Emergence of Self-Organized Moist Convection, as Seen Through the Energy-Cycle in Wavelet Space

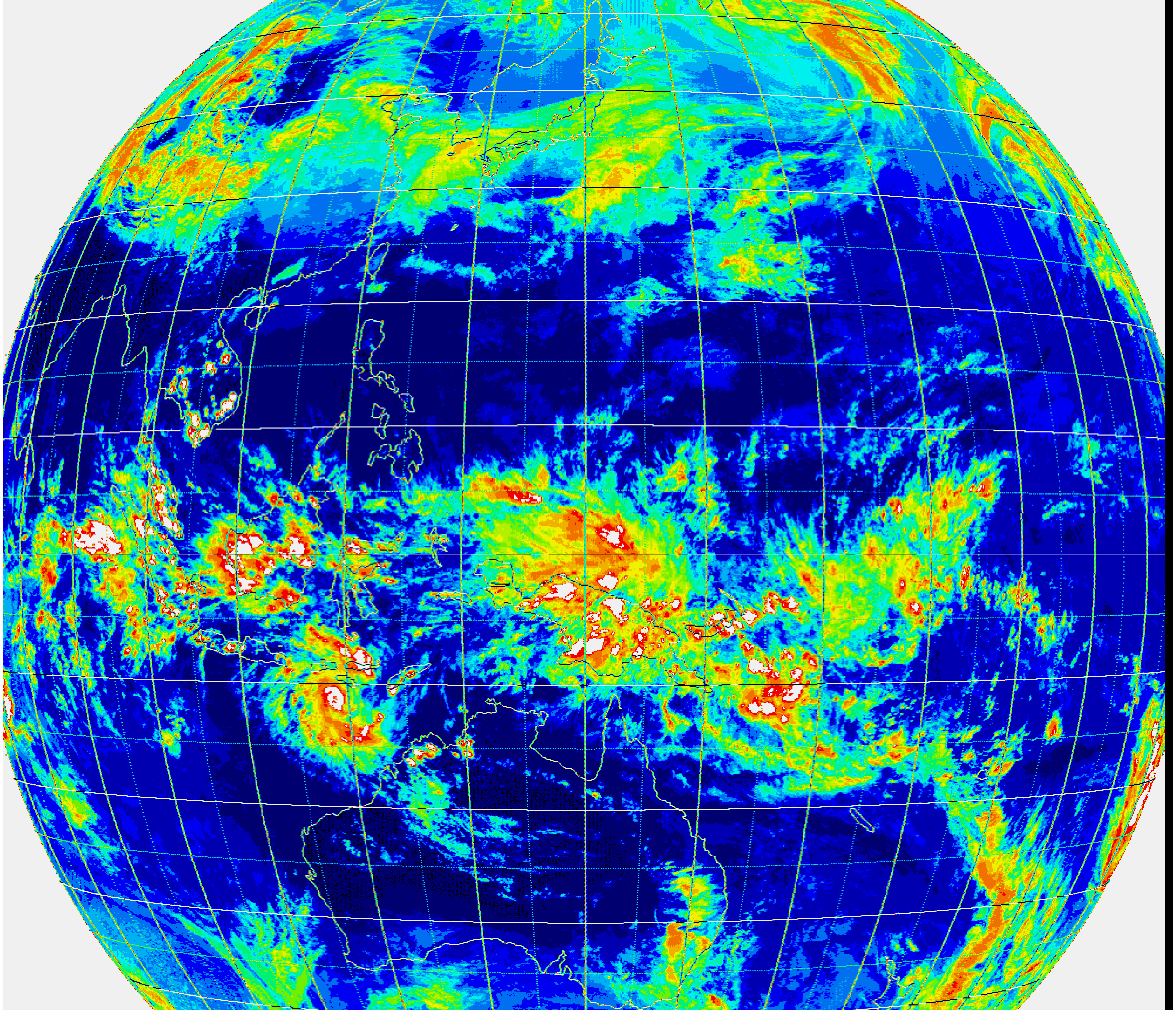
by

Jun-Ichi Yano, Météo France Toulouse

with

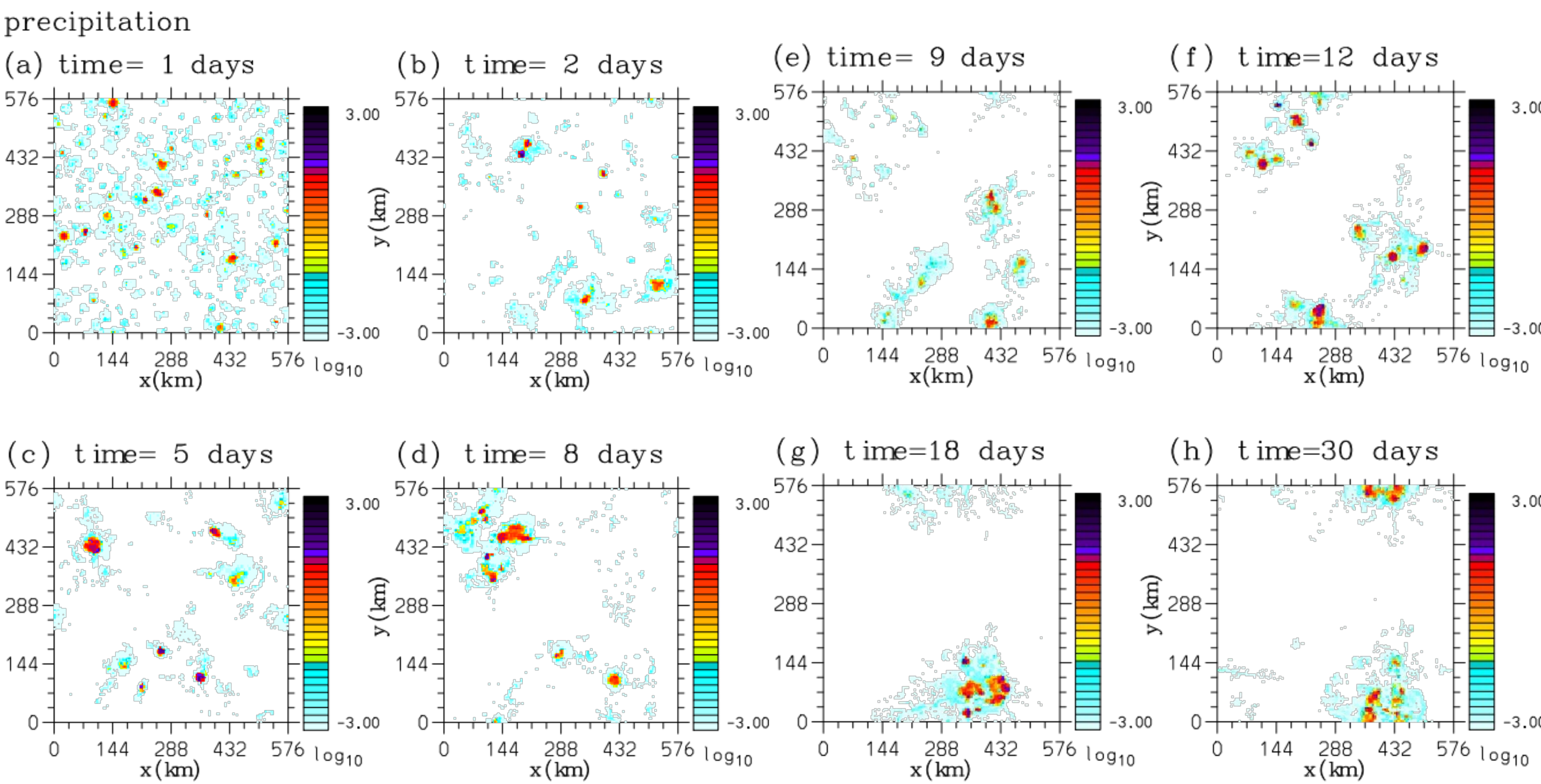
Bob Plant, University of Reading

(**JAMES 2025**)



Numerical Simulation:

Convective Organization:



(Holloway and Woolnough 2016)

Question:

How Convective Organization is formed and Maintained?

Basic Approach:

Analysis of the Energy Cycle in Wavelet Space

Key Concepts:

- **Wavelet**
- **Energy Cycle**

Why Wavelet?:

Organization:

Spatial localization:

Effectively Extracted by Wavelet

Wavelet(Discrete Orthogonal:Meyer)

Two indices: i, j:

localizations:

$k=2^i$, i=0

j=1

i=1

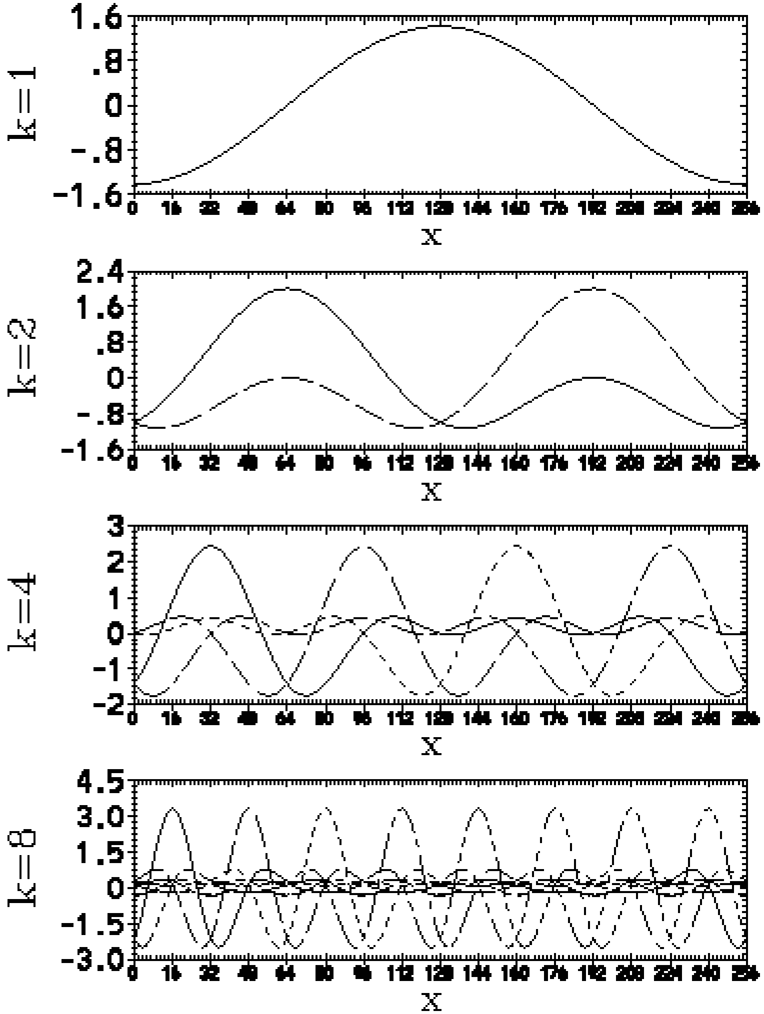
j=1, 2

i=2

j=1, ..., 4

i=3

j=1, ..., 8



i=-1: domain average

Orthogonal&Complete

Fourier Expansion(Discrete Complete Set)
for a finite periodic domain, $[0, L]$:

$$\frac{1}{L}\int_0^L \phi_k(x)\phi_l(x) = \delta_{kl}$$

For any *well–behaved* function, $f(x)$:

$$f(x) = \sum_{k=0}^{\infty} \hat{f}_k \phi_k(x)$$

$$\hat{f}_k = \frac{1}{L}\int_0^L f(x)\phi_k(x)dx$$

wavelet method

$$\frac{1}{L}\int_0^L \phi_{i,j}(x)\phi_{i',j'}(x) = \delta_{ii'}\delta_{jj'}$$

For any *well–behaved* function, $f(x)$:

$$f(x) = \sum_{i=-1}^{i_{max}} \sum_{j=1}^{\mathrm{Max}(1,2^i)} \hat{f}_{i,j}\phi_{i,j}(x)$$

$$\hat{f}_{i,j} = \frac{1}{L}\int_0^L f(x)\phi_{i,j}(x)dx$$

Wavelet (Discrete Orthogonal:Meyer)

Wavelet-based Decomposition:

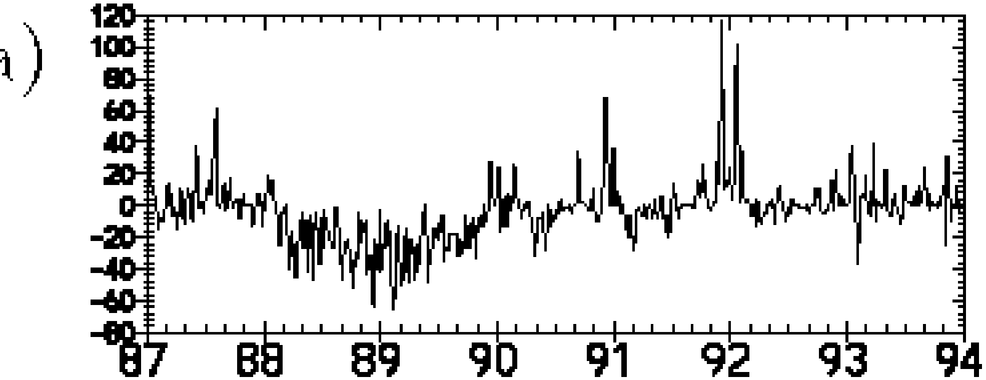
$$\varphi(x) = \sum_{i=-1}^{\ln_2 N-1} \sum_{j=1}^{j_{max}} \tilde{\varphi}_{i,j} \psi_{i,j}(x)$$

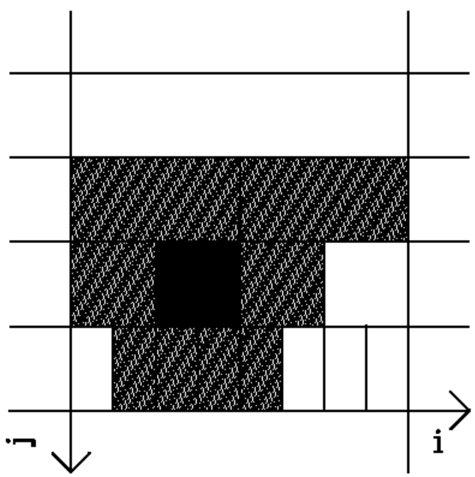
$$\varphi(x,y) = \sum_{i_x=-1}^{\ln_2 N_x-1} \sum_{i_y=-1}^{\ln_2 N_y-1} \sum_{j_x=1}^{j_{x,max}} \sum_{j_y=1}^{j_{y,max}} \tilde{\varphi}_{i_x,i_y,j_x,j_y} \psi_{i_x,j_x}(x) \psi_{i_y,j_y}(y)$$

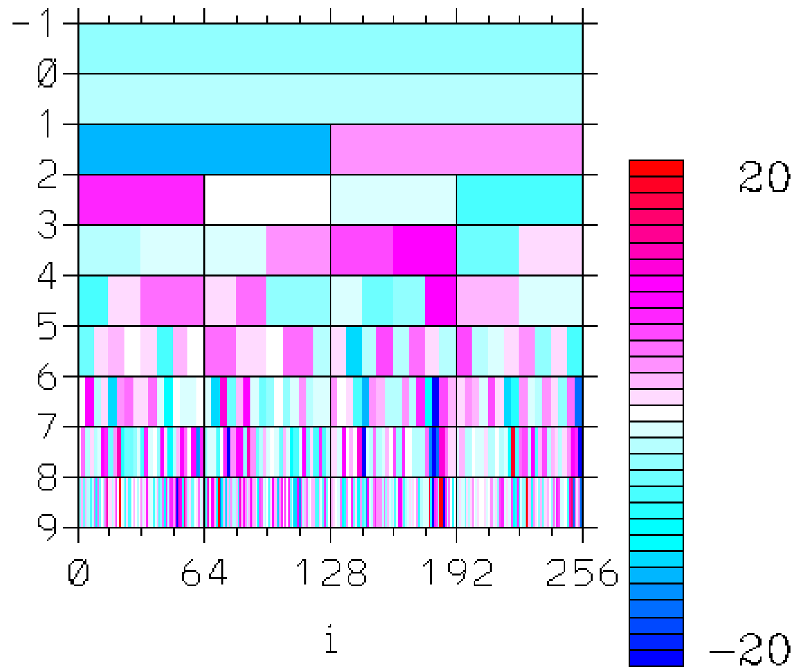
Extraction:

One-Dimensional Demonstration:

Zonal Wind over Western Pacific:



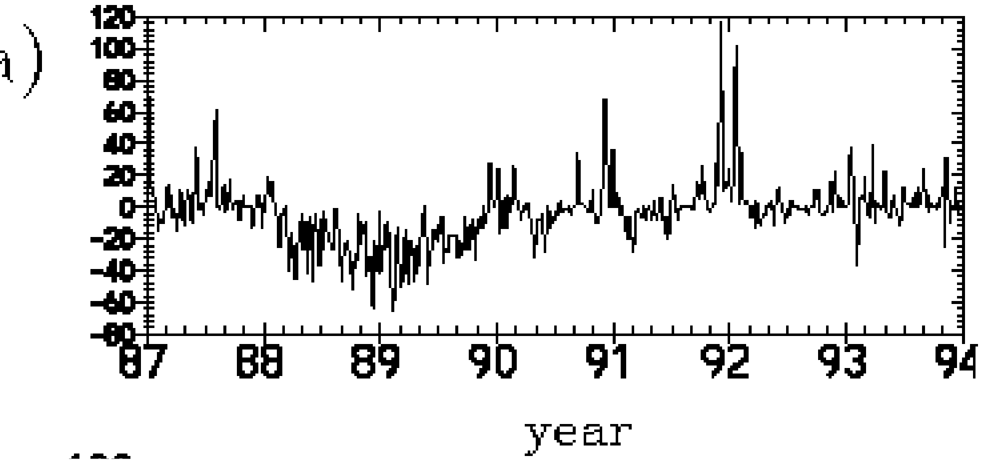


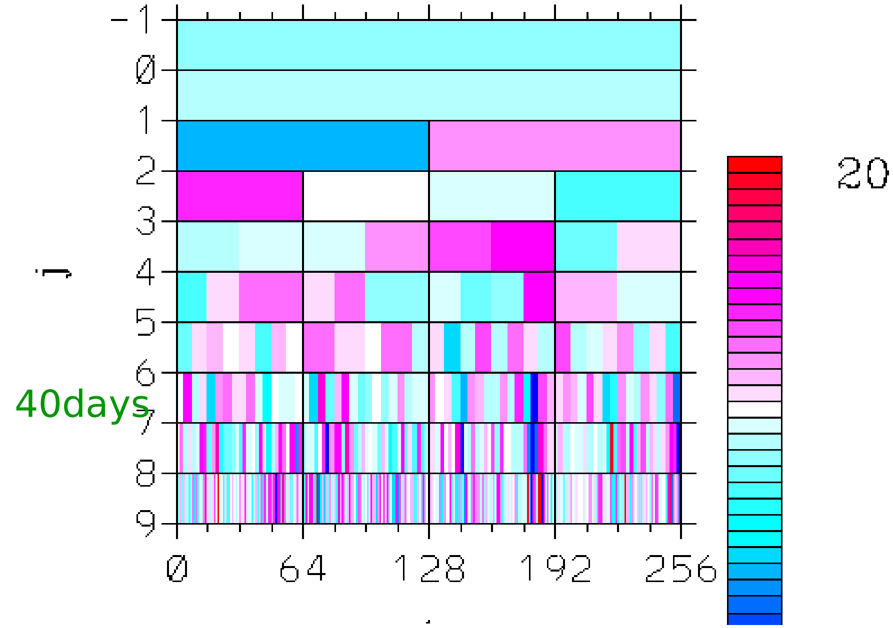


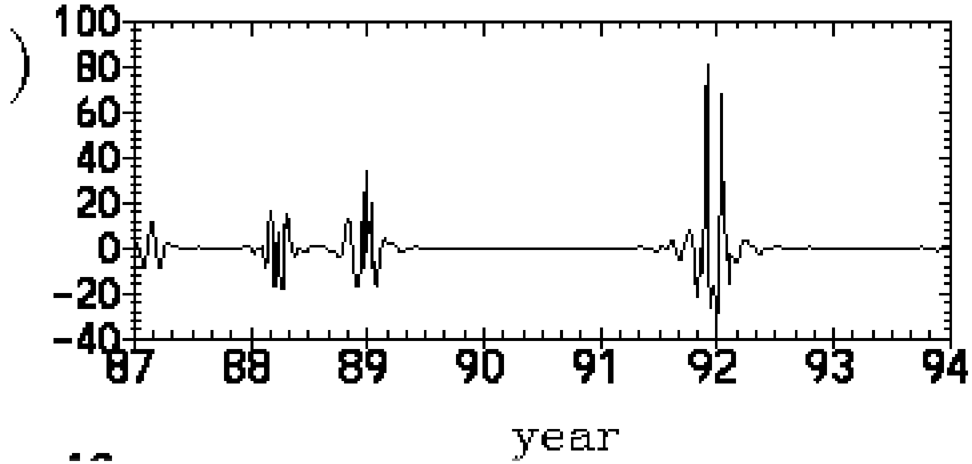
Extraction of Isolated Feat

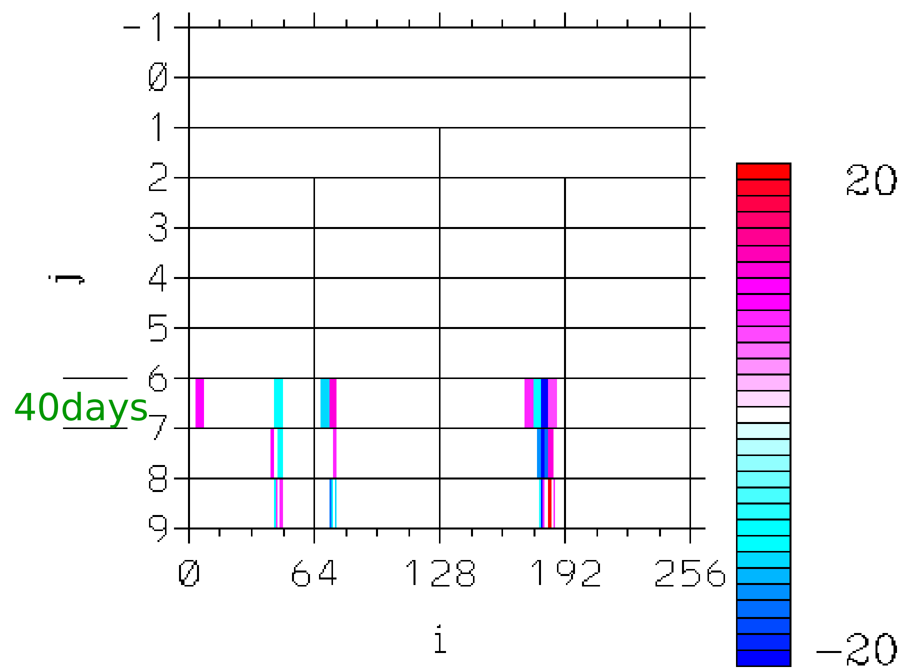
One-Dimensional Demon

Zonal Wind over Westerr

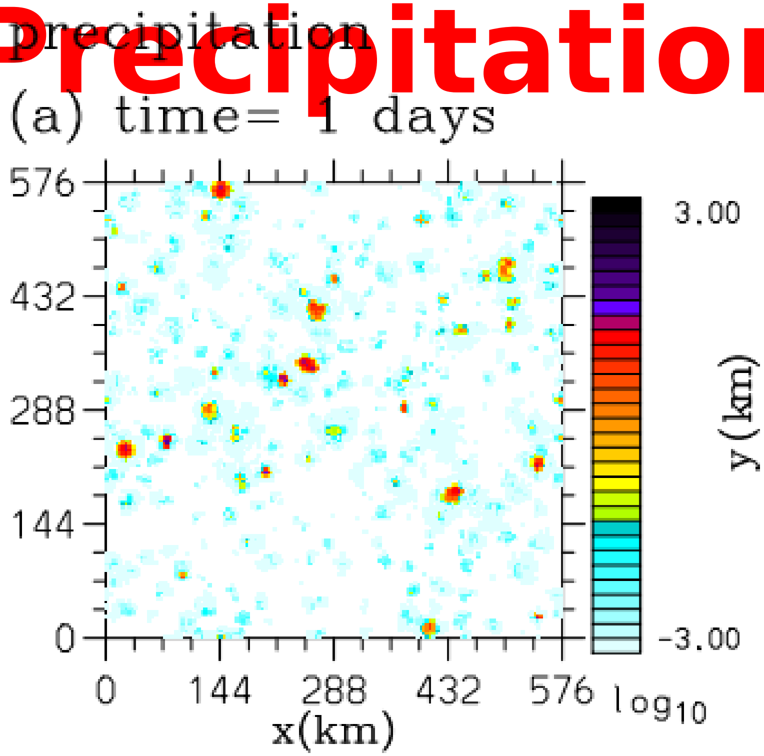


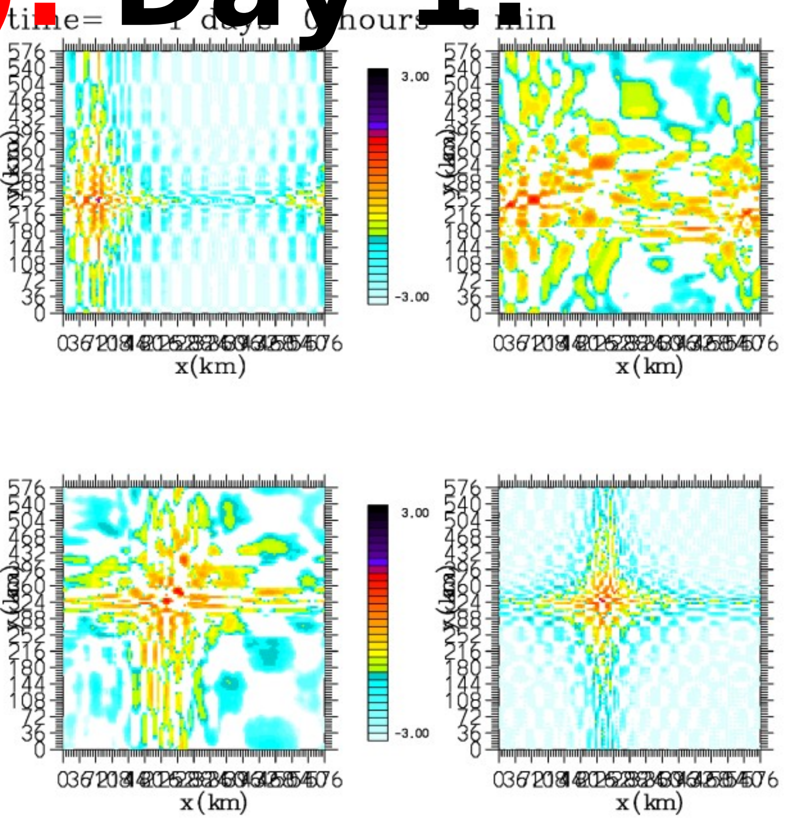




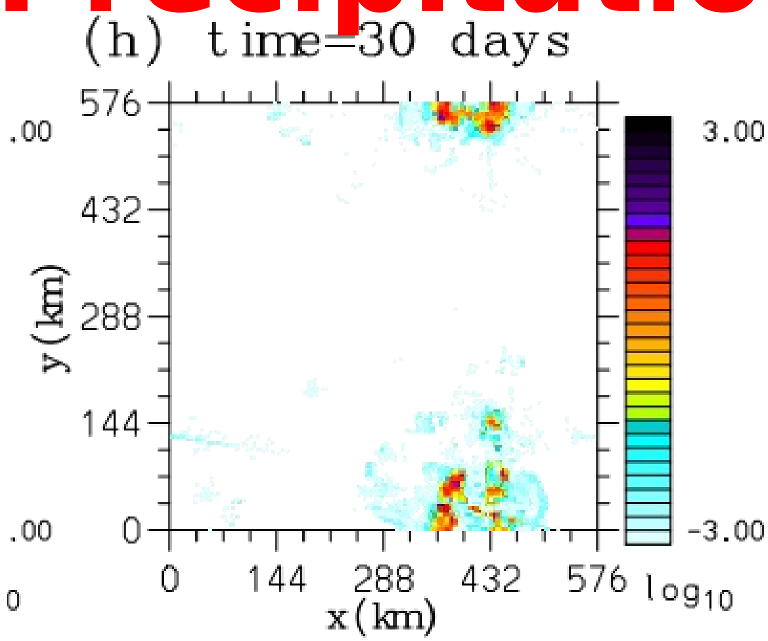


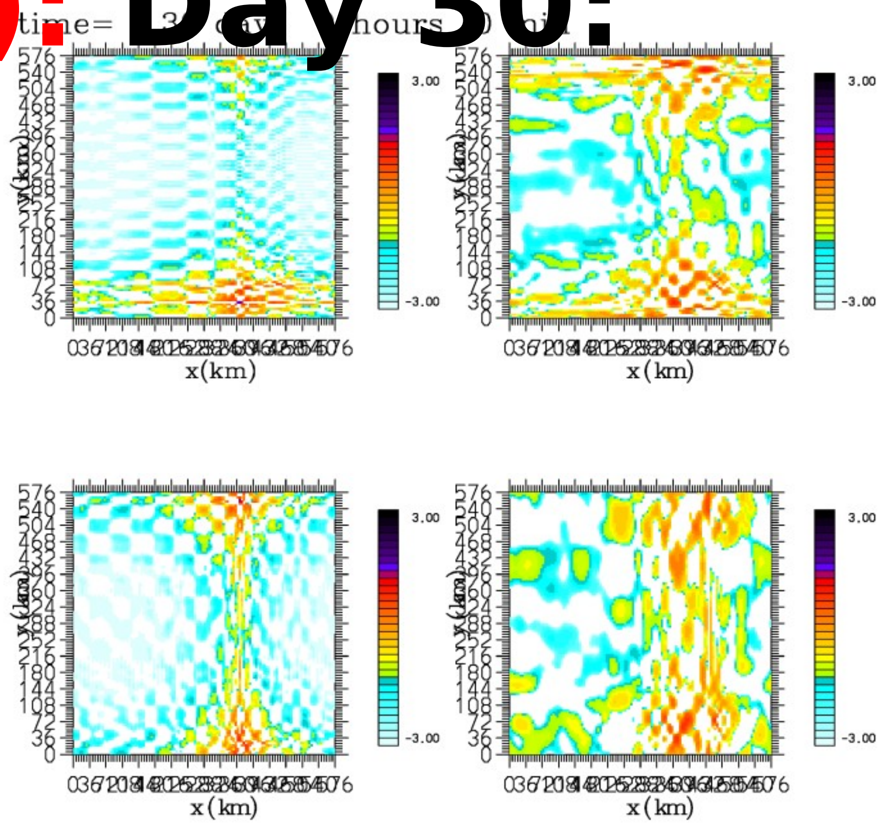
Organized Precipitation Field : First Four Pulse Modes (Precipitation): Day 1:





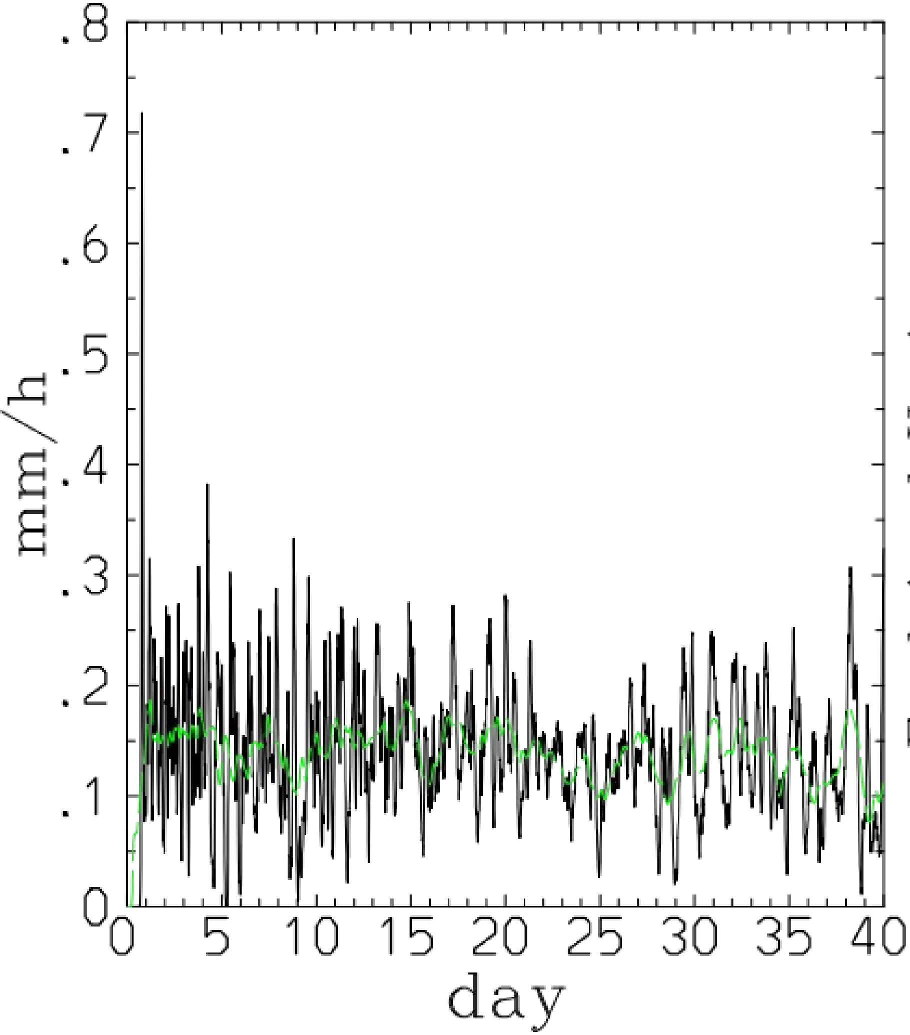
Organized Precipitation Field : First Four Pulse Modes (Precipitation): Day 30:



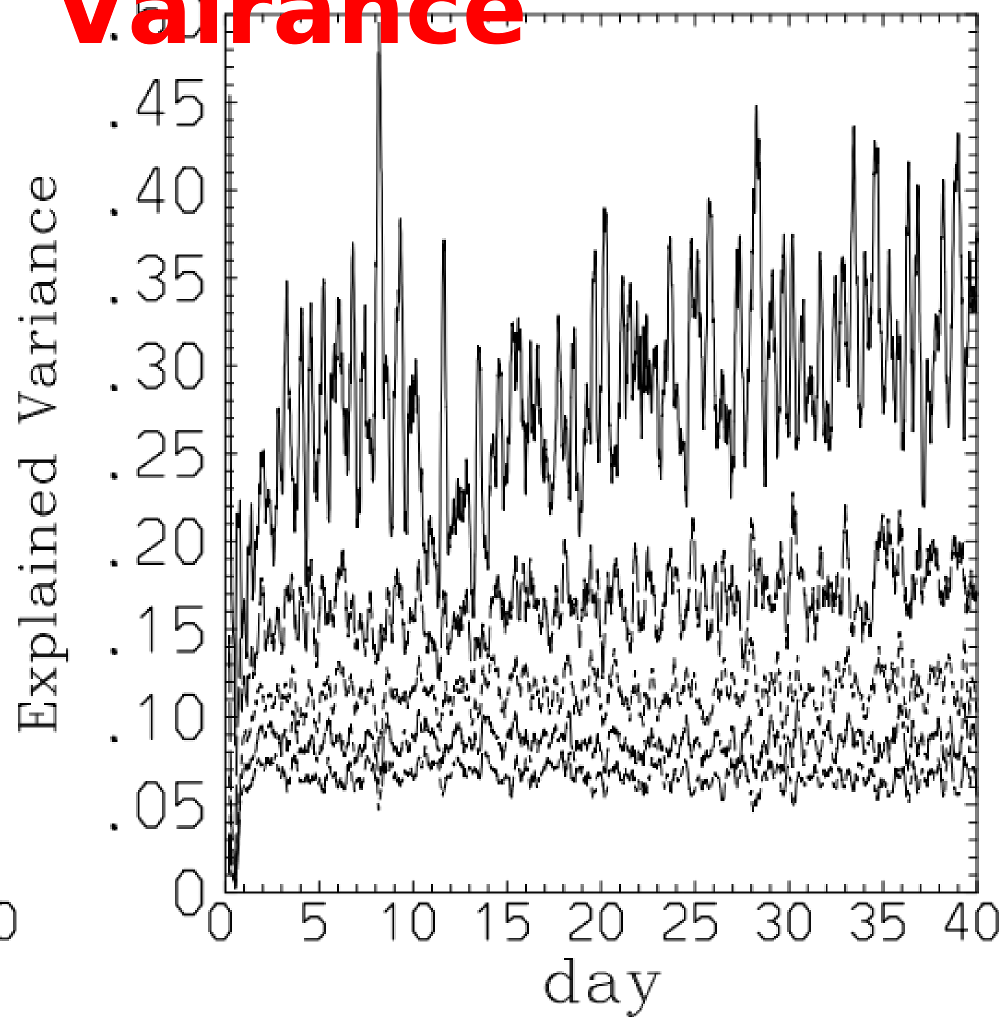


Basic Features :

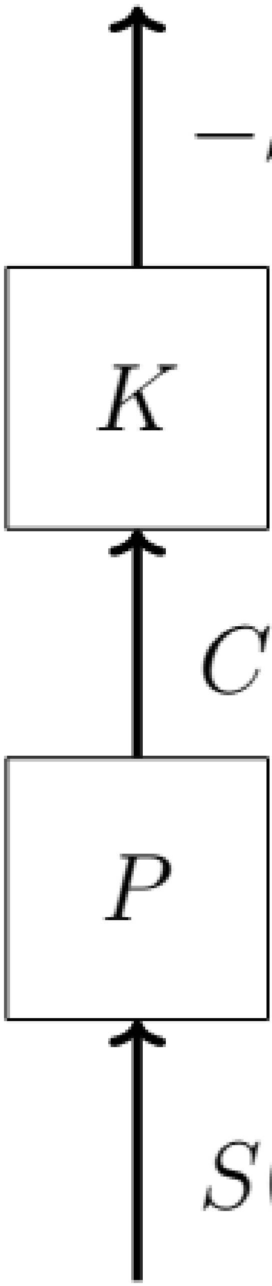
Domain-Averaged Precipitation



First 5 Puleses : Fractional Vairance



Basic Energy Cycle :



$-S(K)$ **Kinetic Energy Sink**

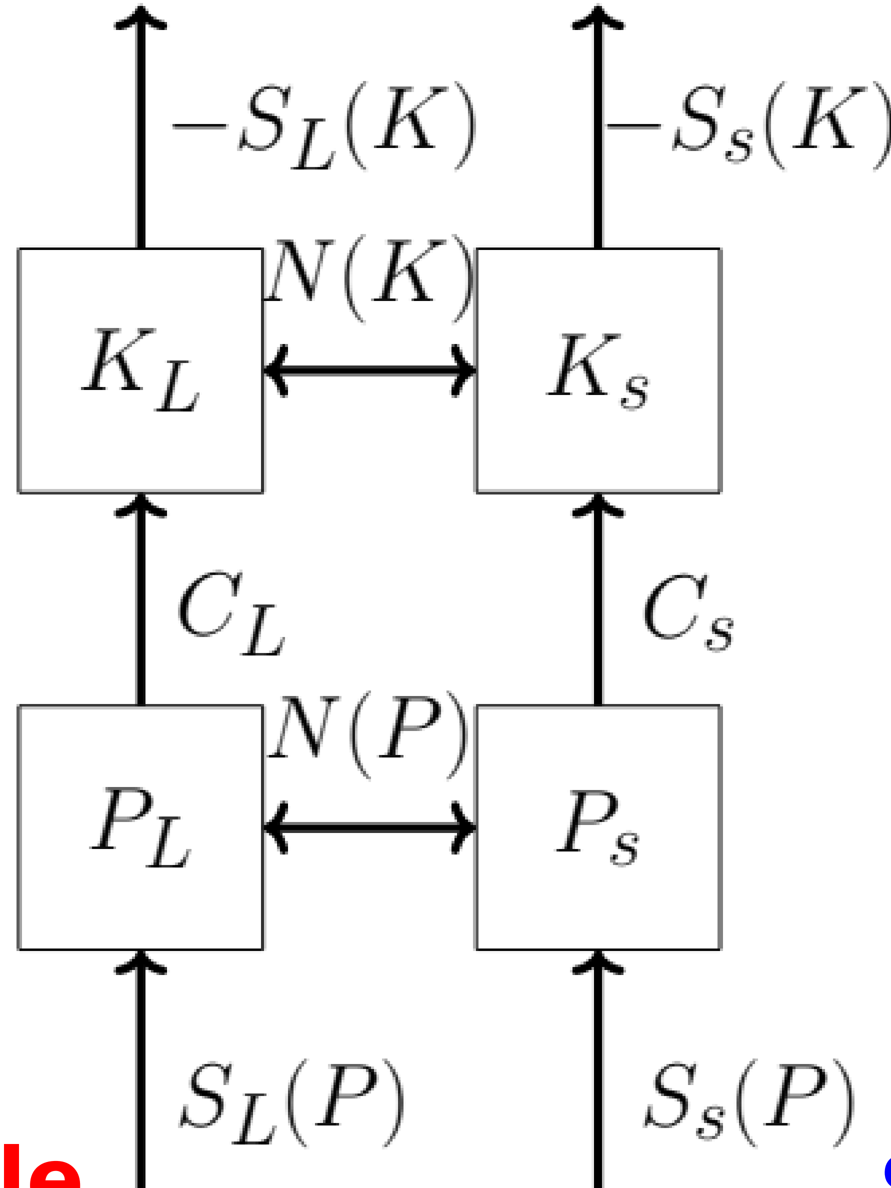
K **Kinetic Energy**

C **Conversion from PE to KE**

P **Available Potential Energy (Lorenz 1955)**

$S(P)$ **Potential Energy Source**

Energy Cycle : Differntiated by Scal



Large Scale

Small Scal

Definitions (Physical Space)

- **Kinetic Energy :** $$K = \frac{\rho}{2}\mathbf{v}^2$$
- **Available Potential Energy :** $$P = \frac{\rho}{2\sigma}\theta_v'^2$$

where $$\sigma = \frac{\bar{\theta}_v}{g}\frac{d\bar{\theta}_v}{dz}.$$

Definitions (Wavelet Space)

- **Kinetic Energy :**

$$\tilde{K}_l = \frac{1}{2}(\widetilde{\rho\mathbf{v}})_l\tilde{\mathbf{v}}_l,$$

- **Available Potential Energy :**

$$\tilde{P}_l = \frac{1}{2\sigma}(\widetilde{\rho\theta'_v})_l\tilde{\theta}'_{vl}.$$

where

$$\sigma = \frac{\bar{\theta}_v}{g}\frac{d\bar{\theta}_v}{dz}.$$

<u>Energy Cycle</u> (Physical Space)

$$\frac{\partial K}{\partial t} + \nabla \cdot (K + \rho c_{pd}\theta_v \pi')\mathbf{v} = C(P,K) + \rho \mathbf{v} \cdot \mathbf{S}^{\mathbf{v}} + c_{pd}\pi' \left(\nabla \cdot \rho \theta'_v \mathbf{v} + \rho w \frac{d\bar{\theta}_v}{dz} \right)$$

$$\frac{1}{\sigma}\frac{\partial \sigma P}{\partial t} + \frac{1}{\sigma}\nabla \cdot \sigma P \mathbf{v} = -C(P,K) + \frac{\rho \theta'_v}{\sigma} S^{\theta_v}$$

Energy Conversion Rate :

$$C(P,K) = \rho g \frac{\theta'_v}{\bar{\theta}_v} w.$$

<u>Energy Cycle</u> (Wavelet Space)

$$\frac{\partial}{\partial t}\tilde{K}_l - \tilde{N}_l(K) = \tilde{F}_l(K) + \tilde{C}_l(P,K) + \tilde{S}_l(K)$$

$$\frac{1}{\sigma}\frac{\partial}{\partial t}\sigma\tilde{P}_l - \tilde{N}_l(P) = -\tilde{C}_l(P,K) + \tilde{S}_l(P)$$

Energy Conversion Rate :

$$\tilde{C}_l(P,K) = \frac{g}{2\bar{\theta}_v}[(\widetilde{\rho w})_l\tilde{\theta}'_{vl} + \tilde{w}_l(\widetilde{\rho\theta'_v})_l]$$

$$\tilde{N}_l = \tilde{N}_{l,H} + \tilde{N}_{l,V},$$

$$\tilde{F}_l(K) = \tilde{F}_{l,H}(K) + \tilde{F}_{l,V}(K)$$

$$\tilde{N}_{l,H}(K) = -\frac{1}{2}[(\widetilde{\rho \mathbf{v}})_l \cdot \langle \mathbf{u} \cdot \nabla_H \mathbf{v} \rangle_l + \langle \nabla_H \cdot \rho \mathbf{u} \mathbf{v} \rangle_l \cdot \tilde{\mathbf{v}}_l]$$

$$\tilde{N}_{l,V}(K) = -\frac{1}{2}\left[\langle \rho \mathbf{v} w \rangle_l \cdot \frac{\partial \tilde{\mathbf{v}}_l}{\partial z} + \tilde{\mathbf{v}}_l \cdot \langle \mathbf{v} \frac{\partial \rho w}{\partial z} \rangle_l\right]$$

$$\tilde{N}_{l,H}(P) = -\frac{1}{2\sigma}[\langle (\mathbf{u} \cdot \nabla_H) \theta'_v \rangle_l \cdot (\widetilde{\rho \theta'_v})_l + \langle \nabla_H \cdot \rho \mathbf{u} \theta'_v \rangle_l \tilde{\theta}'_{vl}]$$

$$\tilde{N}_{l,V}(P) = -\frac{1}{2\sigma}[(\widetilde{\rho \theta'_v})_l \langle w \frac{\partial \theta'_v}{\partial z} \rangle_l + \tilde{\theta}'_{vl} \langle \frac{\partial \rho w \theta'_v}{\partial z} \rangle_l]$$

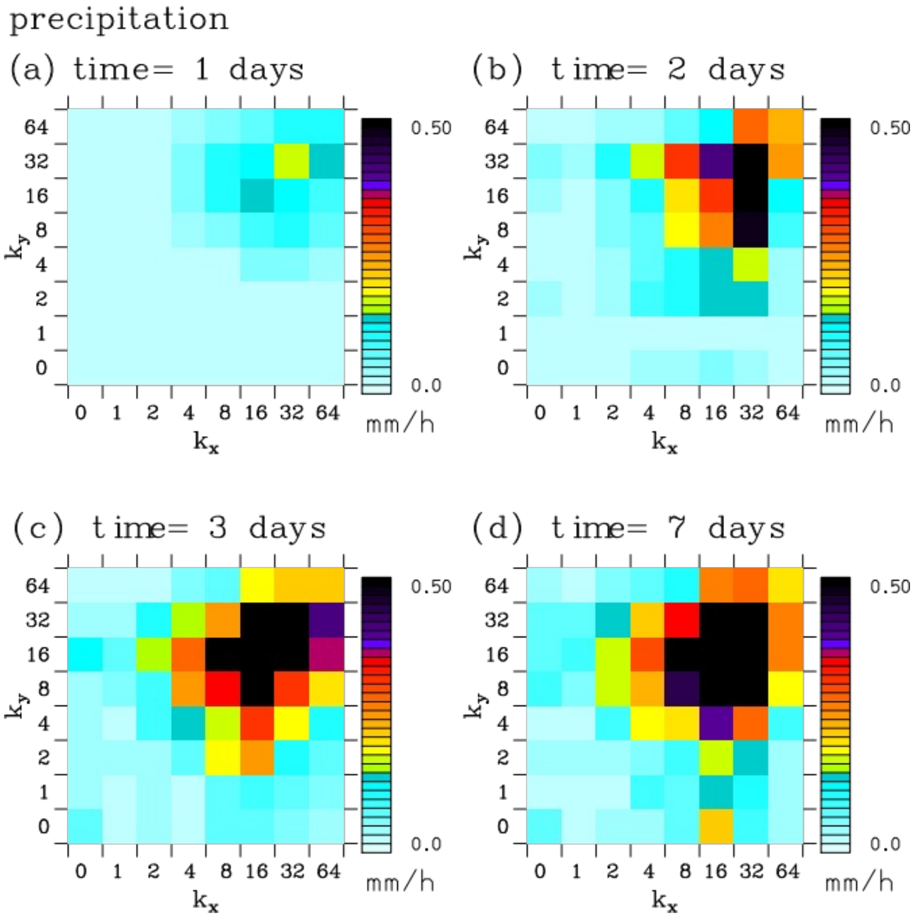
$$\tilde{F}_{l,H}(K) = c_{pd} \tilde{\pi}'_l \langle \nabla_H \cdot \rho \mathbf{u} \theta_v \rangle_l$$

$$\tilde{F}_{l,V}(K) = -c_{pd} \langle \rho w \theta_v \rangle_l \frac{\partial \tilde{\pi}'_l}{\partial z}$$

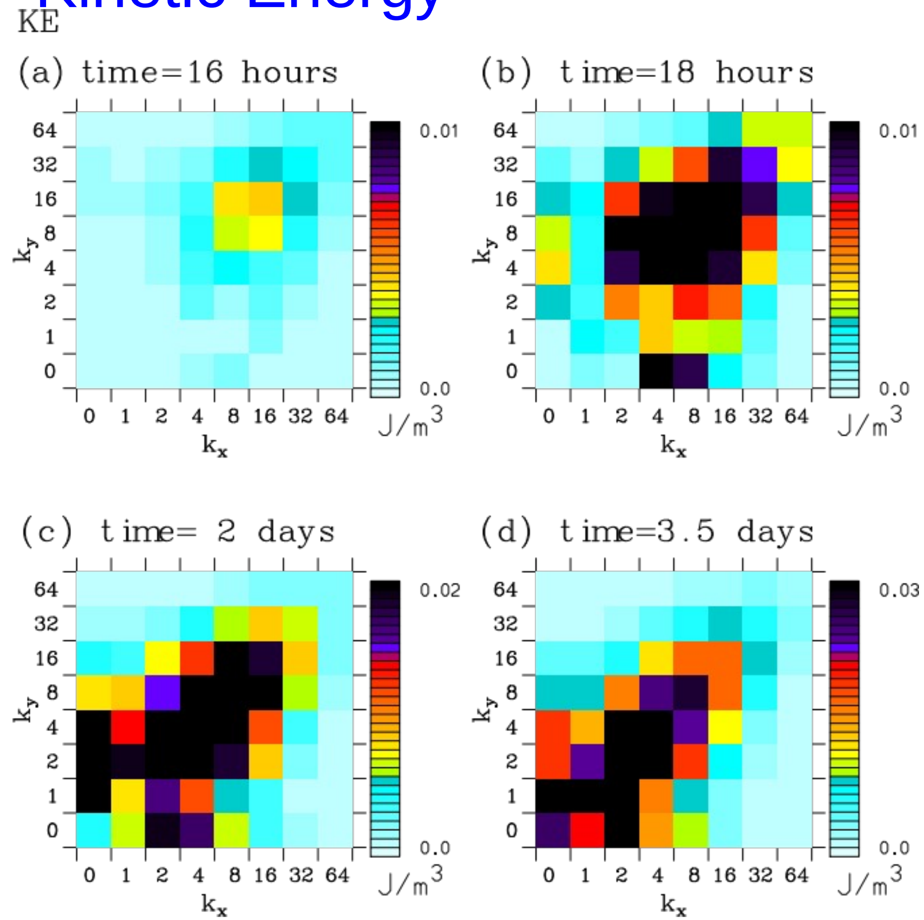
Spectrum Evolution:

$$P(k_x, k_y) = \sum_{j_x=1}^{max(1,k_x)} \sum_{j_y=1}^{max(1,k_y)} \tilde{\varphi}^2_{i_x,i_y,j_x,j_y}$$

Precipitation



Kinetic Energy



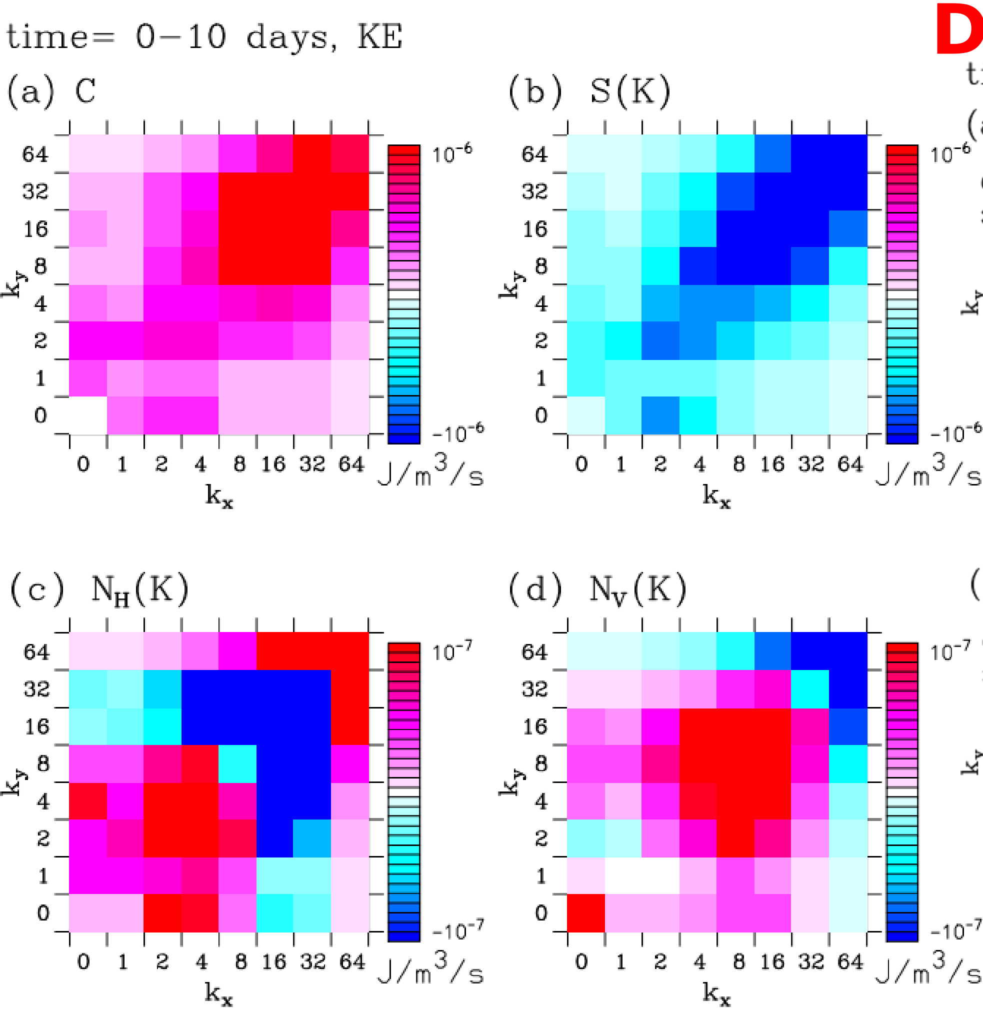
Generation of Convective Organization

Transfer of Kinetic Energy to the Larger Scales

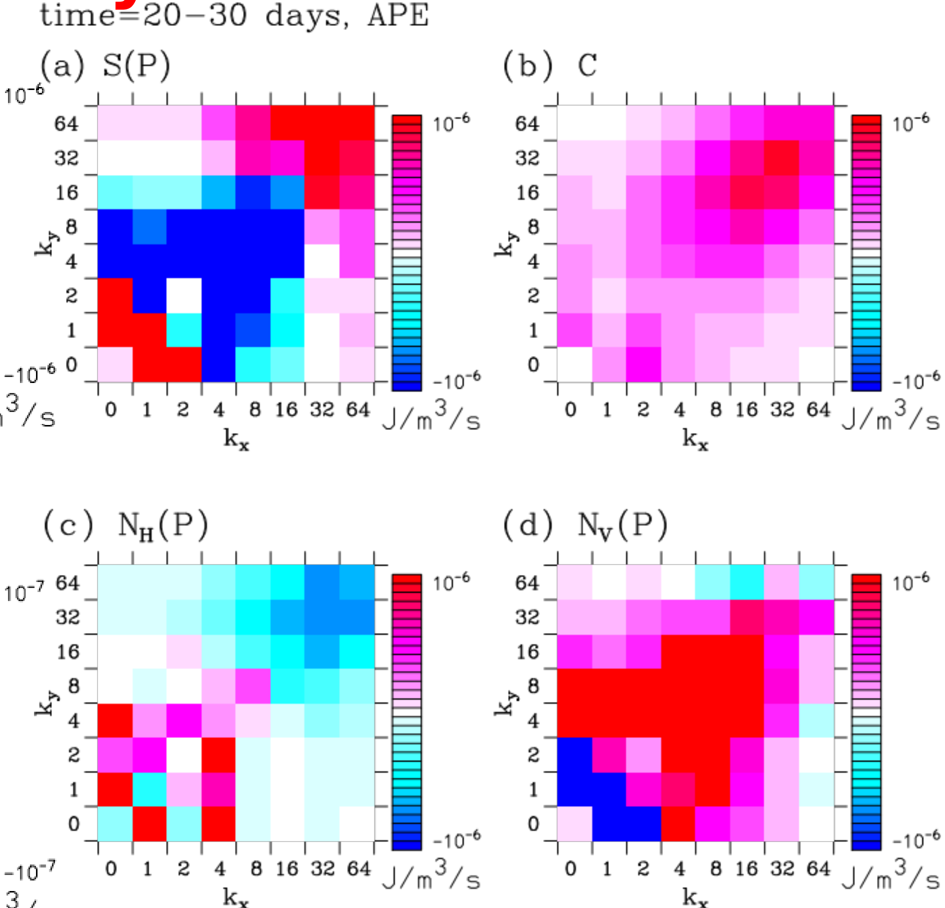
: How This Happens ?

Energy Cycle (Wavelet Space)

Kinetic Energy First 10 Days :



Available Potential Energy Days 20-30:



Process Decomposition:

Choose a Threshold Variable: φ.

High Component:

$$\hat{\mathbf{f}}_l^H = \begin{cases} \hat{\mathbf{f}}_l, & |\hat{\varphi}_l| > \varphi_c \\ 0, & |\hat{\varphi}_l| \leq \varphi_c \end{cases}$$

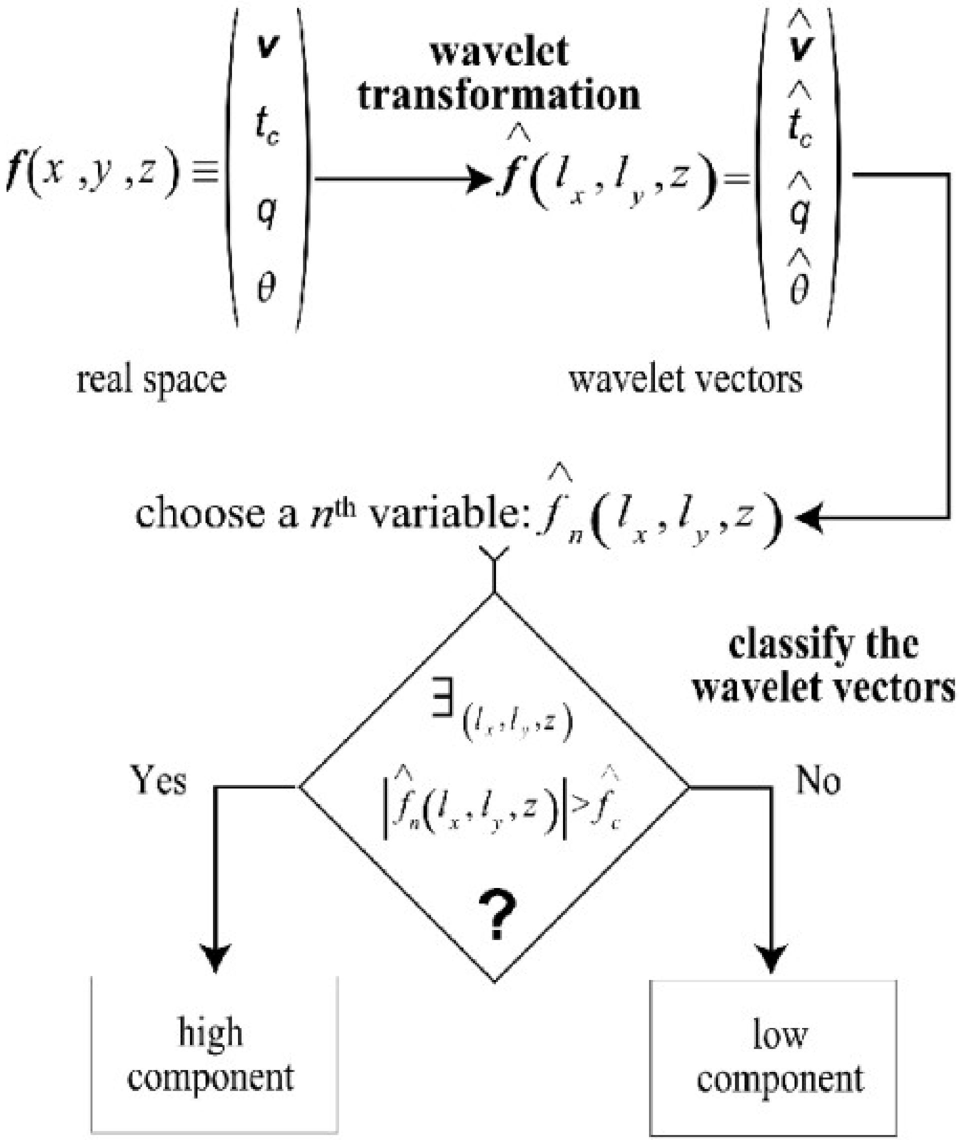
Low Component:

$$\hat{\mathbf{f}}_l^L = \begin{cases} 0, & |\hat{\varphi}_l| > \varphi_c \\ \hat{\mathbf{f}}_l, & |\hat{\varphi}_l| \leq \varphi_c \end{cases}$$

Here,

$$\varphi_c = (\sum_{l=1}^{N-1} \hat{\varphi}_l^2)^{1/2}$$

Process Decomposition:



Process Decomposition:

Convective Organization:

Buoyancy-Production Driven and Non-Buoyancy-Production Driven Components:

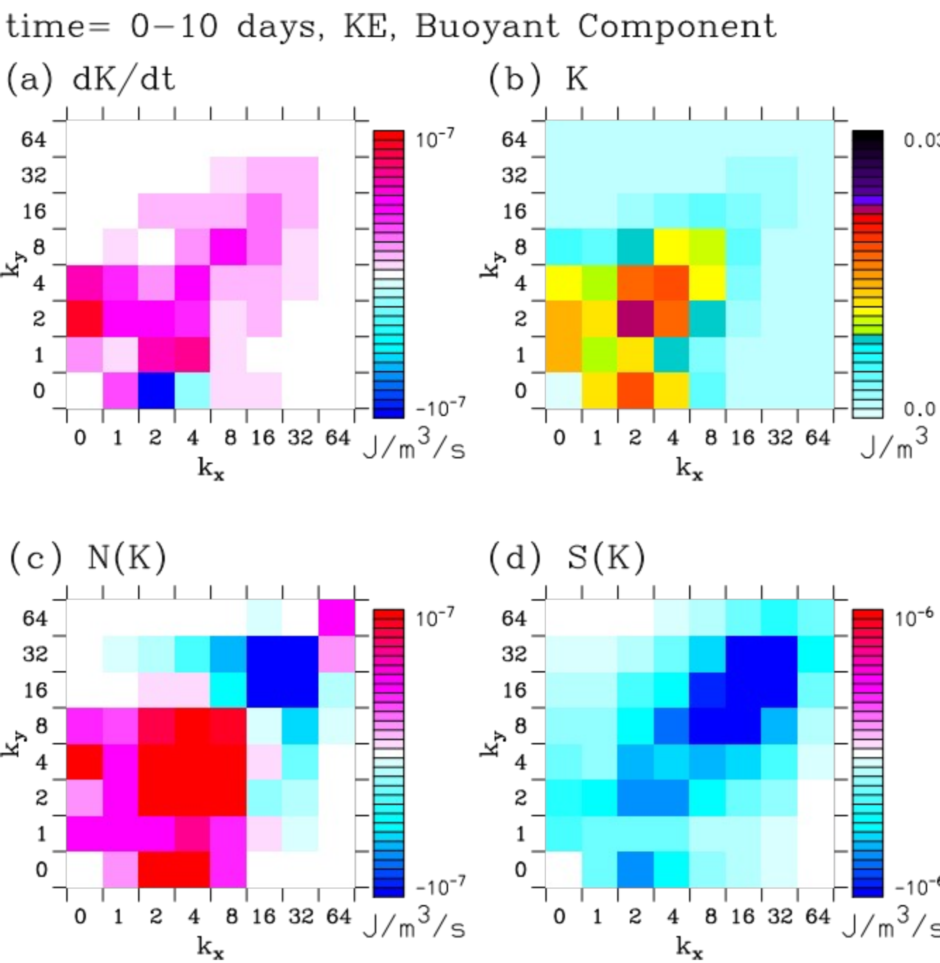
Threshold Variable:

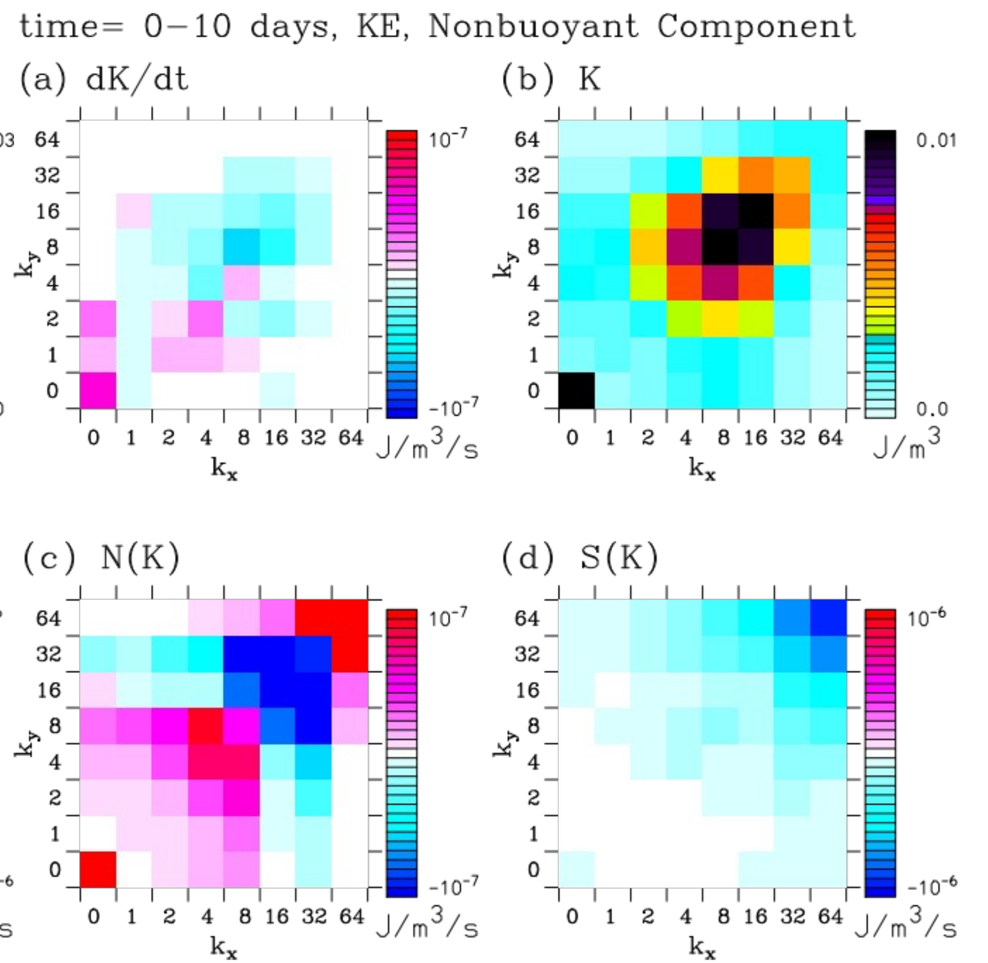
$$\phi = C(P, K)$$

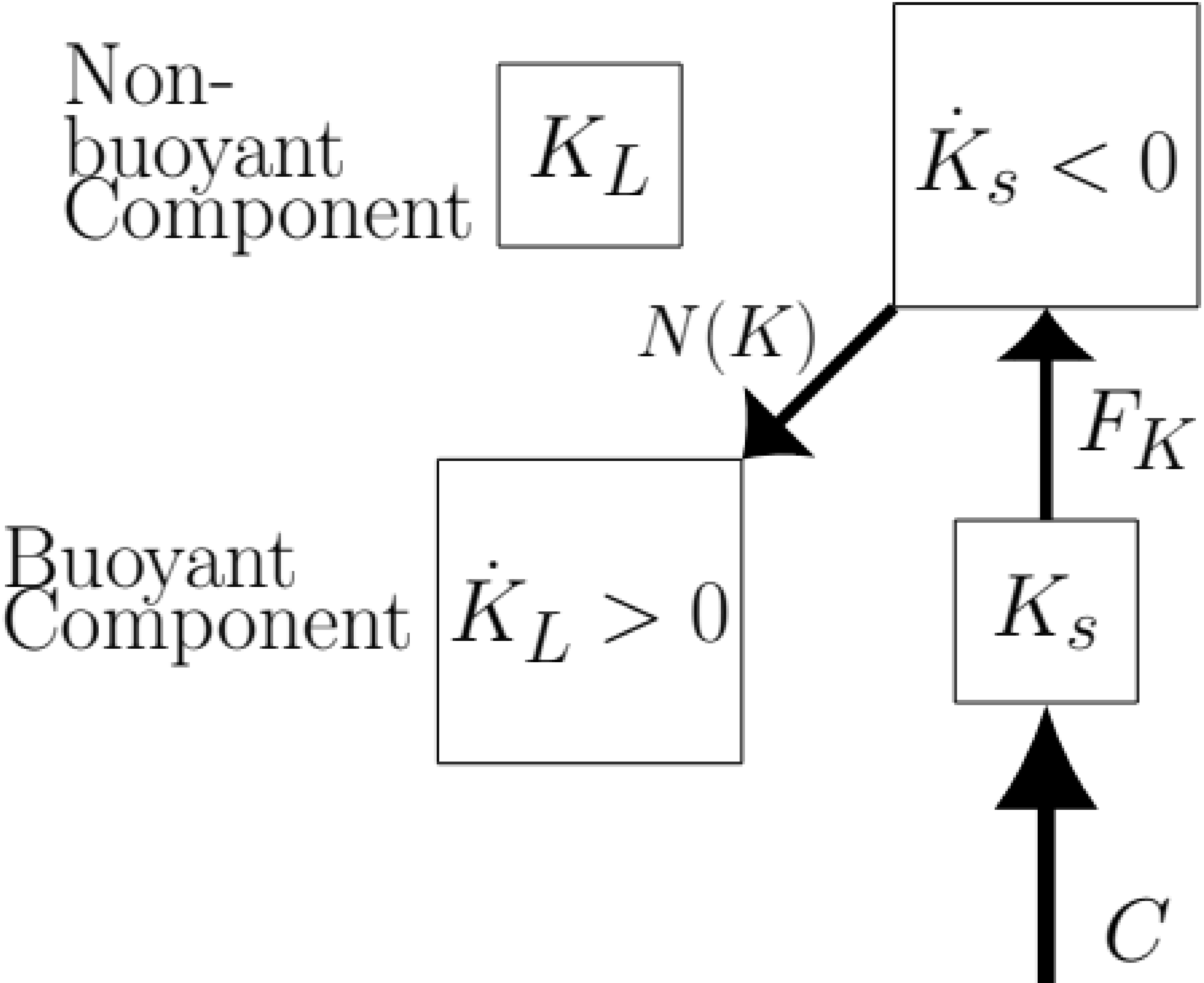
Energy Cycle : Between Two Components :

Kinetic Energy Buoyant Component :

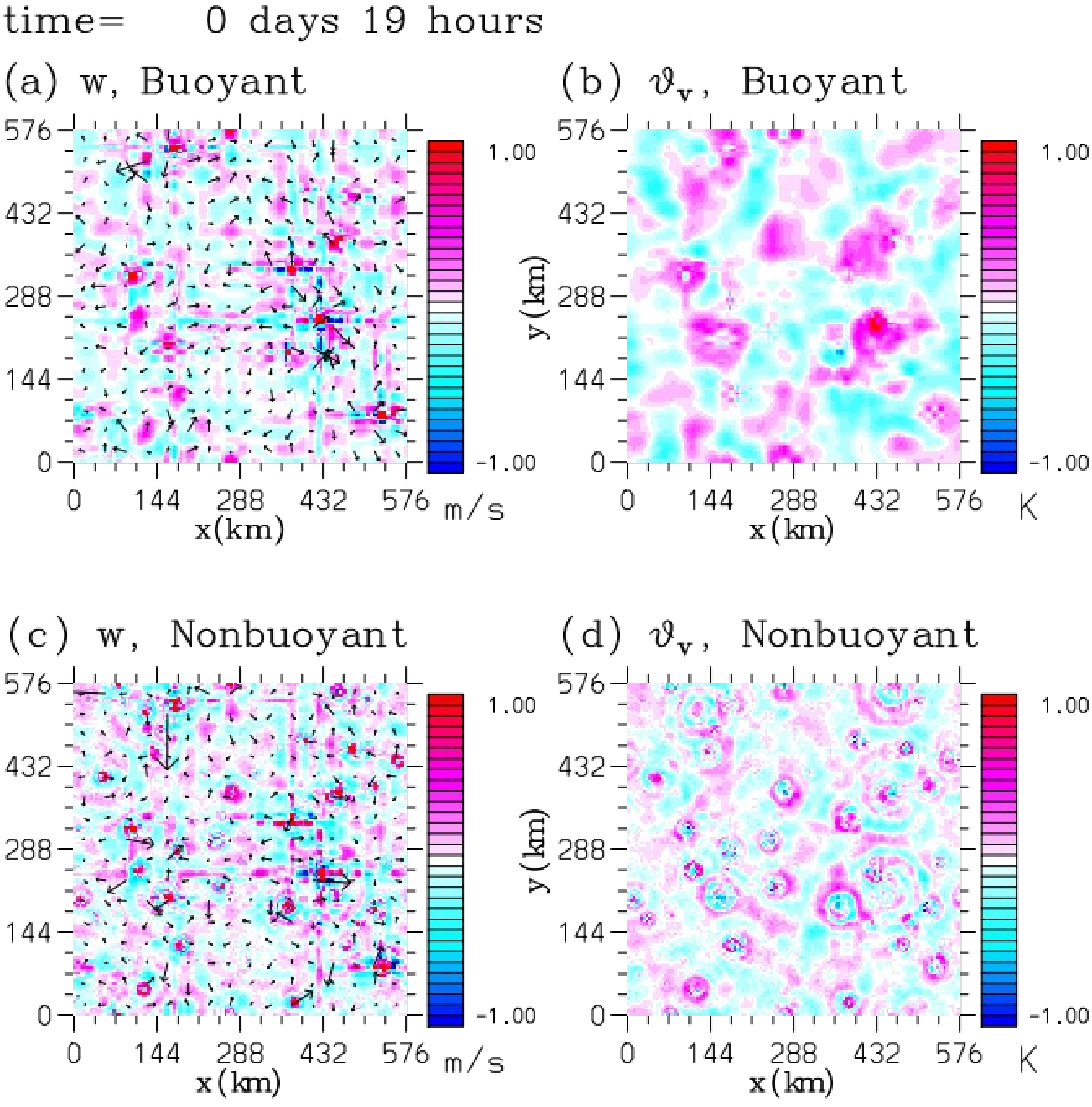
Kinetic Energy NonBuoyant Component :



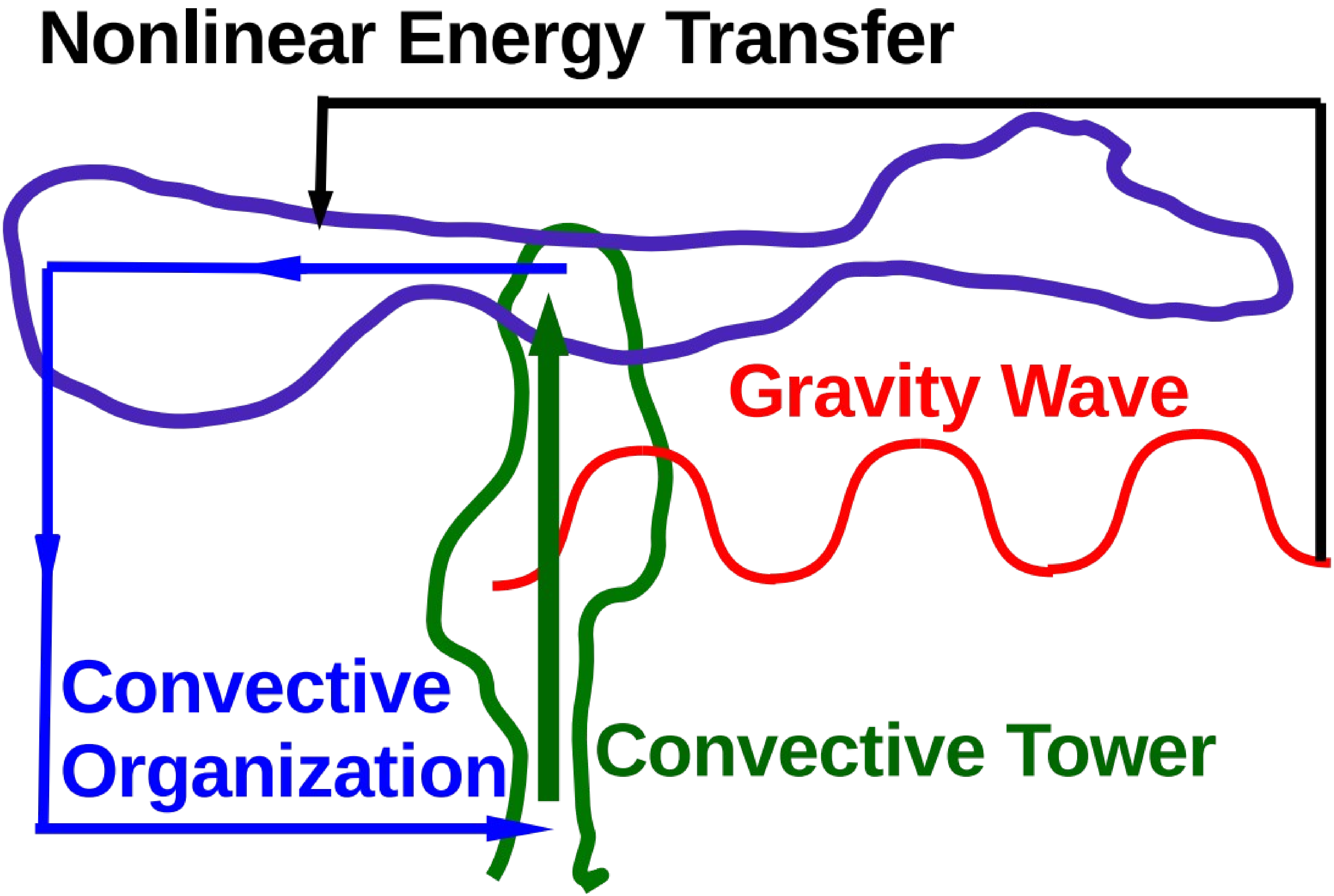




Non-buoyant Component
K_L
$\dot{K}_s < 0$
$N(K)$
F_K
Buoyant Component
$\dot{K}_L > 0$
K_s
C



Gravity Waves


Nonlinear Energy Transfer
Gravity Wave
Convective Organization
Convective Tower

Summary

Wavelet:

- Capacity of Quantifying the “Organization” effectively:
  - Orthogonality and Completeness
  - Automatic Windowing
  - Many Options with Flexibility

Q: What We Want to Quantify?

Possibility To Be Further Pursued:

Process Study in Wavelet Space

e.g., Energy Cycle (cf., Yano and Plant 2025 JAMES)

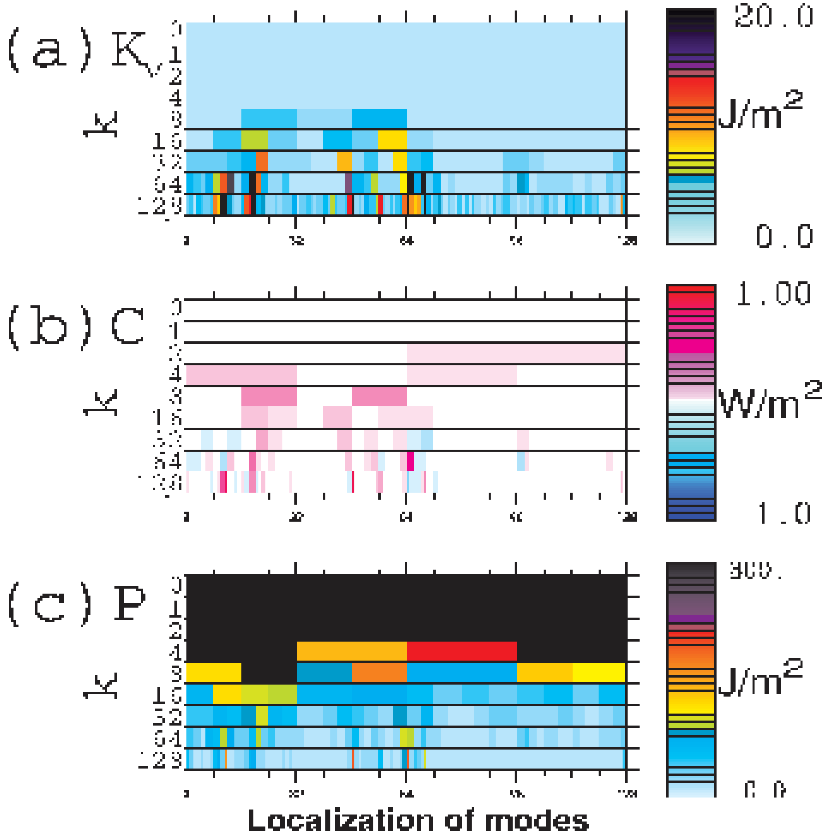
Queestions ?

Comments ?

Exmaple of Process Study in Wavelet Space:

Energy-Conversion Cycle

(Yano et al. 2005, QJ):



Convective Kinetic Energy

Conversion Rate

Available Potential Energy

Compression:

Let $l = l(i, j)$ with $l = 0, \ldots, N-1$:

$$\hat{f}_l^c = \begin{cases} \hat{f}_l, & |\hat{f}_l| > \alpha f_c \\ 0, & |\hat{f}_l| \leq \alpha f_c \end{cases}$$

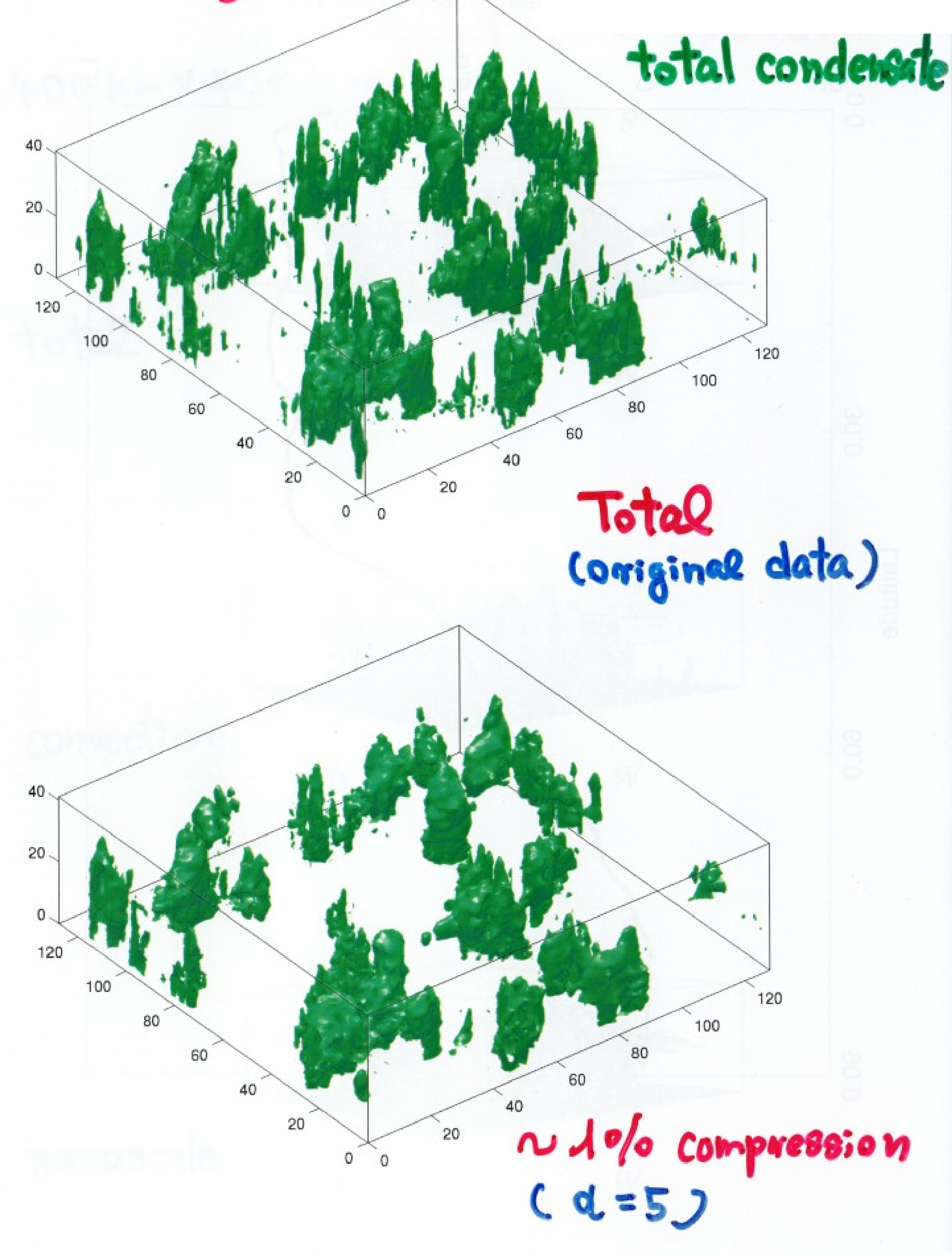
where

$$f_c = (\sum_{l=1}^{N-1} \hat{f}_l^2)^{1/2}$$

Compressed Representation:

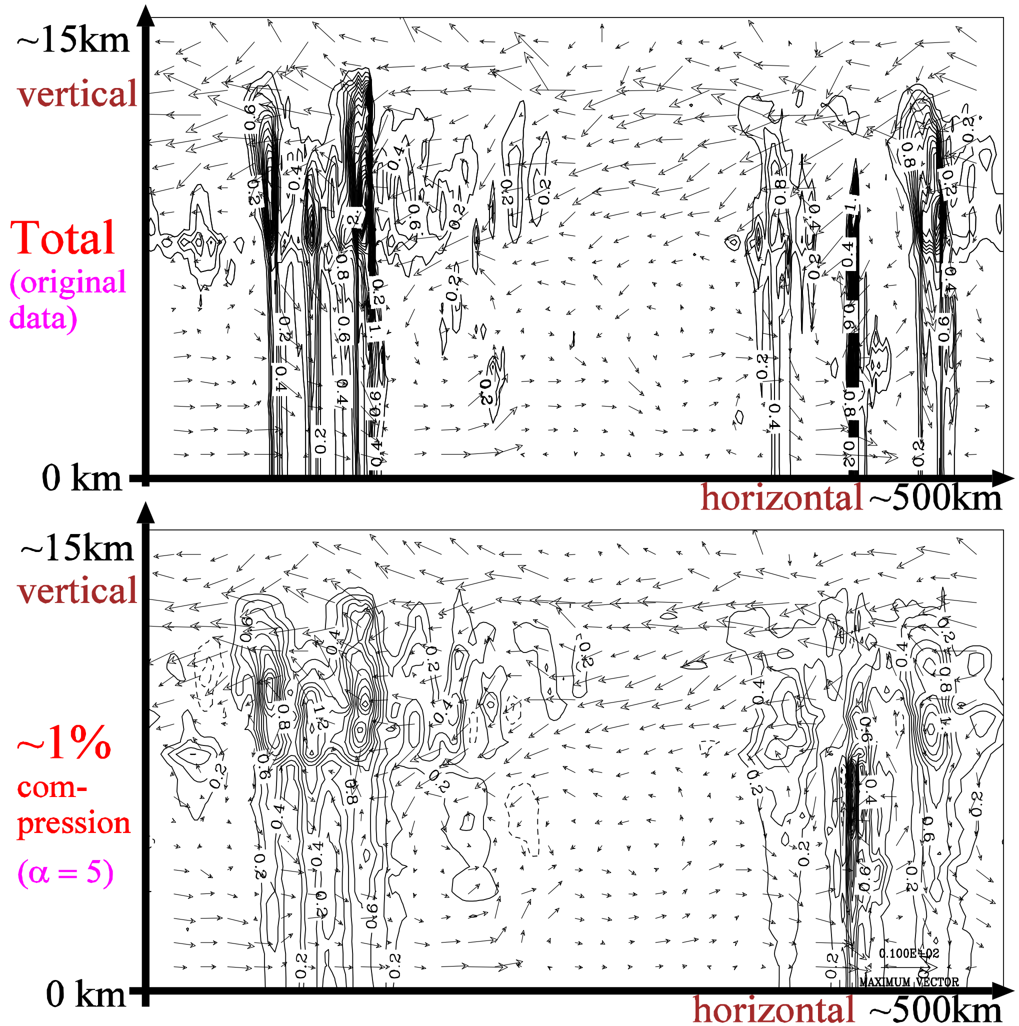
$$f^c(x) = \sum_{l=0}^{N-1} \hat{f}_k^c \phi_k(x)$$

CRM Simulation:
TOGA-COARE:
512 x 512 km^2
x 18km



Wavelet Compression of 3D CRM Data (TOGA period)

Example of a vertical-section total condensate + winds



Decomposition:

Convective and Mesoscale Components:

Threshold Variable:

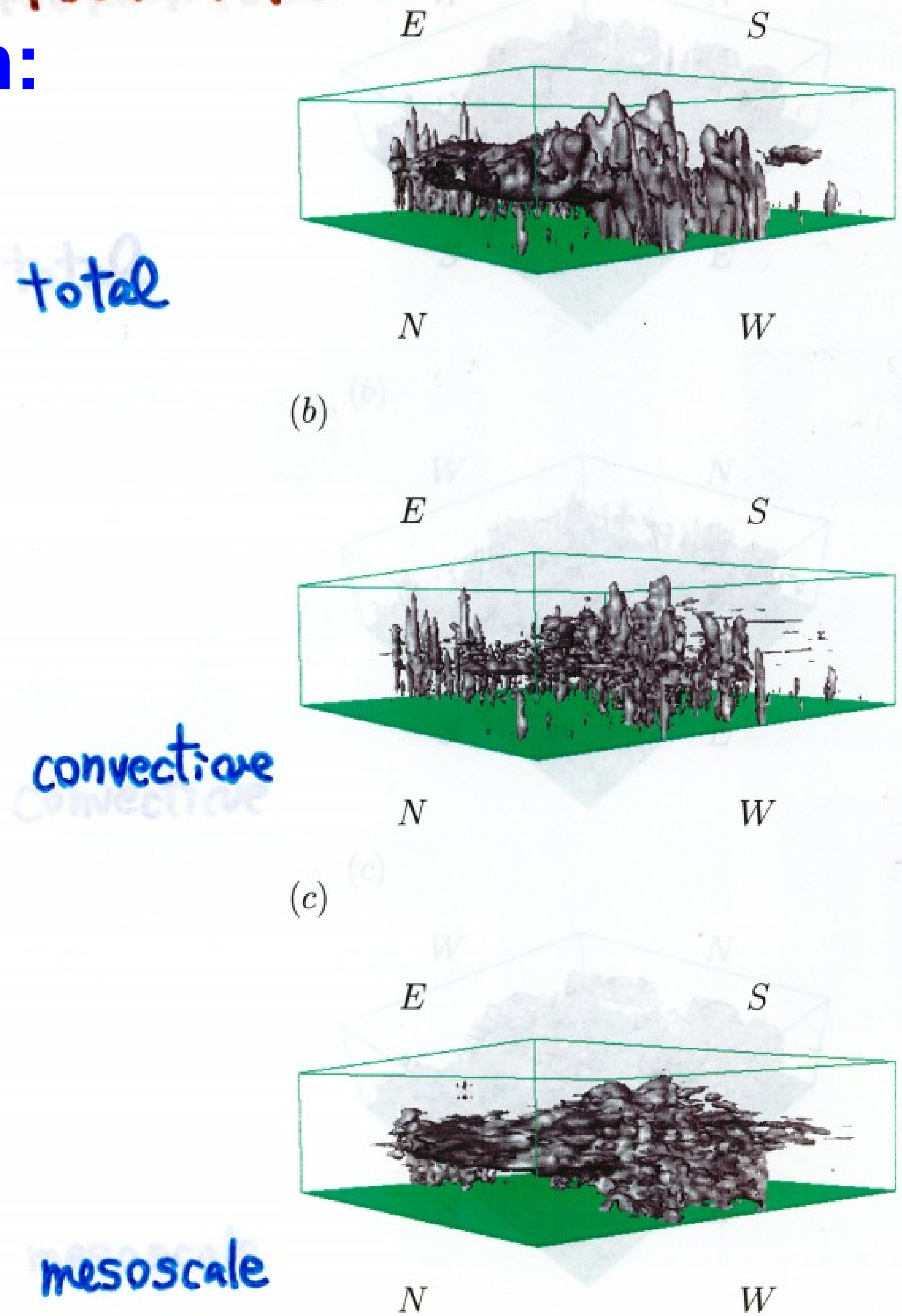
$$\phi = d|\mathbf{v}_H|/dz$$

CRM Simulation:

GATE:

Squall-Line System

400 x 400 km^2

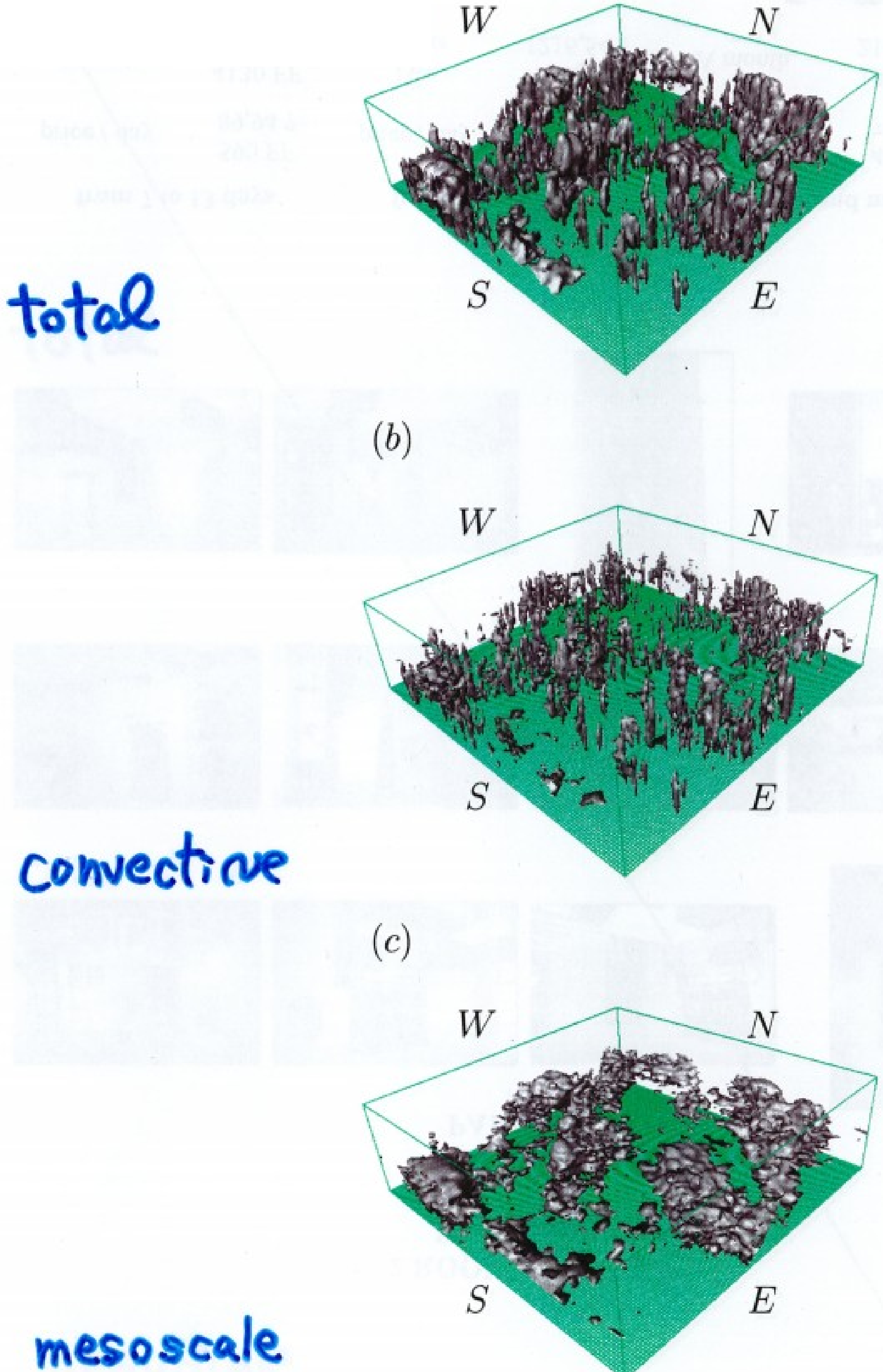


CRM Simulation:

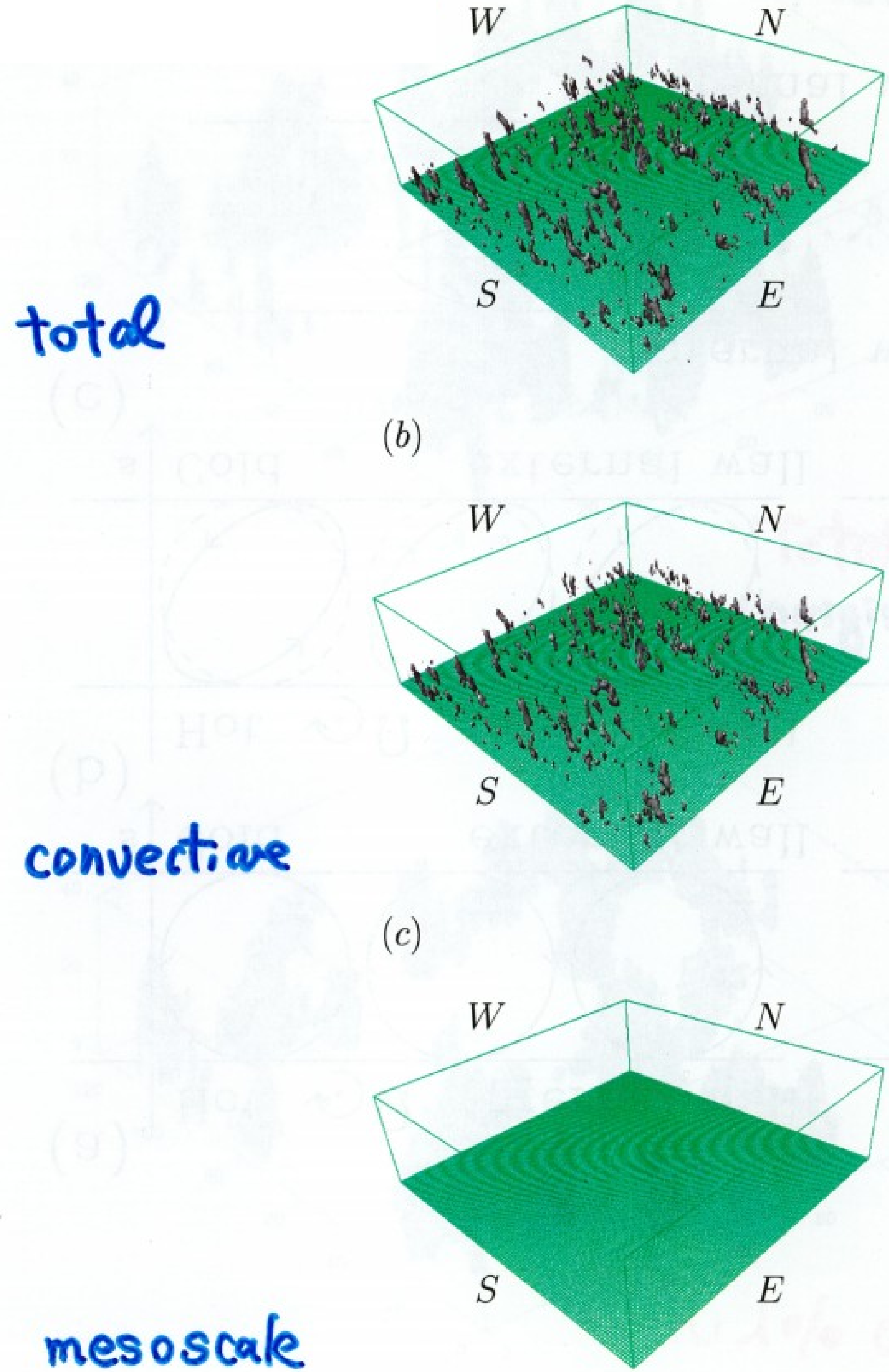
GATE:

Nonsquall-Line System

400 x 400 km^2



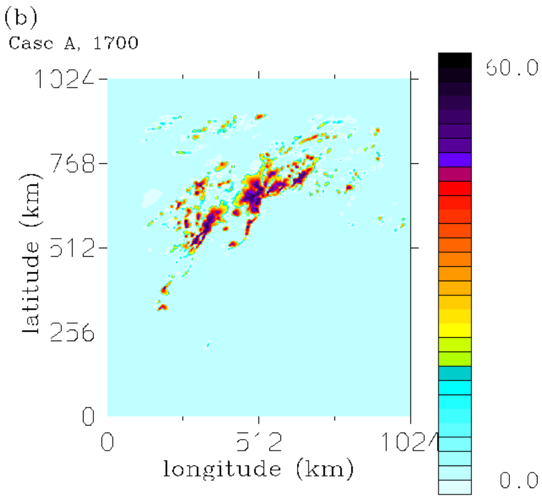
CRM Simulation:
GATE:
Scattered Convection
400 x 400 km^2



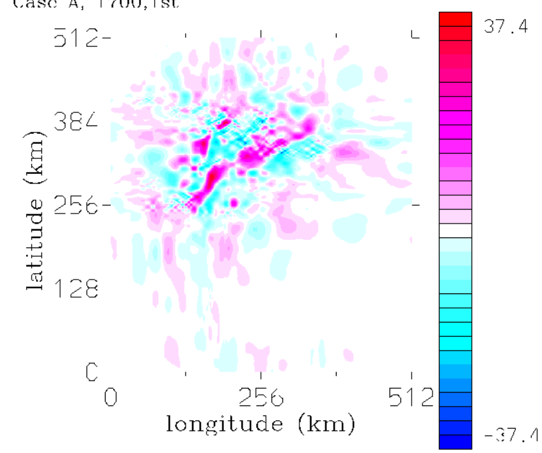
Extraction of Isolated Features:

Two-Dimensional Generalization:

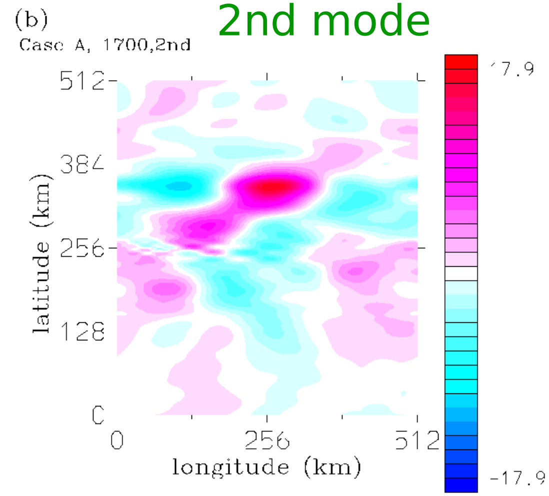
Conceptually Straightforward:



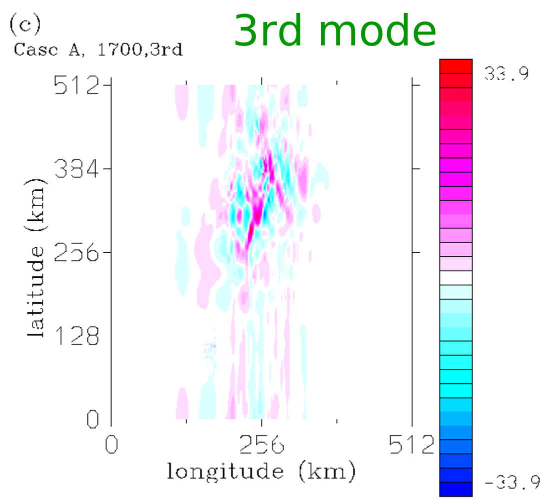
1st mode



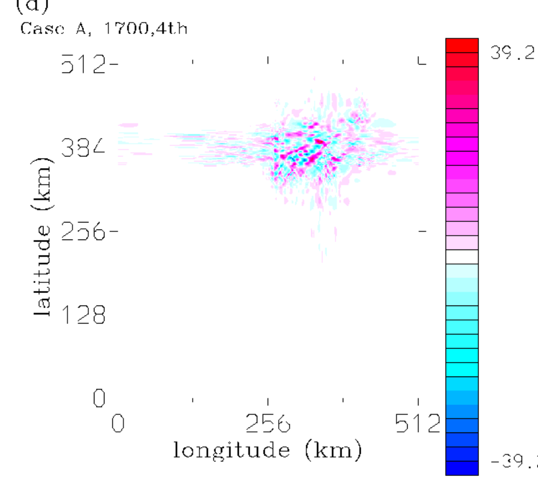
2nd mode



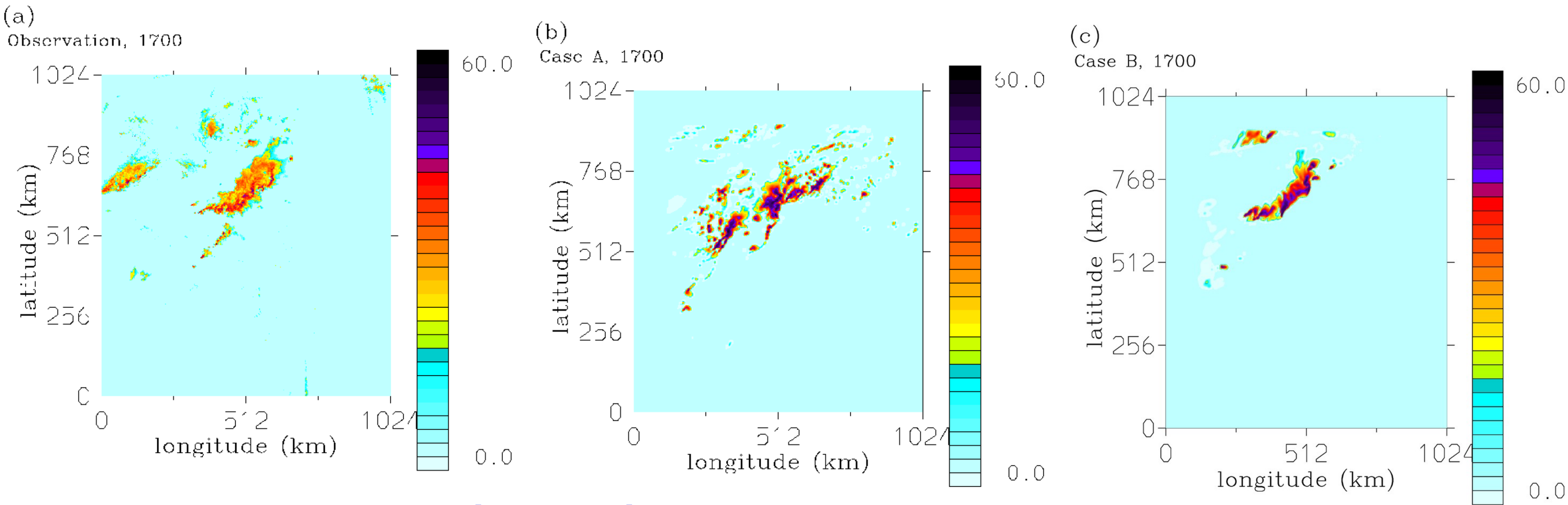
3rd mode



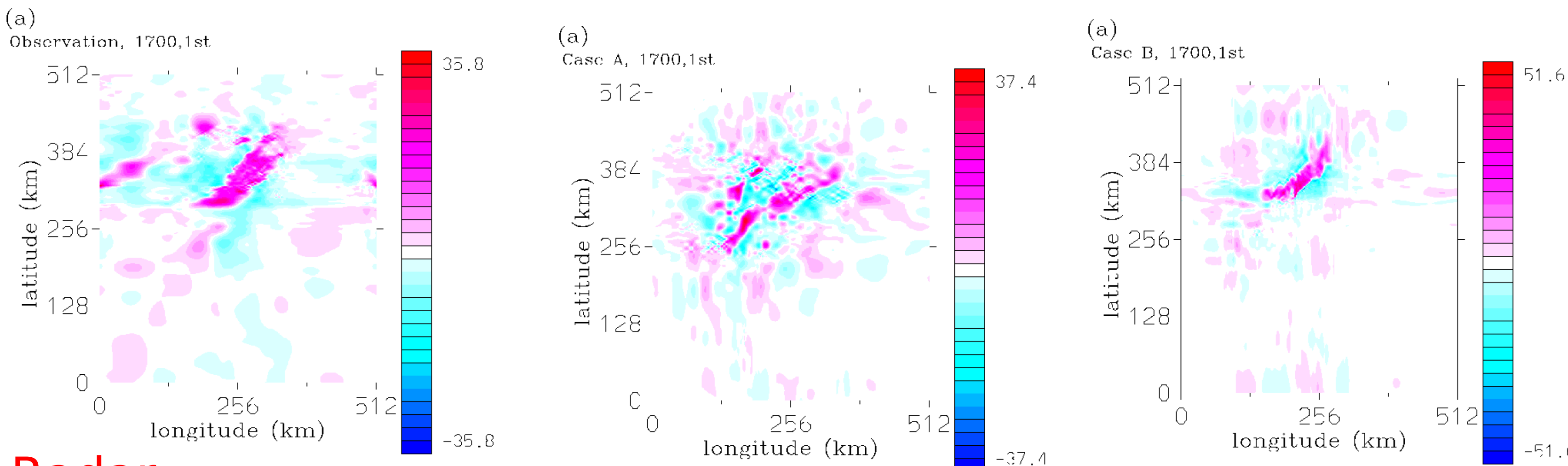
4th mode



Extraction of Isolated Features:



1st Extracted Modes:



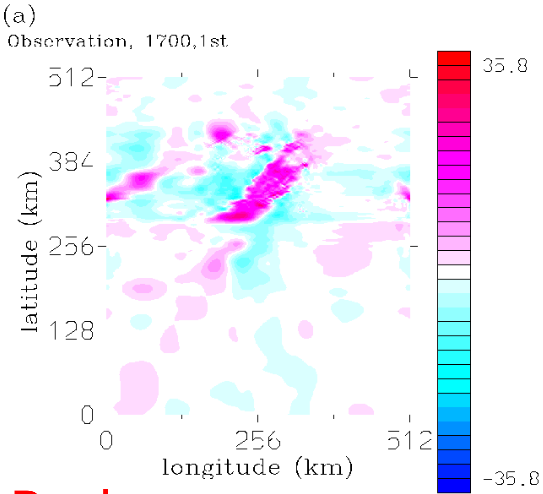
Radar Observation

Forecast A

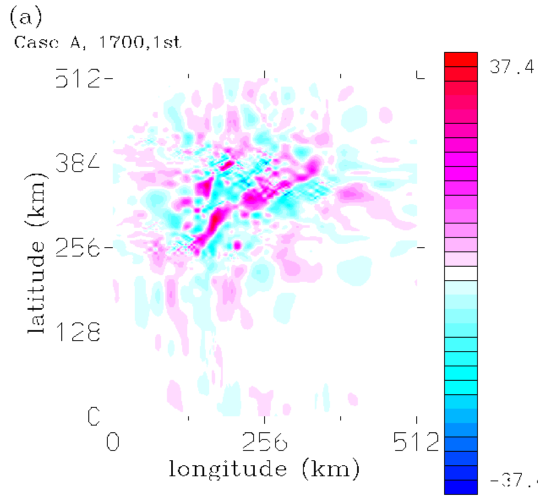
Forecast B

Extraction of Isolated Features:

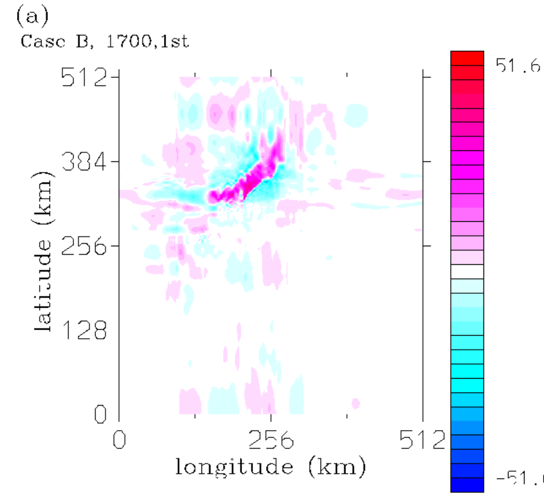
1st Extracted Modes:



Radar Observation

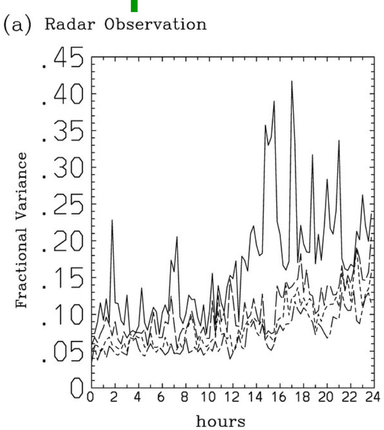


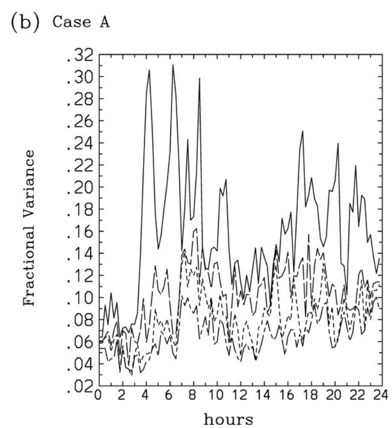
Forecast A

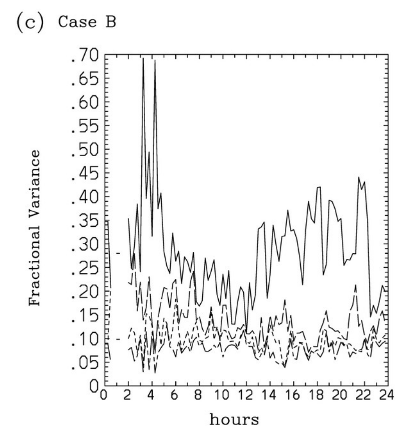


Forecast B

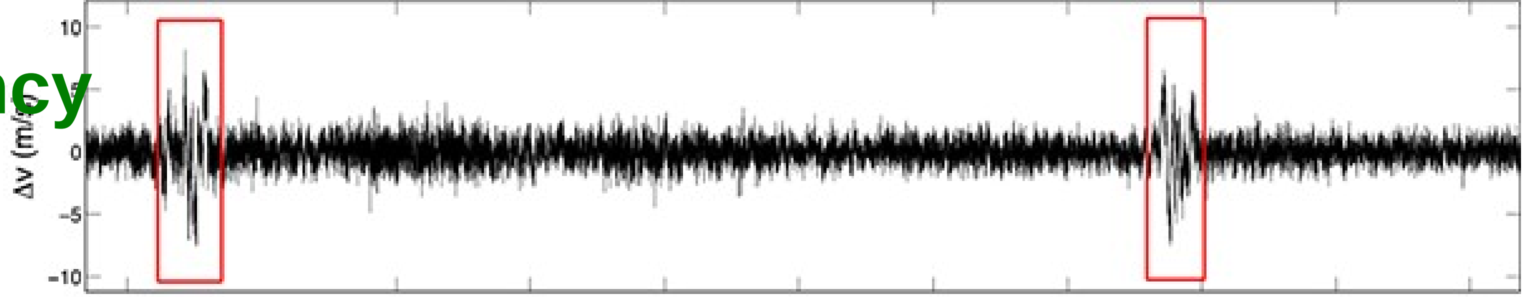
Explained Variance of First 4 Modes:

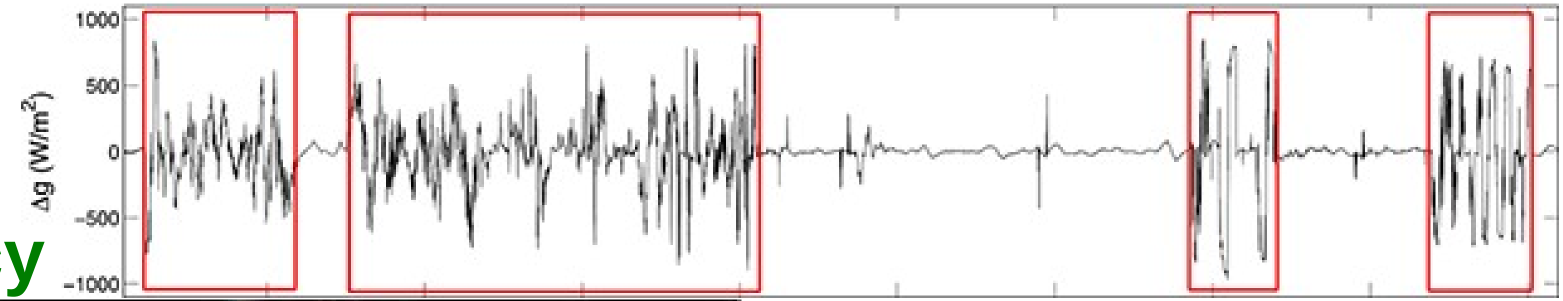






Intermittency





Coherency