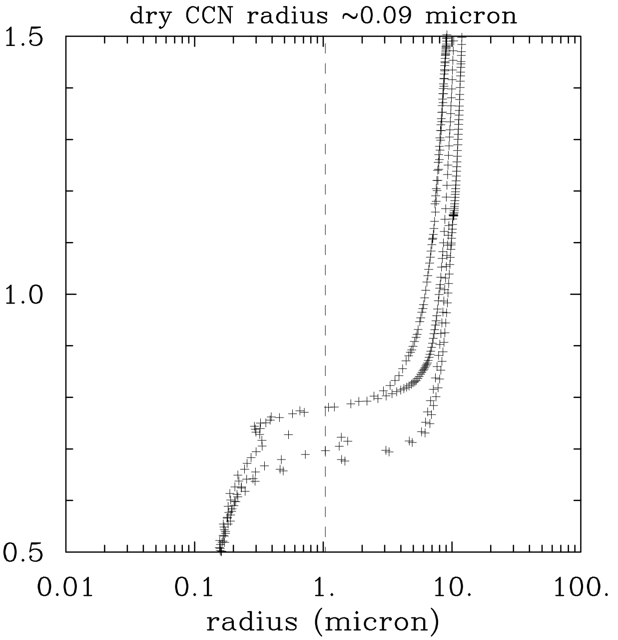
1. Fluctuation supersaturation allows cycles of activation and deactivation:

without turbulence, a droplet is activated once its radius exceeds the critical radius, and then the droplet continues to grow;

with turbulence a droplet can activate or not, or later deactivate.







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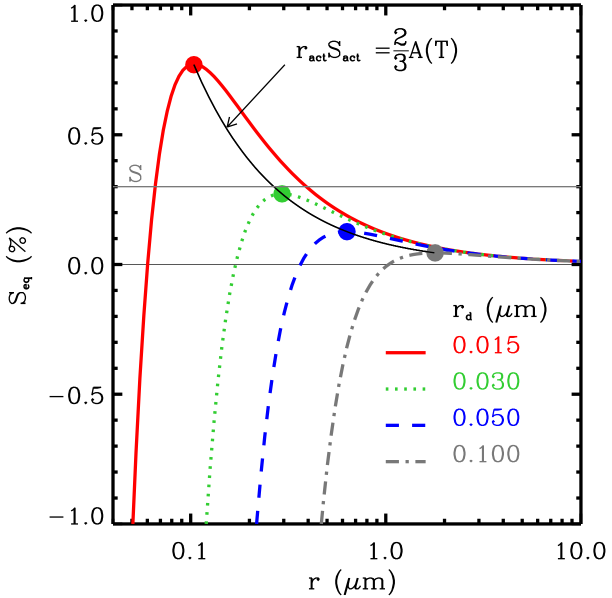

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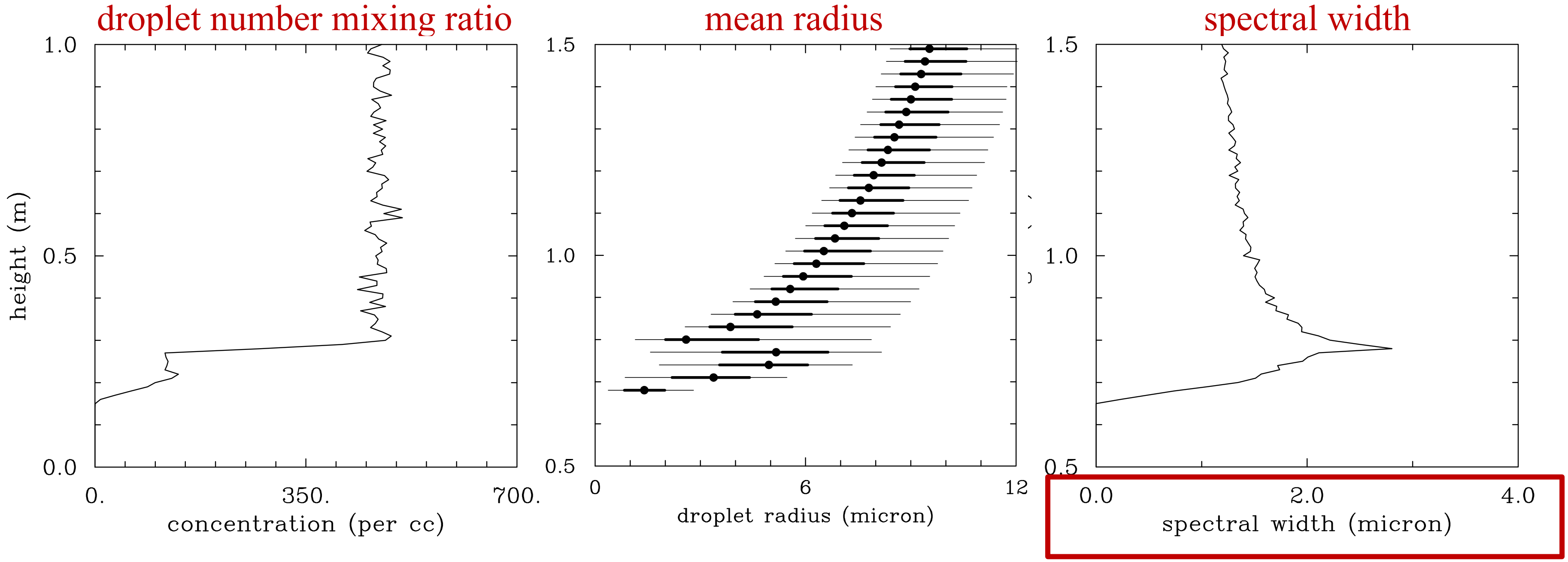
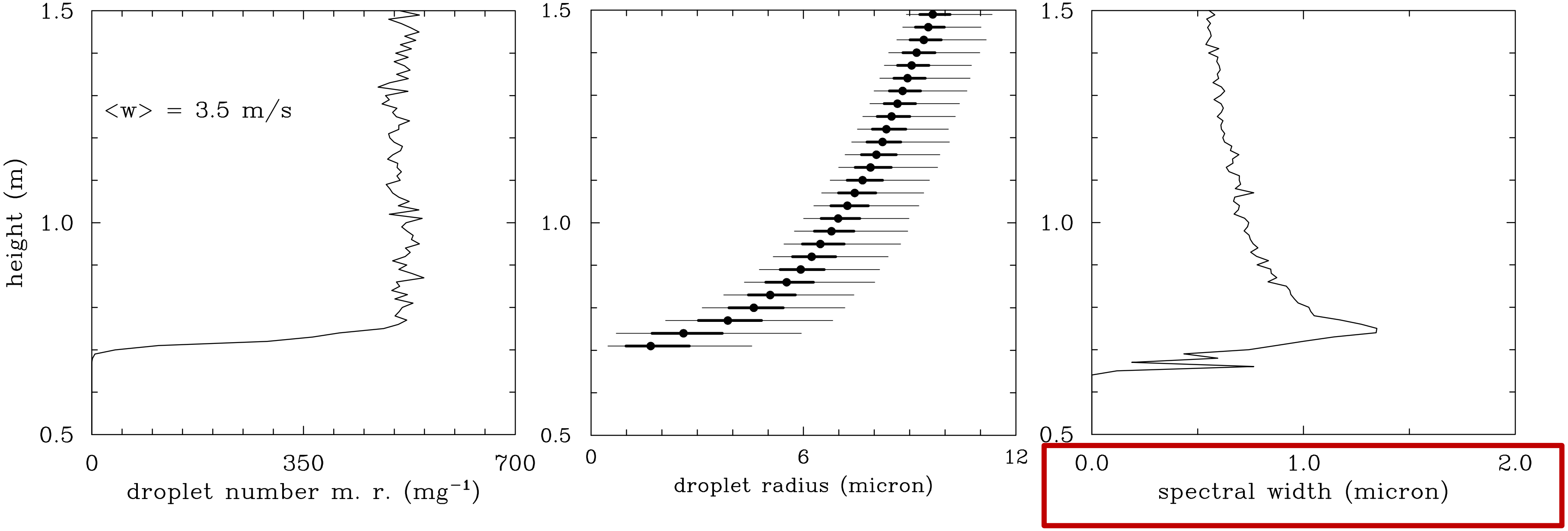
A test simulation in which a droplet grow without S_{eq} after activation, i.e., $dr/dt \sim S$, and not $dr/dt \sim (S - S_{eq})$.

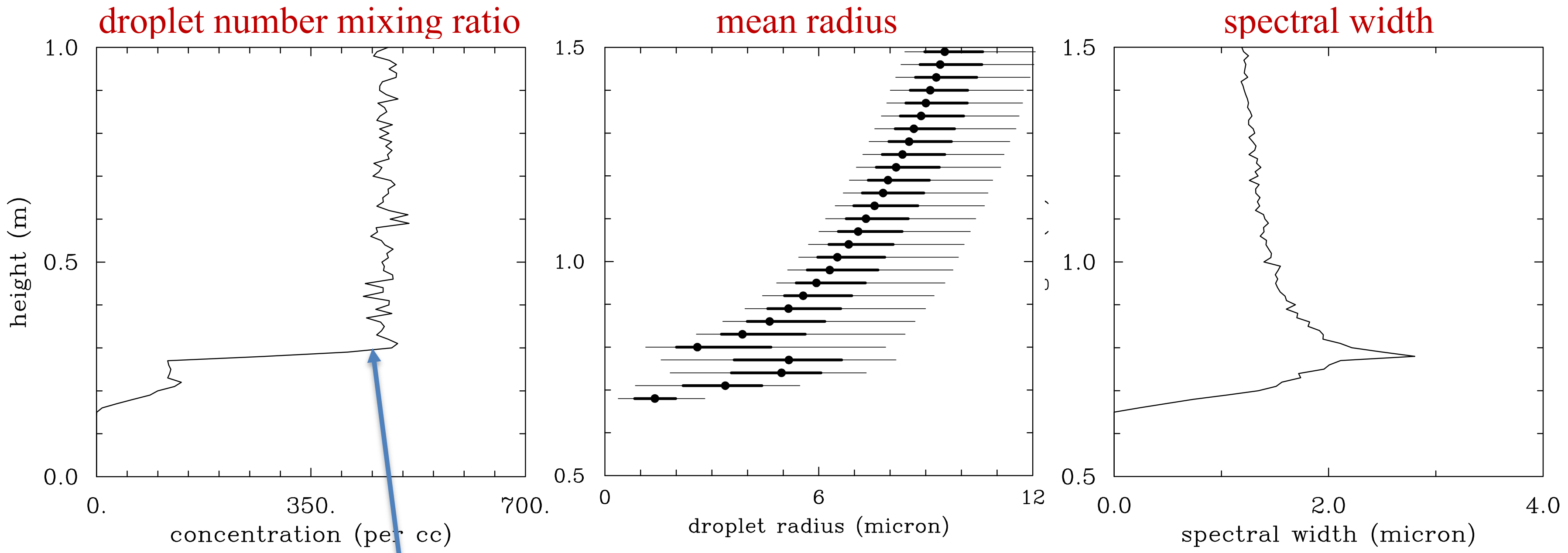
WITH TURBULENCE



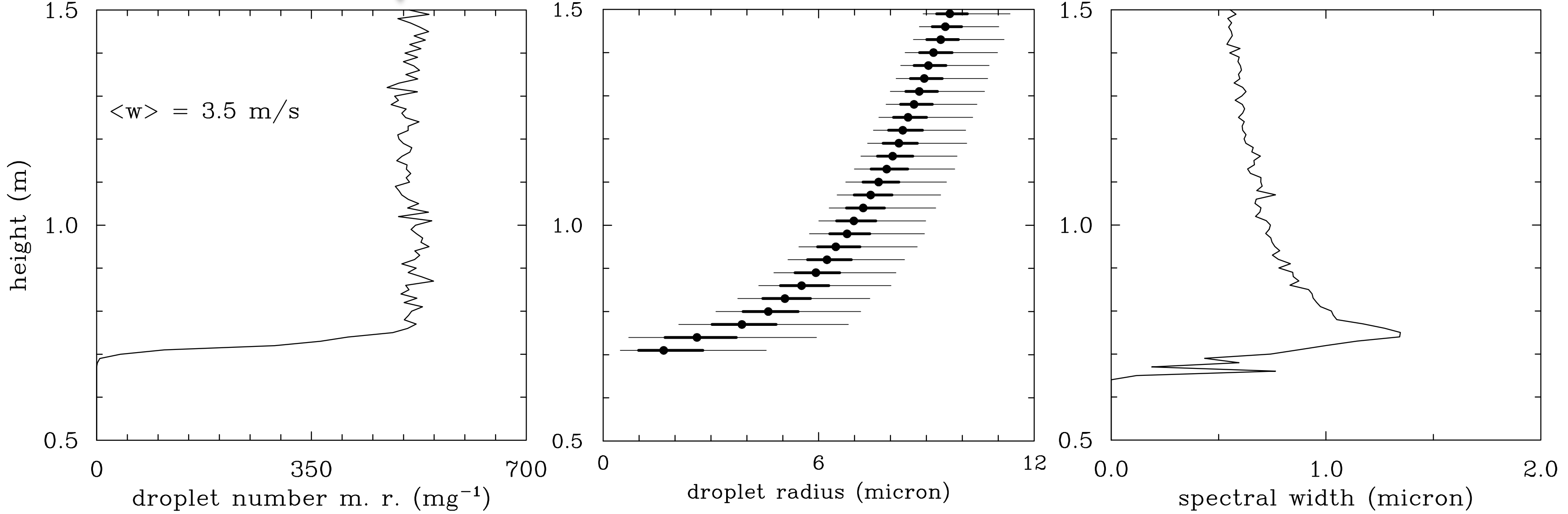
WITH TURBULENCE, BUT SIMPLE GROWTH EQ. ONCE ACTIVATED



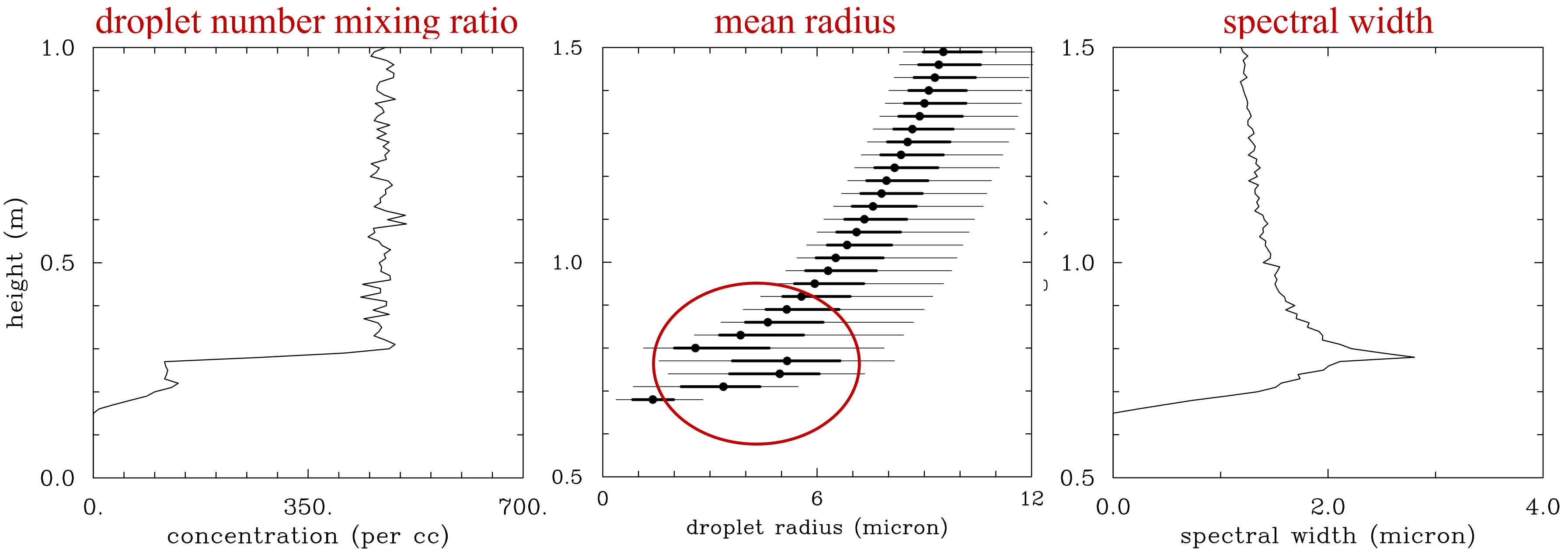
WITH TURBULENCE



WITH TURBULENCE, BUT SIMPLE GROWTH EQ. ONCE ACTIVATED



WITH TURBULENCE

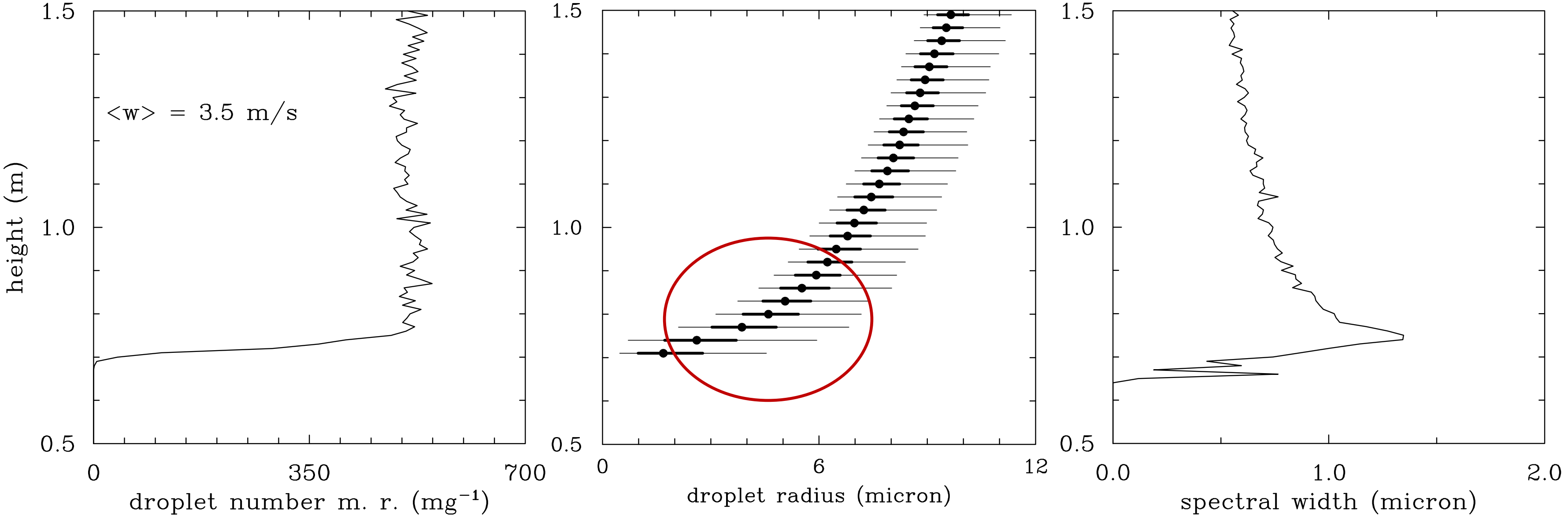


droplet number mixing ratio

mean radius

spectral width

WITH TURBULENCE, BUT SIMPLE GROWTH EQ. ONCE ACTIVATED



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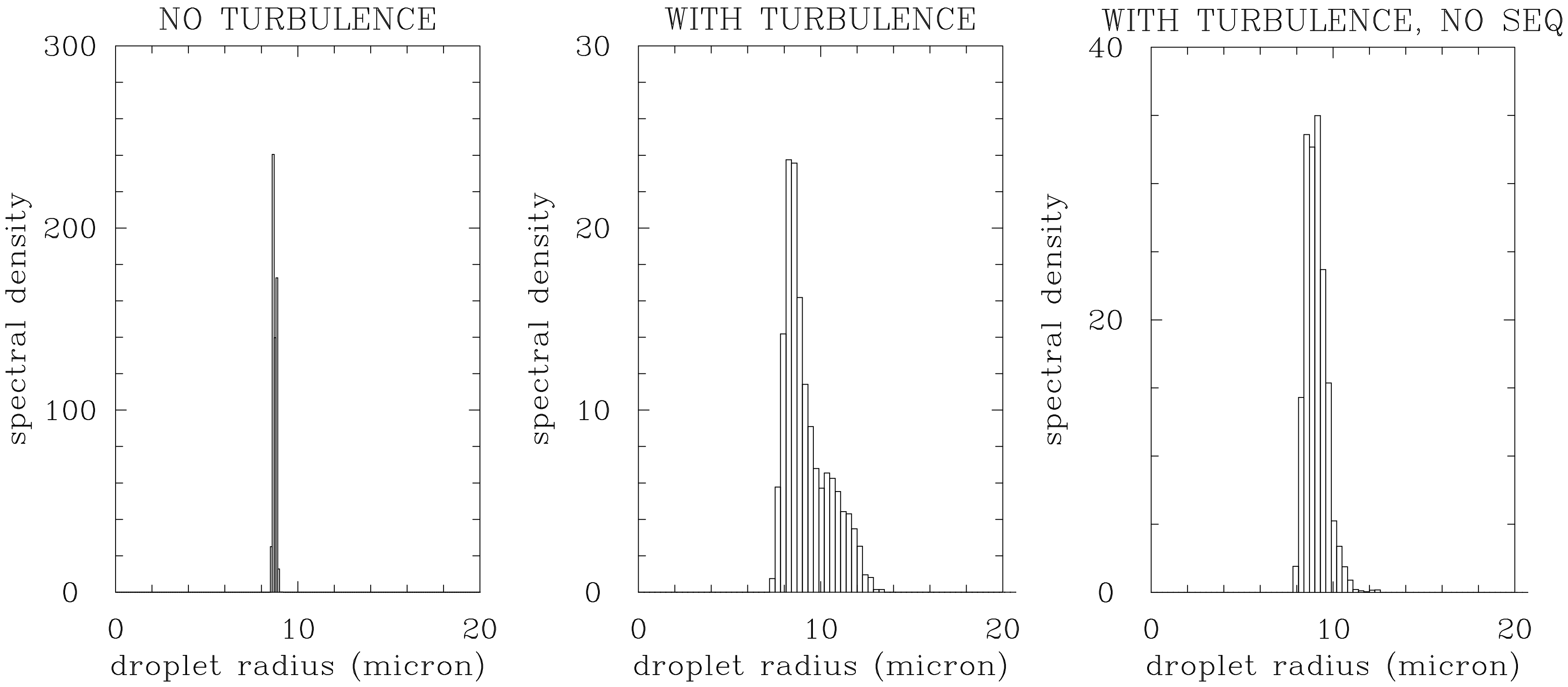

Summary:

Understanding droplet spectral broadening due to diffusional growth in turbulent environments attracted significant attention of turbulence community in recent decades. DNS, scaled-up DNS, and stochastic models have been used to show the impact of turbulence in undiluted cloudy volumes on the adiabatic droplet spectral width.

However, those idealized studies feature a fundamental flaw of periodic domains that allow droplets to circulate in the vertical. If droplet vertical spread is taken into account, the spectral spread saturates at small values. The $t^{1/2}$ scaling (Sardina et al.) comes from the spread of droplet positions as in the random walk model.

Analysis of realistic high-resolution cloud simulations (grid length of ~10 m) applying Lagrangian particle-based microphysics demonstrates the impact of turbulence on the droplet spectral broadening in a simulated cloud.

To understand the impact, a simplified 1D Eulerian-Lagrangian model of a vertical air current was developed. Without turbulence, narrow spectral widths agree with results from the adiabatic parcel simulations. Including the impact of turbulence on droplet growth results in adiabatic spectral widths even larger than simulated by the high-resolution 3D cloud model. This is still work in progress - comments welcome…