Broadening of cloud droplet spectra through eddy hopping: Why did we all have it wrong?

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Plan:

1. Motivation

2. Two significant old papers

3. Embracing the idea and moving forward with initial research

4. Progress as computational resources increase

5. Understanding what is wrong with the approach

6. The way forward

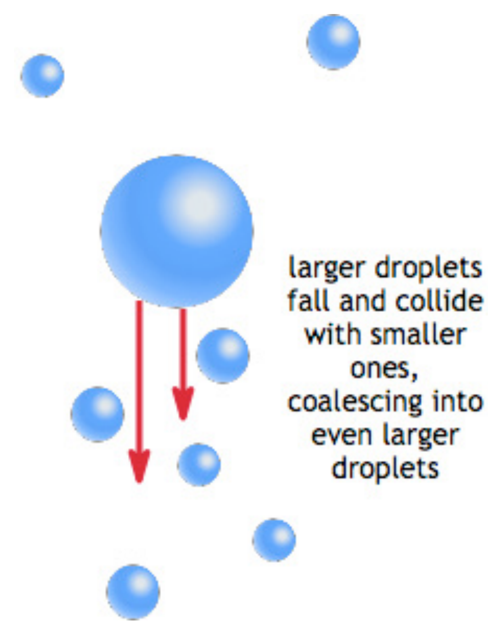
Width of the cloud droplet spectrum in warm clouds is an important parameter.

It affects transfer of solar radiation through a cloud and collision/coalescence that leads to rain formation…

Effective radius r_e:

$$r_e = \frac{\int_0^\infty \pi r^3 \cdot n(r)\, dr}{\int_0^\infty \pi r^2 \cdot n(r)\, dr}$$

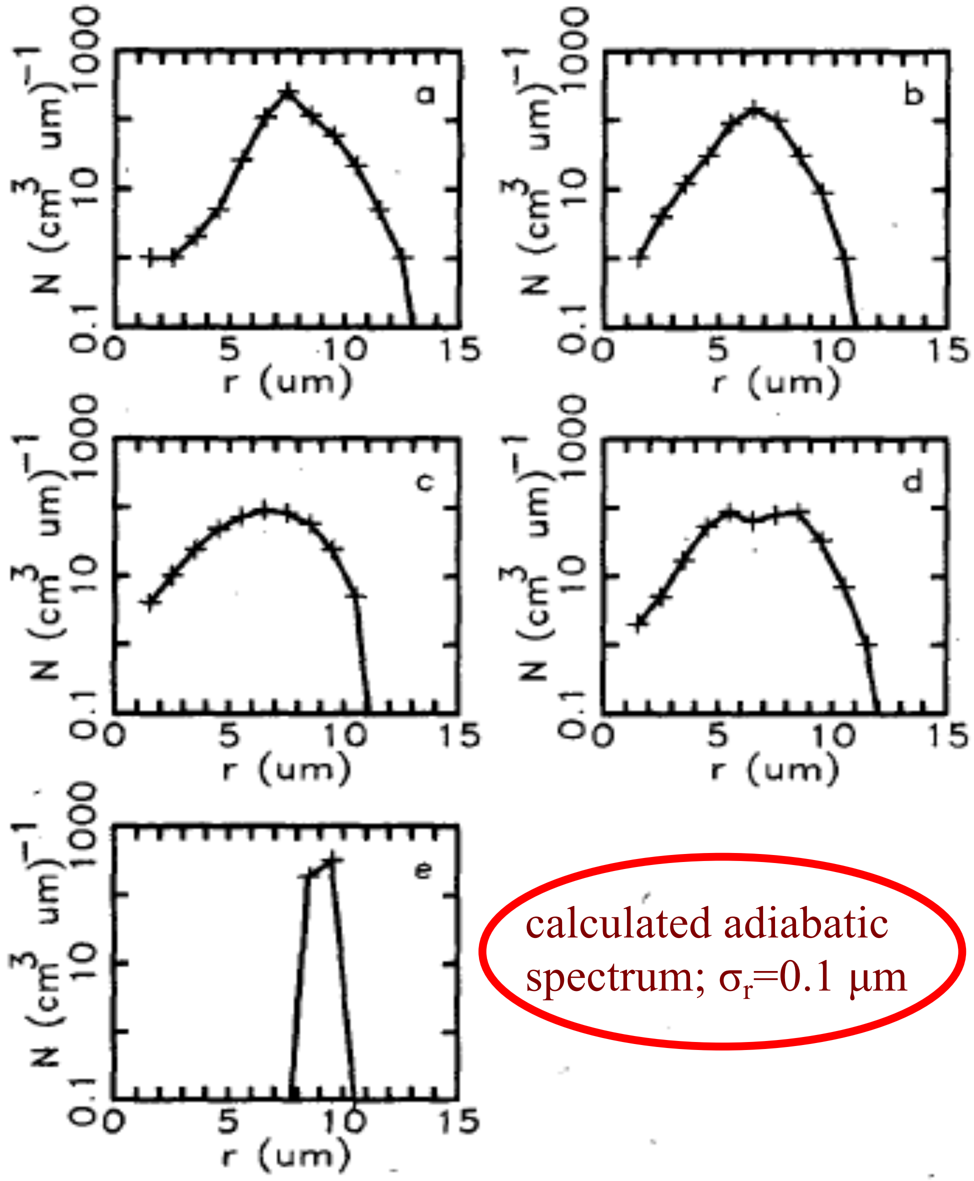
Gravitational droplet collisions:



Observed cloud droplet spectra in cumulus averaged over ~100 m (1 Hz, FSSP data):

observed,
adiabatic fraction
AF ≈ 1; σ_r=1.3 μm

observed, AF ≈ 0.8;
σ_r=1.3 μm



observed, AF ≈ 0.8;
σ_r=1.8 μm

observed, AF ≈ 1;
bimodal

calculated adiabatic
spectrum; σ_r=0.1 μm

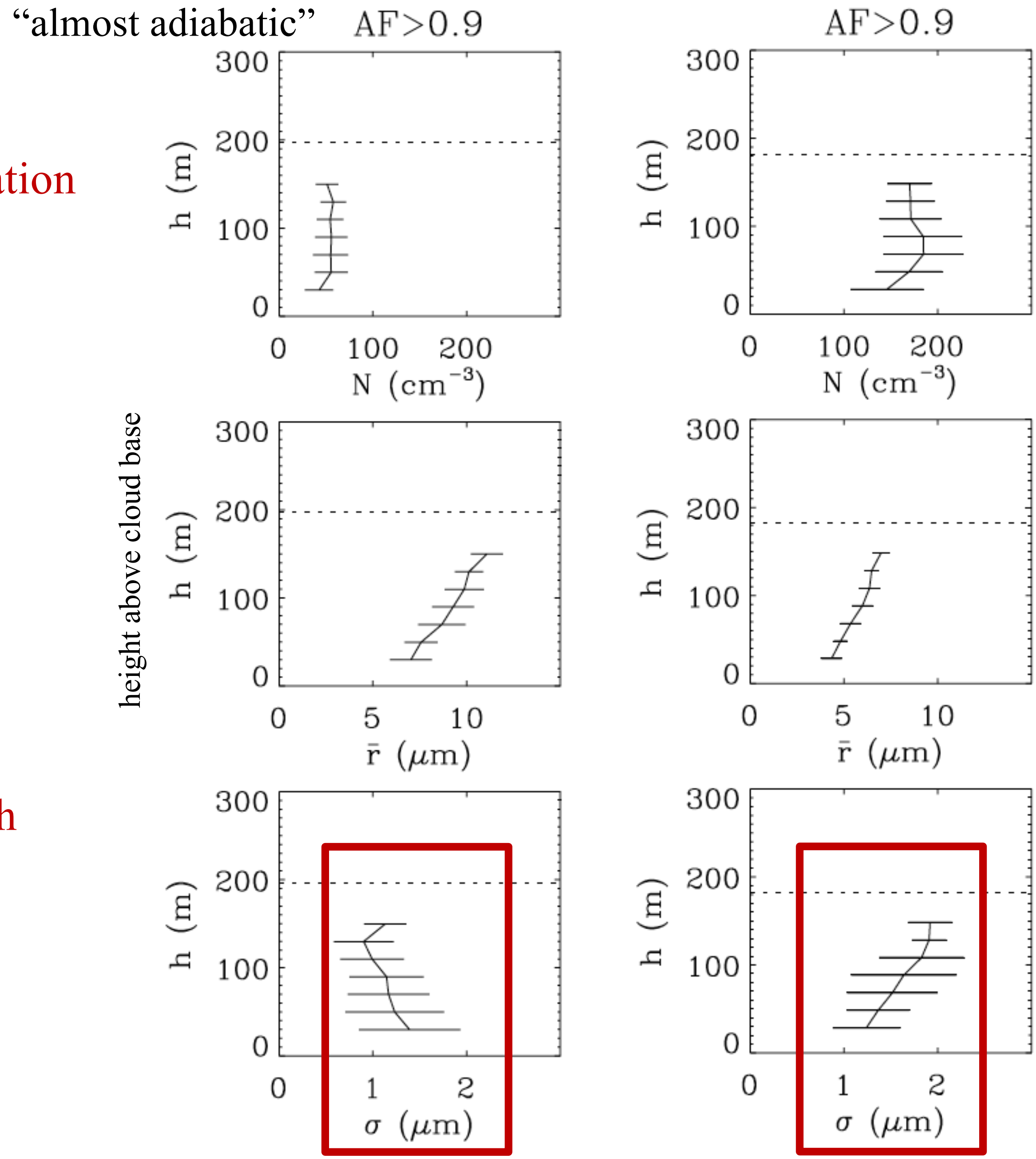
(Jensen et al. *JAS* 1985)

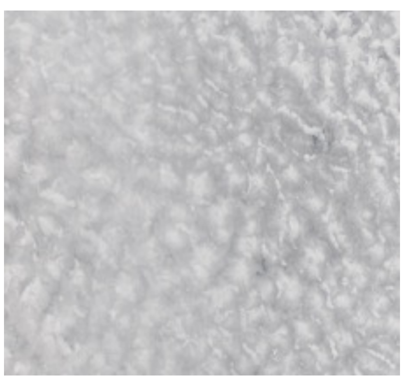
Observed cloud droplet spectra in stratocumulus averaged over ~10m (10 Hz, Fast FSSP):

droplet concentration

mean radius

spectral width





(Pawlowska et al. *GRL* 2006)

Paper #1, *JAS* 1989

Growth of Cloud Drops by Condensation: A Criticism of Currently Accepted Theory and a New Approach

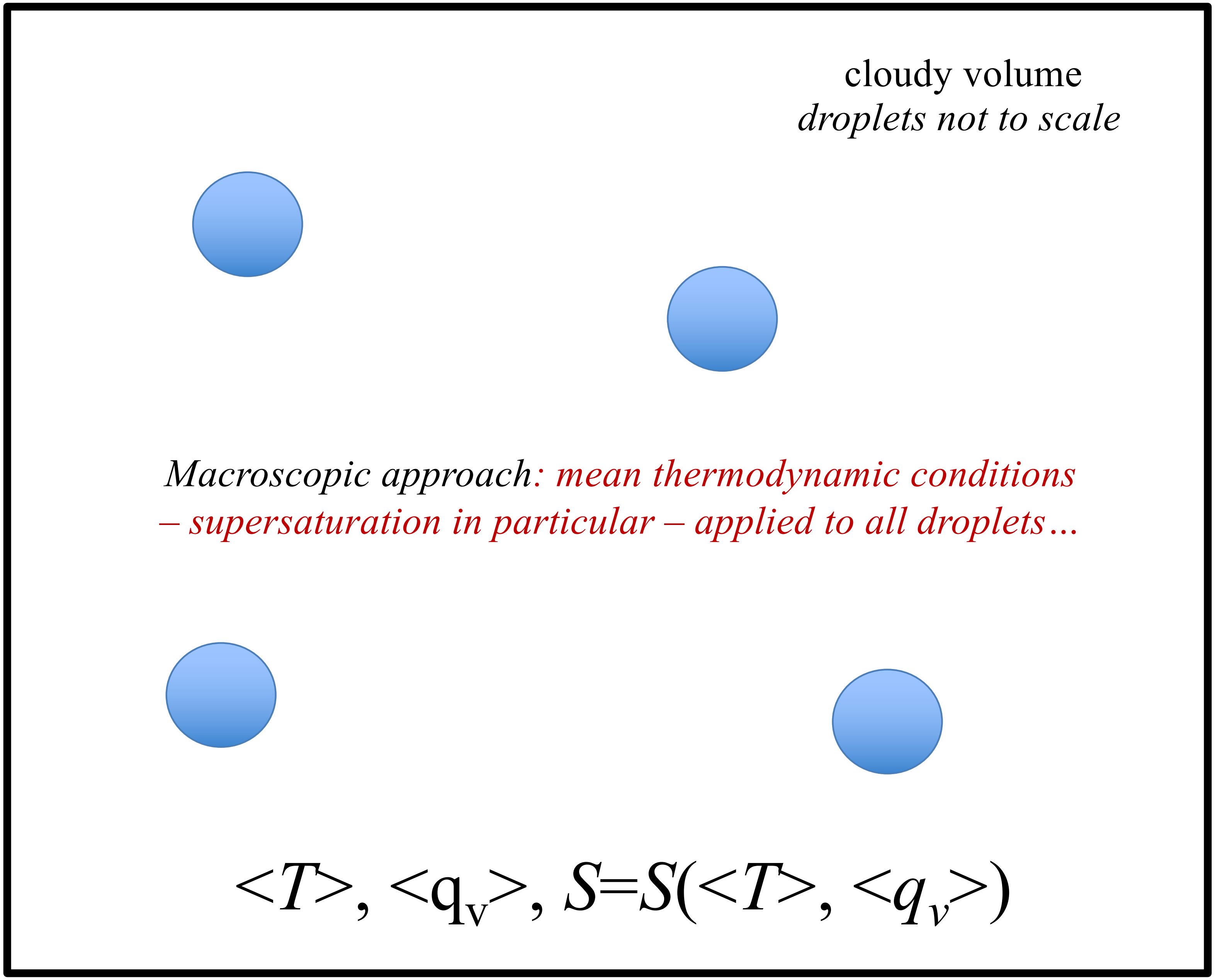
R. C. SRIVASTAVA

Laboratory for Atmospheric Probing, Department of the Geophysical Sciences, The University of Chicago, Chicago, Illinoi:

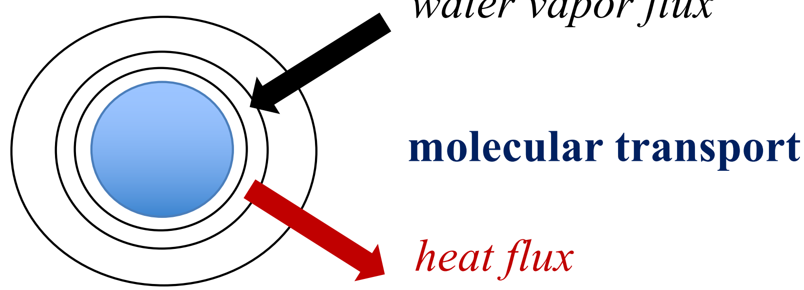
(Manuscript received 10 March 1988, in final form 10 August 1988)

ABSTRACT

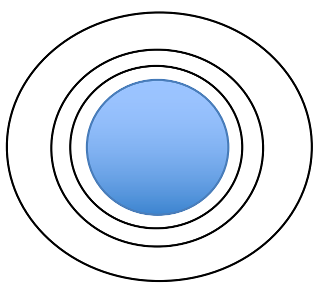
The currently accepted theory of the growth of cloud drops by condensation employs an equation for the rate of increase of drop mass and an equation for the supersaturation. The latter equation gives the average supersaturation over a large volume, or the macroscopic supersaturation. Use of this supersaturation in the equation for the growth of cloud drops is criticized. In a first approach at a microscopic theory, the average supersaturation over the volume occupied by a drop, called the microscopic supersaturation, is used to calculate the growth of the drop. The microscopic supersaturation can differ from drop to drop due to randomness in their spatial distribution and is affected differently by fluctuations of vertical air velocity than the macroscopic supersaturation. In a second approach at a microscopic theory, the diffusion equations for water vapor and heat, together with appropriate boundary conditions, are solved for an assemblage of drops. It is shown again that a microscopic supersaturation may be defined for calculating drop growth and that this supersaturation can also differ from drop to drop and responds differently to vertical air velocity fluctuations than the macroscopic supersaturation. In the microscopic approaches both the random distribution of drops and vertical air velocity fluctuations can affect the growth of cloud drops by condensation; this is in contrast to conclusions drawn from the currently accepted theory. Estimates of the variance of the microscopic supersaturation are given. It is shown that diffusive interactions between drops in a population can be neglected if the dimensionless parameter $[(l_*/r_0)\sqrt{(D\tau)}]$, where l_* is the volume fraction of the drops, r_0 is a typical drop radius, D is the diffusivity, and τ is the age of the diffusion process, is very small compared to unity.

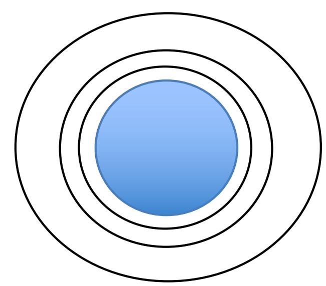

cloudy volume
droplets not to scale
Macroscopic approach: mean thermodynamic conditions
– supersaturation in particular – applied to all droplets…
$<T>, <q_v>, S=S(<T>, <q_v>)$

droplets not to scale



Microscopic approach: each droplet experiences different mean thermodynamic conditions – supersaturation in particular…





Paper #2, JAS 1989

Effects of Variable Droplet Growth Histories on Droplet Size Distributions. Part I: Theory

WILLIAM A. COOPER

National Center for Atmospheric Research, Boulder, Colorado*

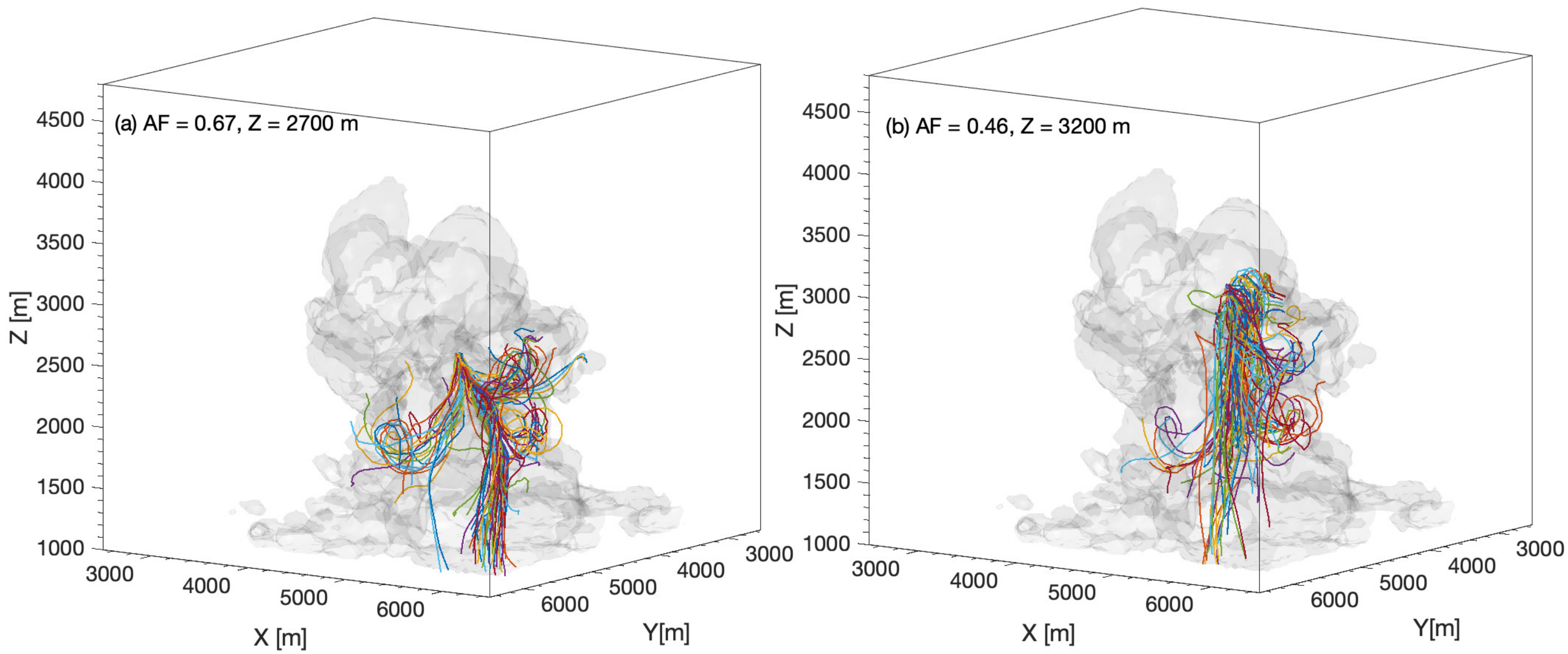
(Manuscript received 4 April 1988, in final form 14 November 1988)

ABSTRACT

A theoretical framework is developed that permits estimation of the effects of fluctuating supersaturation on the development of cloud droplet size spectra. The studies focus on the role of turbulent fluctuations in vertical wind and in the microphysical environments in which droplets grow, and represent the effects of droplets mixing together that have encountered different trajectories through the cloud. It is contended that the effects can be analyzed in terms of two contributions to the variance in supersaturation history, one dependent on the average microphysical environment (specifically, integral radius) of the near environment in which a droplet grows, and the other dependent on the correlation between the integral radius and the updraft along the droplet trajectory. Variations in the possible trajectories that all end at a given point (and so form the droplet spectrum there) are used to estimate the possible widths of droplet spectra, and methods of testing these predictions using experimental data are also proposed. Possible broadening effects due to fluctuations in the updraft at cloud base are also analyzed. It is suggested that simple turbulent motions in a stochastically varying cloud may provide significant broadening of the cloud droplet spectrum if those motions are accompanied by a variable microphysical structure produced by dry-air entrainment.

"Eddy hopping" term was first used in Grabowski and Wang, *ARFM* 2013…

Chandrakar et al. *JAS* 2021



Trajectories of selected “super-droplets” arriving at a given location (different in right and left panels) above the cloud base. Example of “eddy hopping” at the cloud scale…

Can “eddy hopping” in a turbulent cloud lead to increase of spectral width even if there is no cloud dilution (i.e., AF close to 1)?

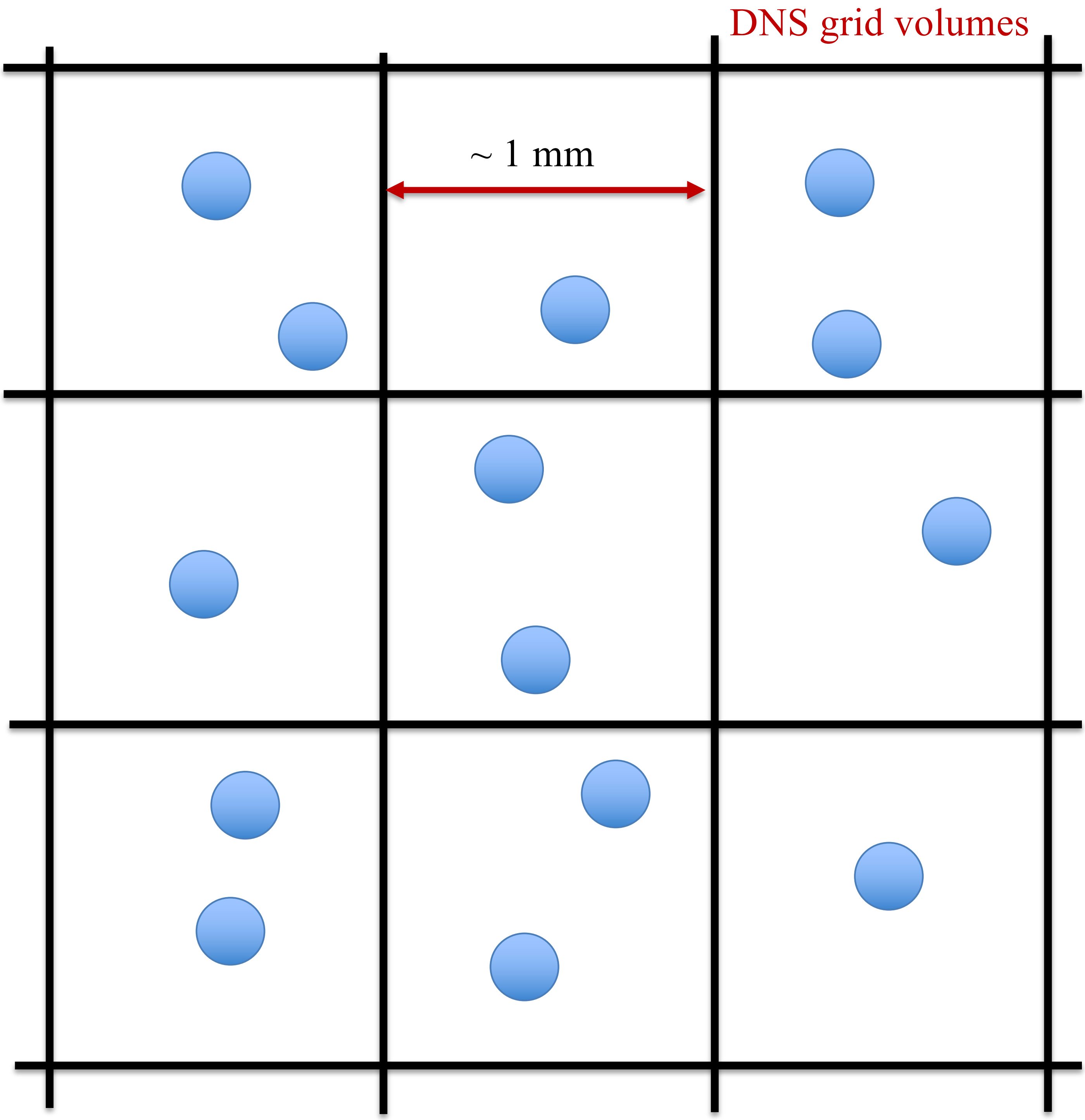
“Eddy hopping” term was first used in Grabowski and Wang, *ARFM* 2013…

Can small-scale turbulence explain the width of the droplet spectra in undiluted cloudy volumes?

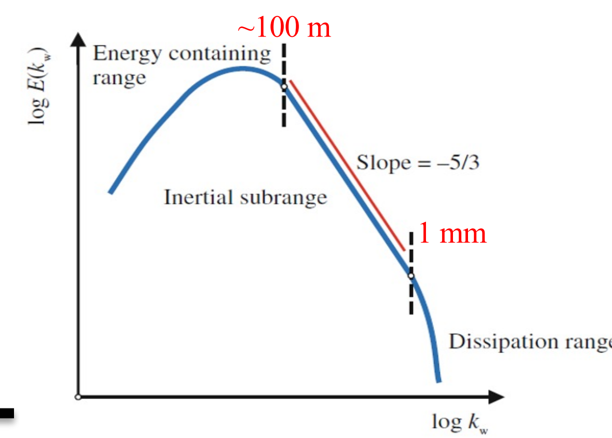
If we use direct numerical simulation (DNS) to simulate small-scale homogeneous isotropic turbulence with cloud droplets, then (in reference to the two papers):

- droplets follow different trajectories through a turbulent cloudy volume (Cooper)

- each droplet grows in conditions of its immediate environment (Srivastava)



Droplets not to scale:
10 micron = 0.01 mm !



Can small-scale turbulence explain the width of the droplet spectra in undiluted cloudy volumes?

Microscopic Approach to Cloud Droplet Growth by Condensation. Part I: Model Description and Results without Turbulence

JAS 2001

P. A. VAILLANCOURT* AND M. K. YAU

Department of Atmospheric and Oceanic Sciences, McGill University, Montreal, Quebec, Canada

W. W. GRABOWSKI

National Center for Atmospheric Research, Boulder, Colorado

Microscopic Approach to Cloud Droplet Growth by Condensation. Part II: Turbulence, Clustering, and Condensational Growth

P. A. VAILLANCOURT, M. K. YAU, AND P. BARTELLO

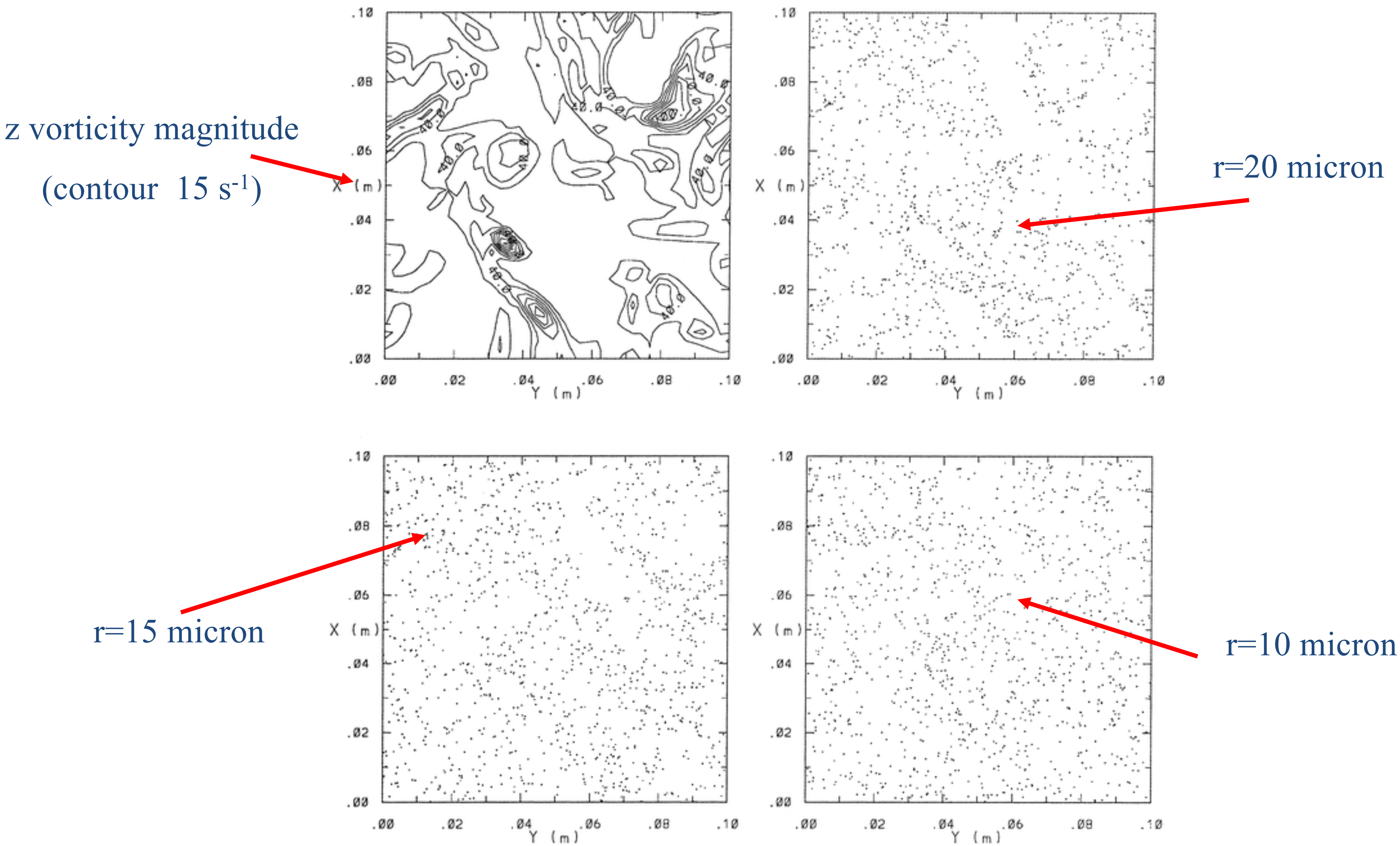
McGill University, Montréal, Québec, Canada

JAS 2002

W. W. GRABOWSKI

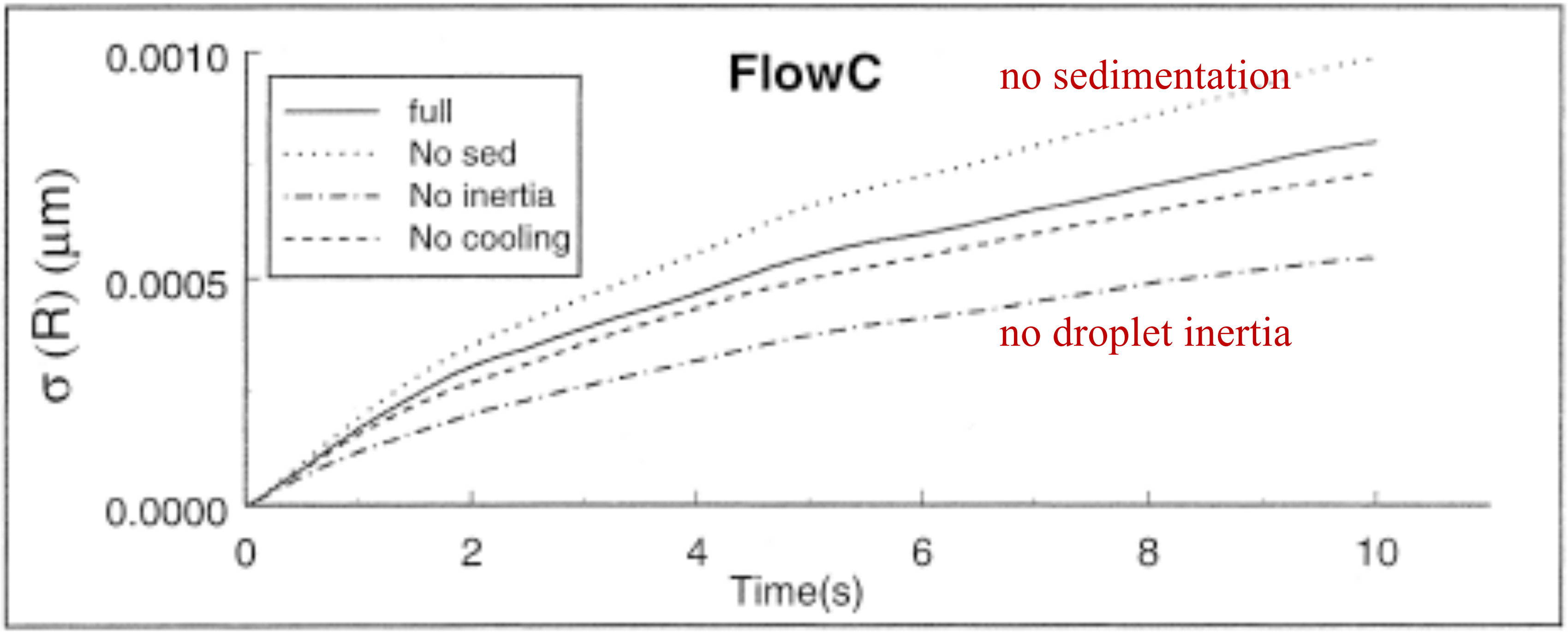
National Center for Atmospheric Research, Boulder, Colorado

Direct numerical simulations (DNS) of homogeneous isotropic turbulence with cloud droplets *growing by the diffusion of water vapor* for conditions relevant to cloud physics (ε=160 cm^2s^{-3})



Note the domain size: about 1 liter…

Vaillancourt et al. *JAS* 2002



Main conclusion: centimeter-scale turbulence has a small effect for the diffusional growth…

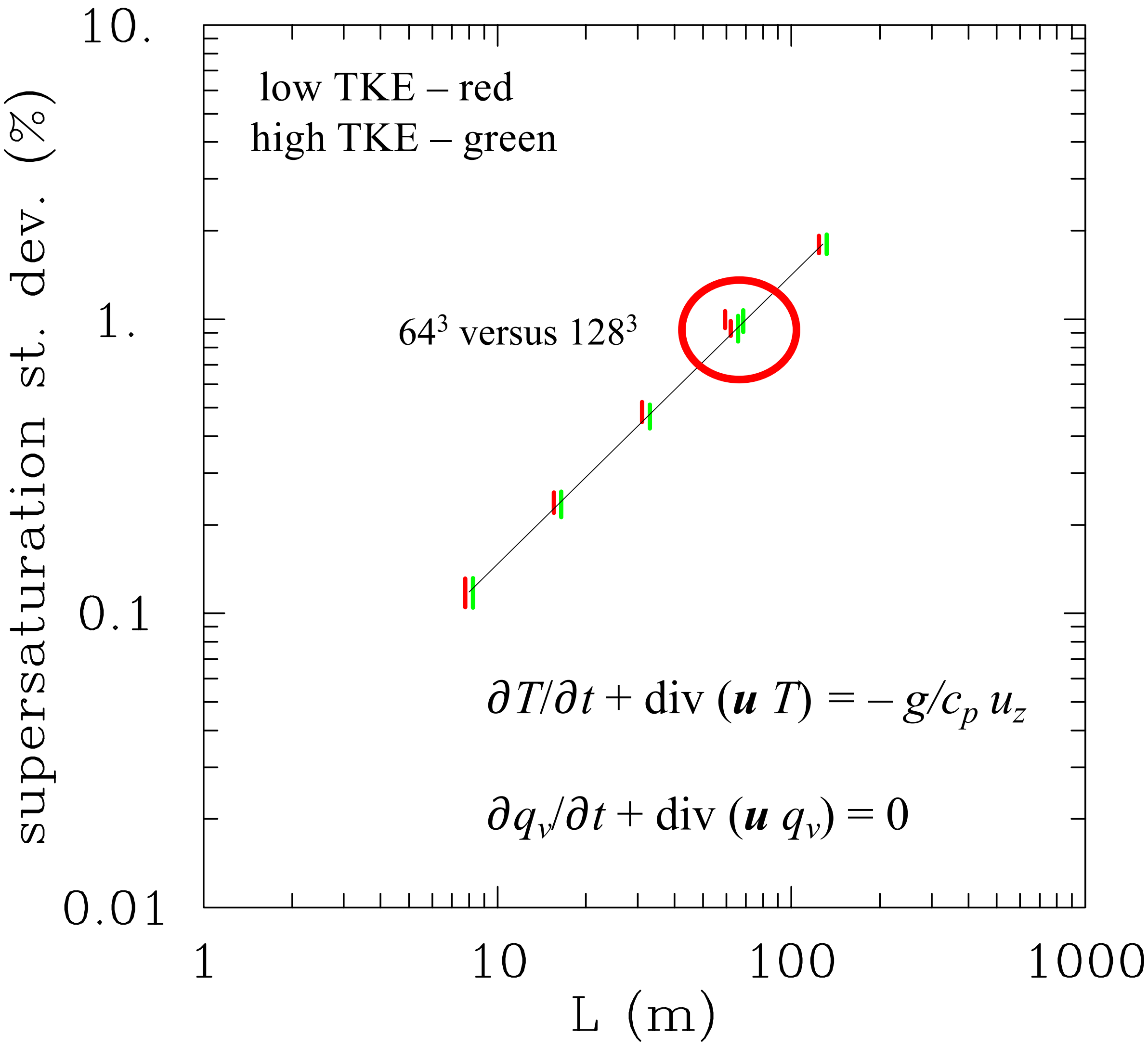
Vaillancourt et al. *JAS* 2002

Why does the domain size matters?

What to do to have something like DNS but with a larger domain?

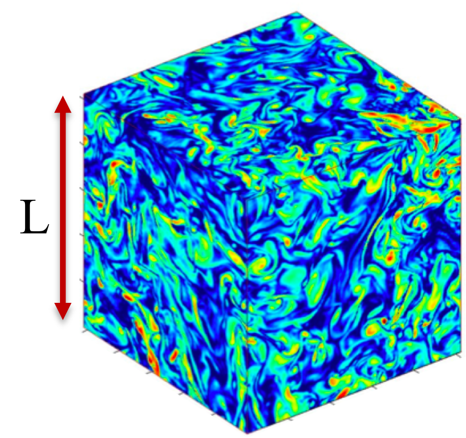
Simulations without droplets and with no mean ascent:

"Implicit LES" simulations of homogeneous isotropic turbulence…



low TKE: eddy dissipation 10 $cm^2 s^{-3}$

high TKE: eddy dissipation 1000 $cm^2 s^{-3}$



- Domain size affects magnitude of supersaturation fluctuations.
- TKE dissipation has an insignificant impact.
- Reynolds number (i.e., 64^3 versus 128^3 simulations) has an insignificant impact.

Grabowski et al. (*JAS* 2022a)

In turbulent simulations, often the key issue is about the Reynolds number *Re*. *Re* depends on the resolved range of scales, from the TKE input scale L to the TKE dissipation scale η.

$$\frac{L}{\eta} \sim Re^{3/4}$$

This is only marginally relevant to the problem considered here as the largest scales dominate the supersaturation fluctuations…

Cloud Droplet Growth by Condensation in Homogeneous Isotropic Turbulence

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AGNESE SEMINARA

School of Engineering and Applied Sciences, Harvard University, Cambridge, Massachusetts

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Department of Physics, and Department of Mathematics and Computer Science, Eindhoven University of Technology, Eindhoven, Netherlands, and Istituto per le Applicazioni del Calcolo, Rome, and Istituto Nazionale di Fisica Nucleare, Ferrara, Italy

Label	N_{drops}/V (cm^{-3})	LWC (g m^{-3})	τ_s (s)	R (μm)	St
Series 1	130	1.2	2.5	13	3.5×10^{-2}
Series 2	130	0.07	7	5	5×10^{-3}

Label	N^3	L (cm)	T_L (s)	Re_λ	ε (m^2 s^{-3})	σ_s^0 (%)	v_{rms} (m s^{-1})	N_{drops} ($\times 10^5$)
A	64^3	9	2.0	40	10^{-3}	1.5×10^{-3}	4×10^{-2}	0.93
B	128^3	18	3.5	65	9.0×10^{-4}	3.4×10^{-3}	5.0×10^{-2}	8.2
C	256^3	38	5.5	105	10^{-3}	6.1×10^{-3}	7.0×10^{-2}	71.2
D	512^3	70	7.6	185	1.1×10^{-3}	1.2×10^{-2}	1.0×10^{-1}	320

Lanotte et al. *JAS* 2009

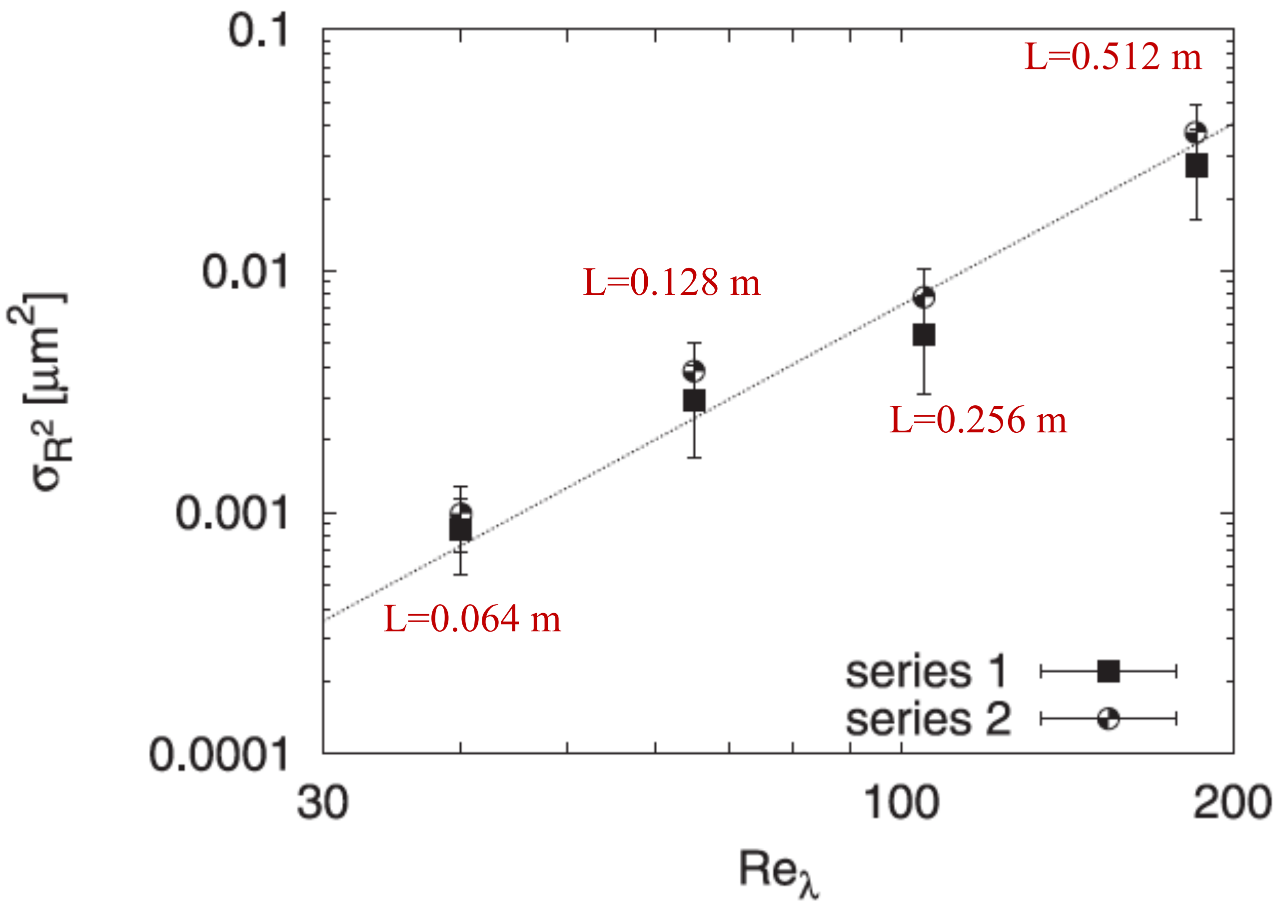

FIG. 3. Log–log plot of the spreading of droplet size distribution $\sigma_{R^2}(T_L)$ for the square radius R^2, measured after one large-scale eddy turnover time T_L as a function of the Reynolds number Re_λ.

Non-turbulent parcel: constant R^2 standard deviation; $\sigma_{R^2} = \text{const}$

$$\frac{dR^2}{dt} \sim S$$

Lanotte et al. *JAS* 2009

Theory and simulations with droplets and with no mean ascent:

initially monodisperse droplets with radius of 13 microns

PRL **115,** 184501 (2015) PHYSICAL REVIEW LETTERS week ending 30 OCTOBER 2015

Continuous Growth of Droplet Size Variance due to Condensation in Turbulent Clouds

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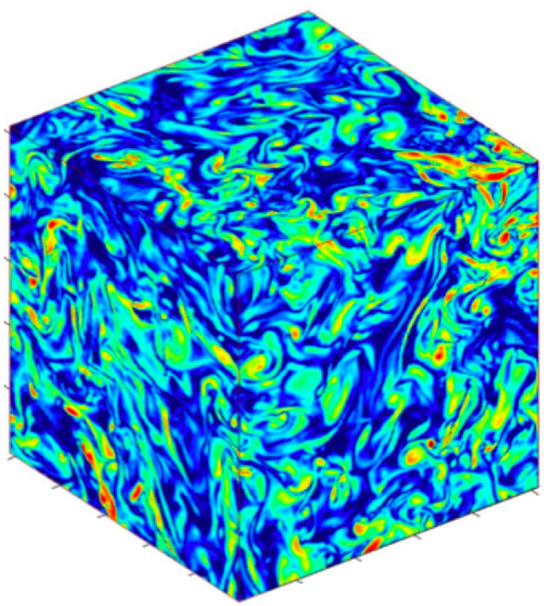
(Received 15 December 2014; revised manuscript received 21 May 2015; published 29 October 2015)



$$\sigma_{R^2} \sim t^{1/2}$$

no mean ascent: $\langle R \rangle = \text{const}$

no turbulence: $\sigma_{R^2} = \text{const}$

$$\frac{d < (R^{2'})^2 >}{dt} = \frac{d\sigma^2_{R^2}}{dt} = 4A_3 < s' R^{2'} >$$

When phase relaxation time is small compared to the turbulence time scale:

$$\langle \tau_s \rangle \ll T_0$$

$$T_0 \sim L / E^{1/2}$$

Sardina et al. (*PRL* 2015)

then $\sigma_{R^2} \sim t^{1/2}$.

$\langle \tau_s \rangle \sim 1$ sec

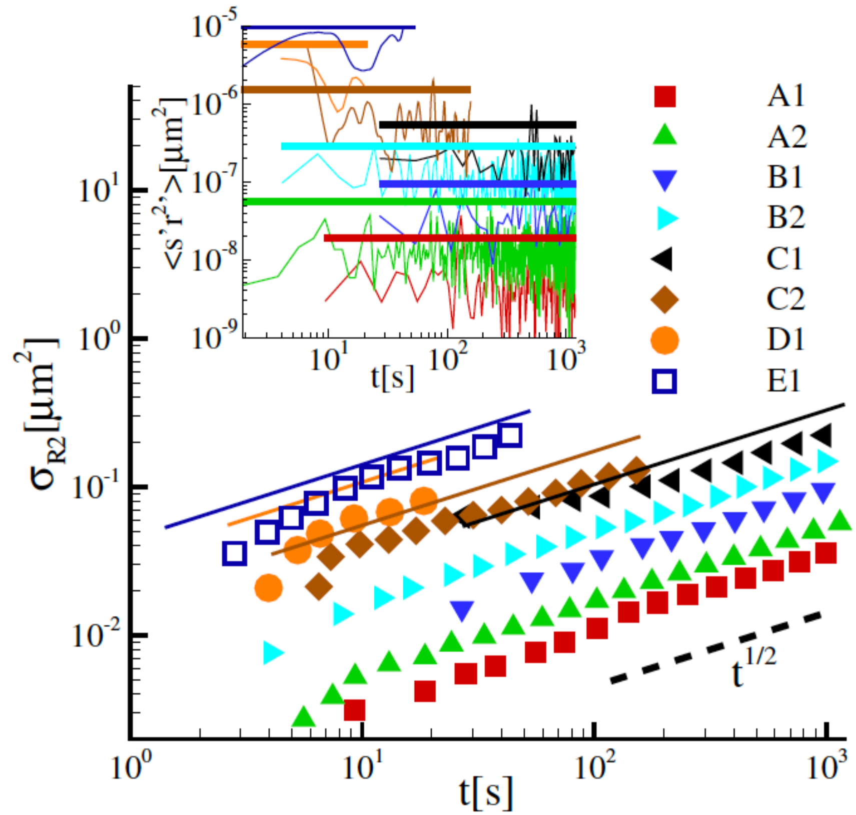
TABLE 1. Time constant characterizing supersaturation. (Values of $\tau = 1/(a_2 I)$ s for $p = 771$ mb, $T = 4.3°C$)

Radius (μm)	Droplet concentration (cm^{-3}) 100	300	500	1000
2	14.1	4.7	2.8	1.4
3	8.7	2.9	1.7	0.87
5	4.9	1.6	0.98	0.49
10	2.3	0.77	0.46	0.23

Politovich and Cooper JAS 1988

TABLE I. Parameters of the simulations. The resolution N, the cloud size L_{box}, the root mean square of the turbulent velocity fluctuations v_{rms}, and $T_L = L_{\text{box}}/v_{\text{rms}}$ an approximation of the large turbulent scales. T_0 indicates the integral time $T_0 = (\pi/2v_{\text{rms}}^3)\int[E(k)/k]dk$ with k the wave number and $E(k)$ the turbulent kinetic energy spectra [29]. The total number of droplets is indicated by N_d.

Label	N^3	L_{box} [m]	v_{rms} [m/s]	T_L [s]	T_0 [s]	Re_λ	N_d
DNS $A1/2$	64^3	0.08	0.035	2.3	0.64	45	6×10^4
DNS $B1/2$	128^3	0.2	0.05	4	0.95	95	9.8×10^5
DNS $C1/2$	256^3	0.4	0.066	6	1.5	150	9×10^6
DNS D1	1024^3	1.5	0.11	14	3	390	4.4×10^8
DNS E1	2048^3	3	0.12	30	4	600	$3. \times 10^9$
LES F1	512^3	100	0.7	142	33	5000	1.3×10^{14}



Sardina et al. (*PRL* 2015)

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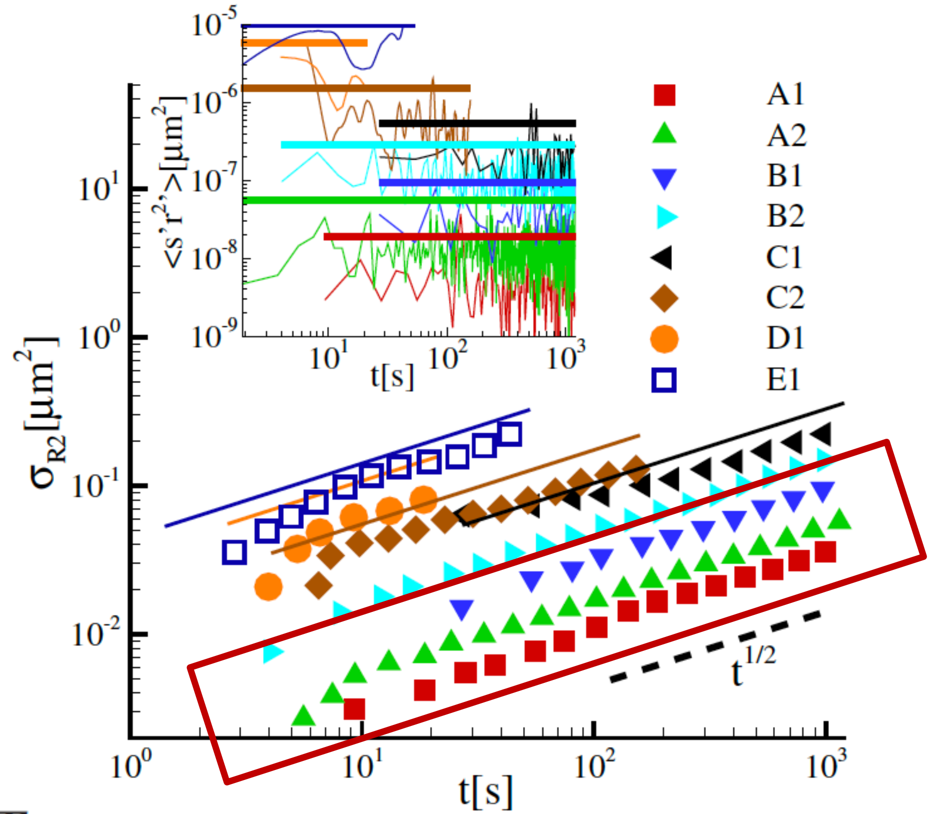

$$\langle \tau_s \rangle \ll T_0$$

Why the $t^{1/2}$ scaling still applies when this condition is not valid?

https://doi.org/10.5194/acp-2020-159
This is just a preview and not the published preprint.

Atmospheric Chemistry and Physics Discussions
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Diffusional growth of cloud droplets in homogeneous isotropic turbulence: DNS, scaled-up DNS, and stochastic model

Lois Thomas[1,2], Wojciech W. Grabowski[3], and Bipin Kumar[1]
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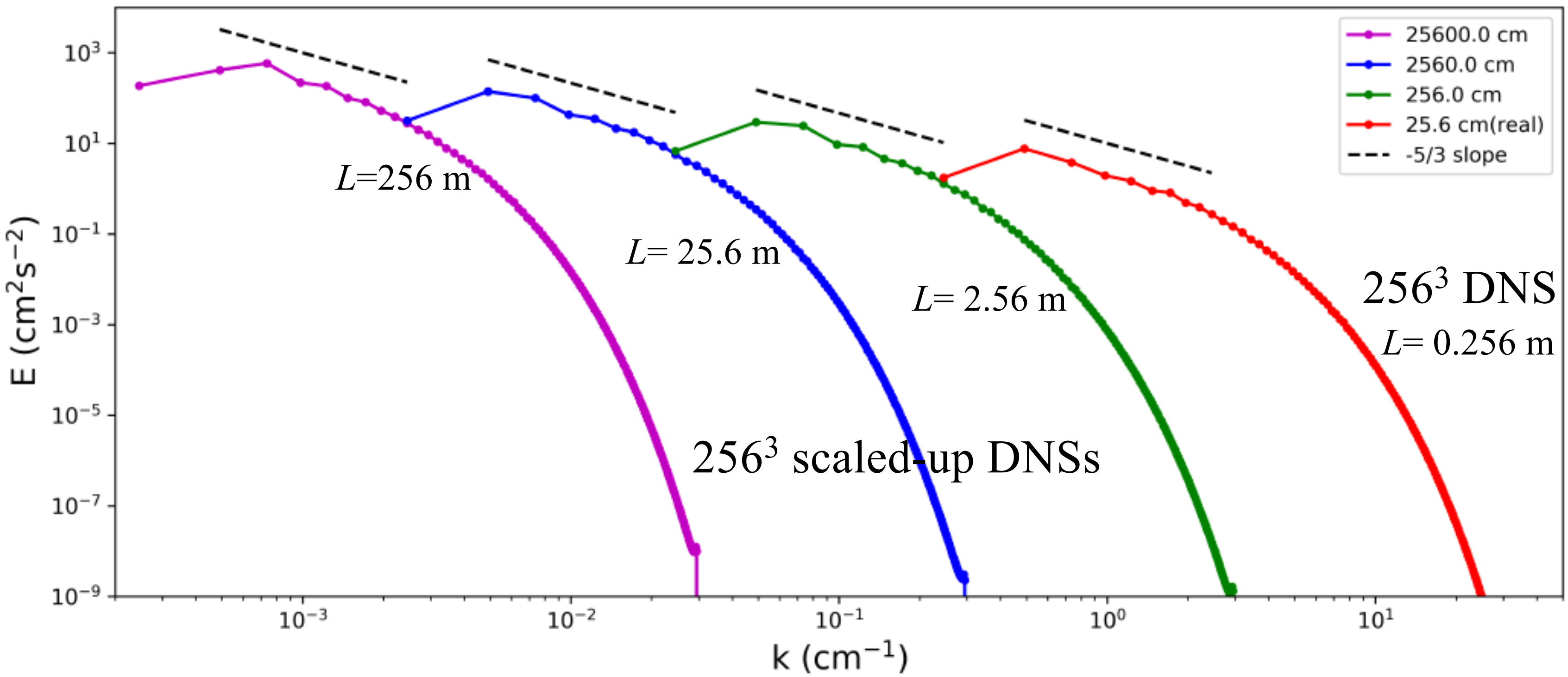
Thomas et al. *ACP* 2020



$$\frac{\eta}{L} \sim Re^{-3/4}$$

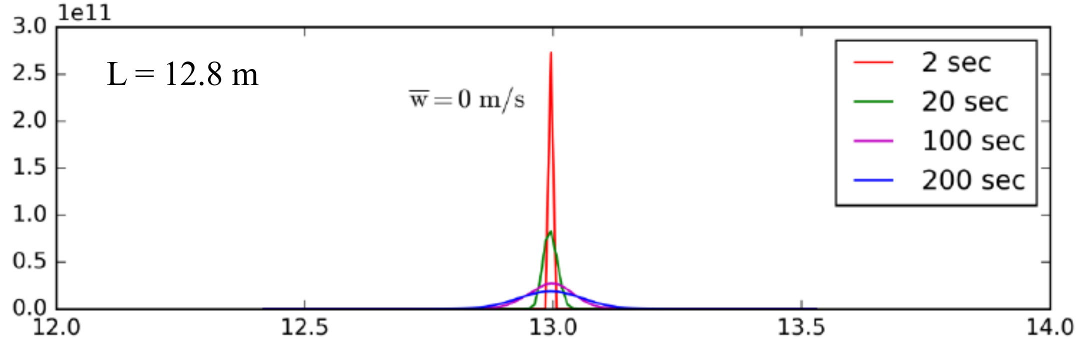
$$E \sim (L\varepsilon)^{2/3}$$

$$\nu_2 = \nu_1 \left(\frac{L_2}{L_1}\right)^{4/3}$$

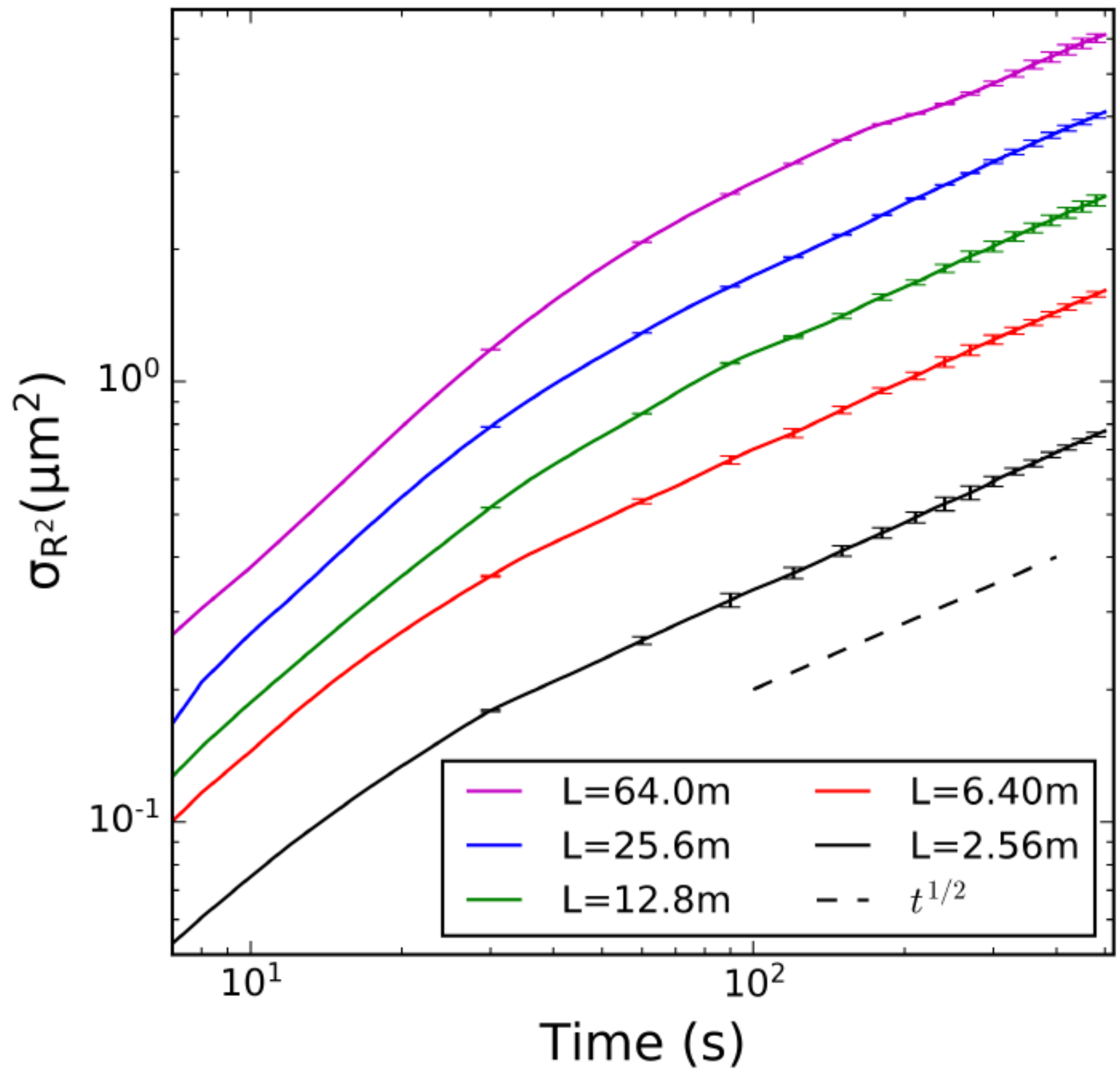
$\varepsilon = 10\ cm^2\ s^{-3}$

1 - DNS
2- scaled-up DNS

Thomas et al. *ACP* 2020



Evolution of the droplet distribution and variance of radius squared in scaled-up 256^3 DNS simulations with domain sizes L increasing from 2.56 m to 64 m. A small ensemble with different multiplicities is run for each L.



Thomas et al. *ACP* 2020

Is there anything we have not yet considered?

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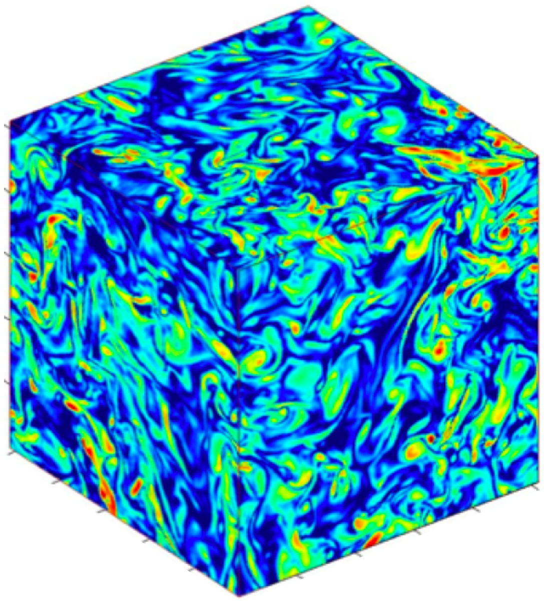
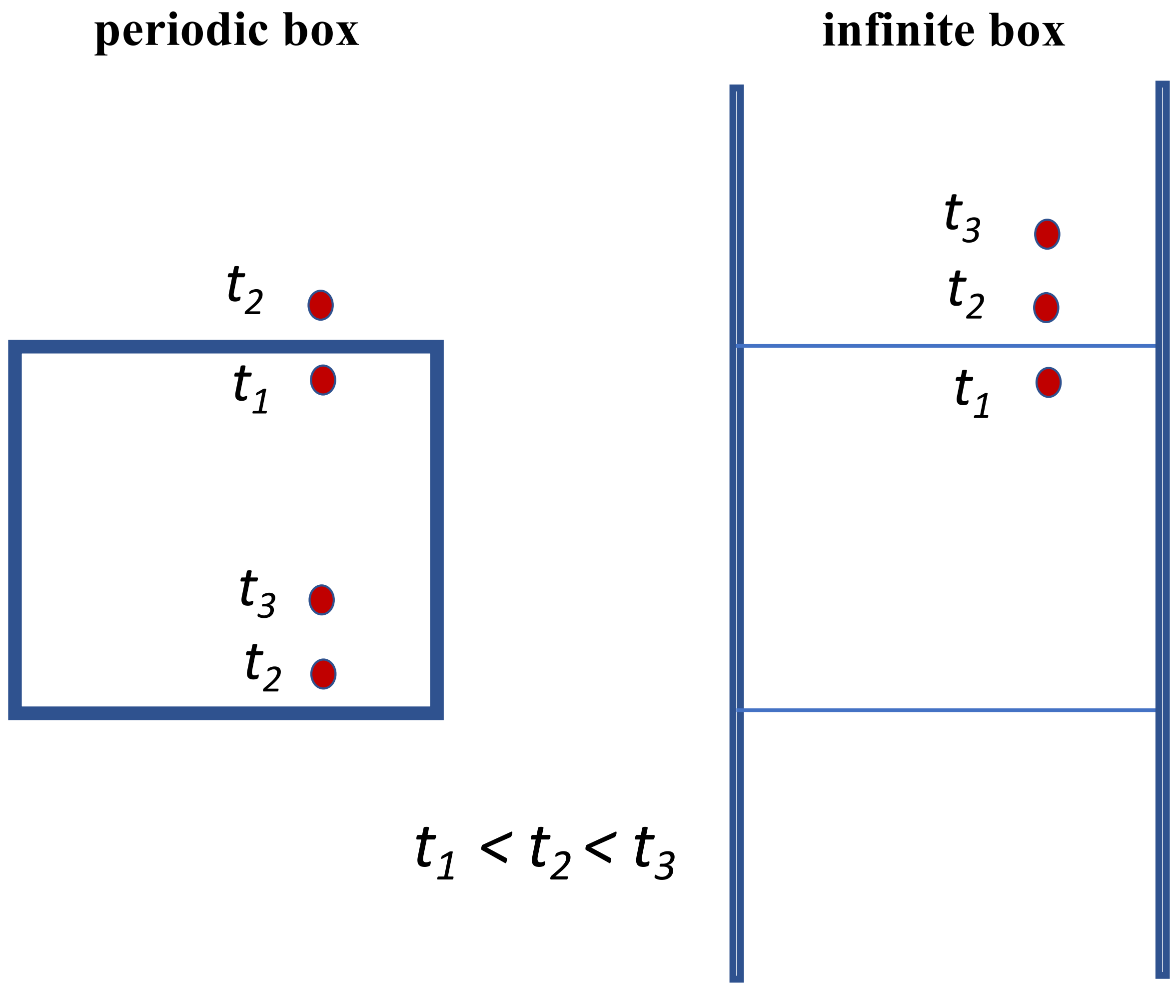
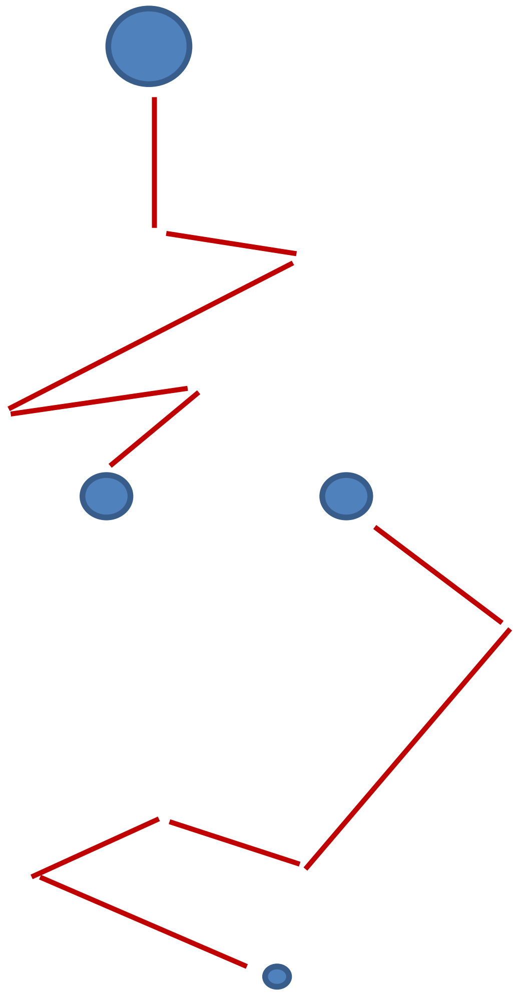




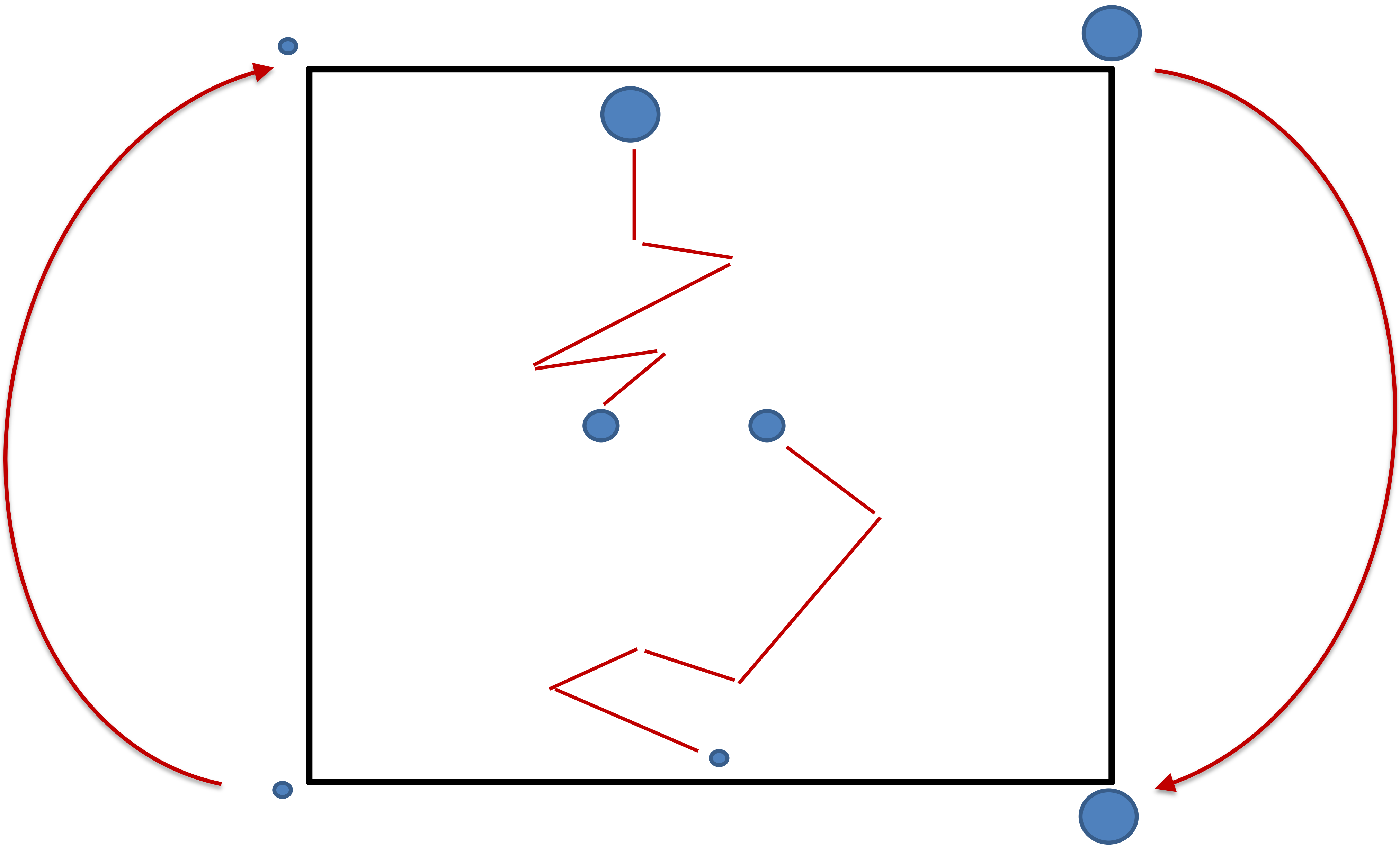


$$\partial T/\partial t + \mathrm{div}\,(\boldsymbol{u}\ T) = L_v/c_p\ C_d - g/c_p\ u_z$$

$$\partial q_v/\partial t + \mathrm{div}\,(\boldsymbol{u}\ q_v) = -\ C_d$$

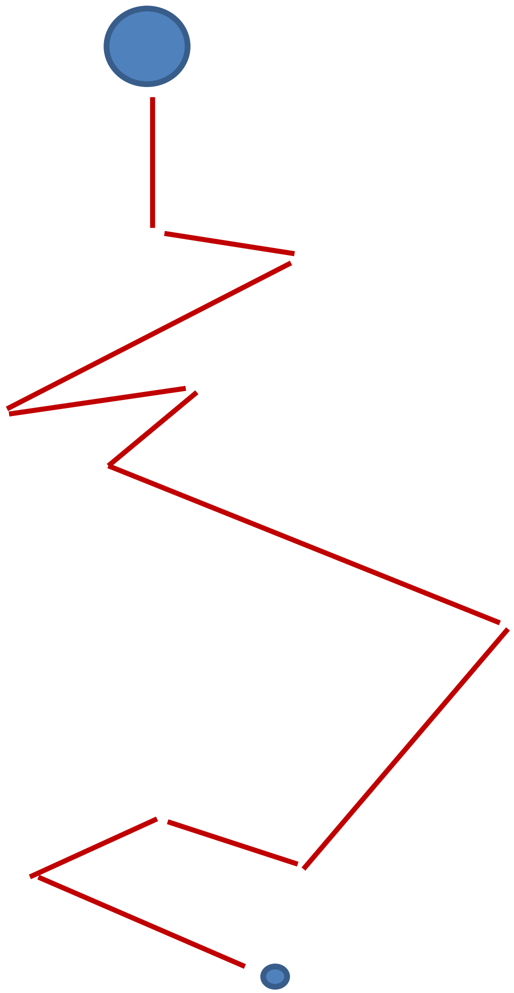
$$\partial S/\partial t + \mathrm{div}\,(\boldsymbol{u}\ S) = A\ u_z - S/\tau$$

$$S \approx A\ u_z\ \tau$$



In a periodic domain, droplets can circulate the domain
and continue growing/evaporating…

In the stochastic model, we have not considered droplet vertical position…



$$\frac{dr^{2\prime}}{dt} \sim S'$$

$$\frac{dS_i'}{dt} = a_1 w' - \frac{S_i'}{\tau_{\mathrm{relax}}}$$

$$w'(t+\delta t) = w'(t)e^{-\delta t/\tau} + \sqrt{1-e^{-2\delta t/\tau}}\,\sigma_{w'}\psi$$

e.g., Bartlett and Jonas *QJ* 1972,
Grabowski and Abade *JAS* 2016

Stochastic model for droplet growth by condensation and following droplet position in the vertical:

$$\frac{dr^{2\prime}}{dt} \sim S'$$

$$\frac{dS_i'}{dt} = a_1 w' - \frac{S_i'}{\tau_{\mathrm{relax}}}$$

$$w'(t+\delta t) = w'(t)e^{-\delta t/\tau} + \sqrt{1 - e^{-2\delta t/\tau}}\,\sigma_{w'}\psi$$

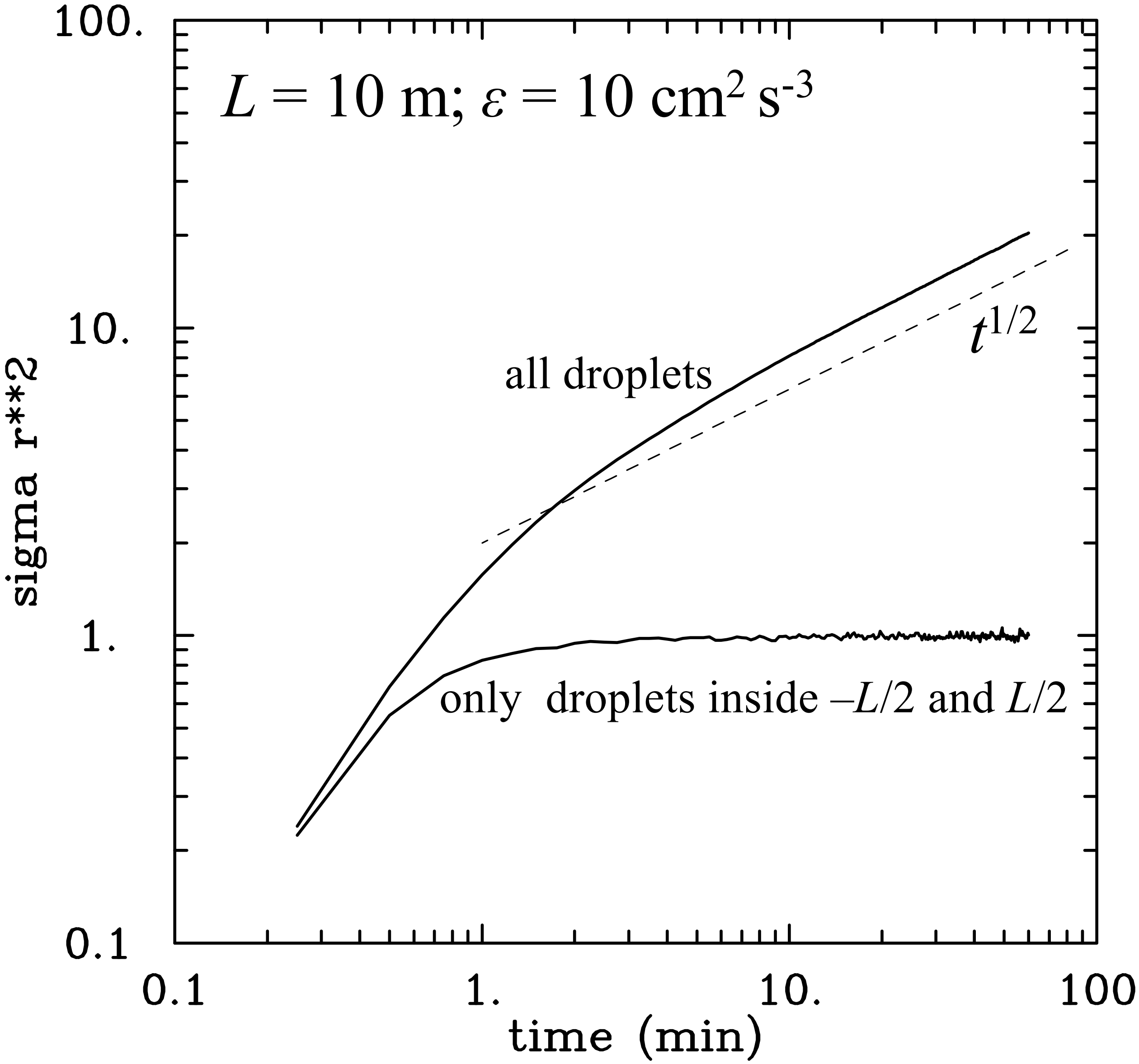
$$\sigma_{w'}^2 = \frac{2}{3}E$$

E - TKE

ψ - Gaussian random number

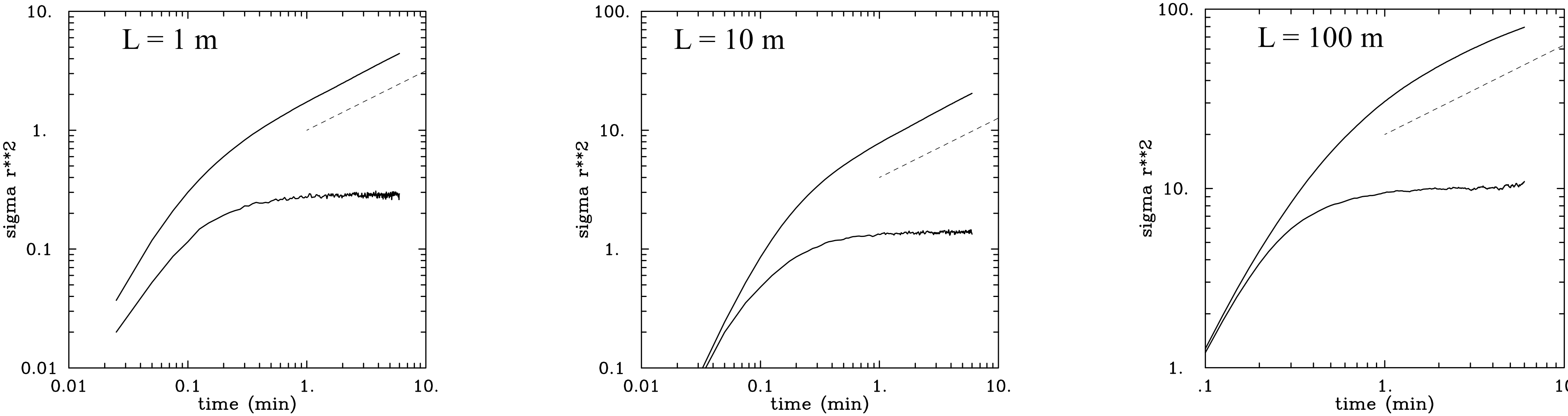
$$\tau = \frac{L}{(2\pi)^{1/3}}\left(\frac{C_\tau}{E}\right)^{1/2} \qquad E = \left(\frac{L\varepsilon}{C_E}\right)^{2/3}$$

keeping track of height changes: $z(t+\delta t) = z(t) + w'\,\delta t$

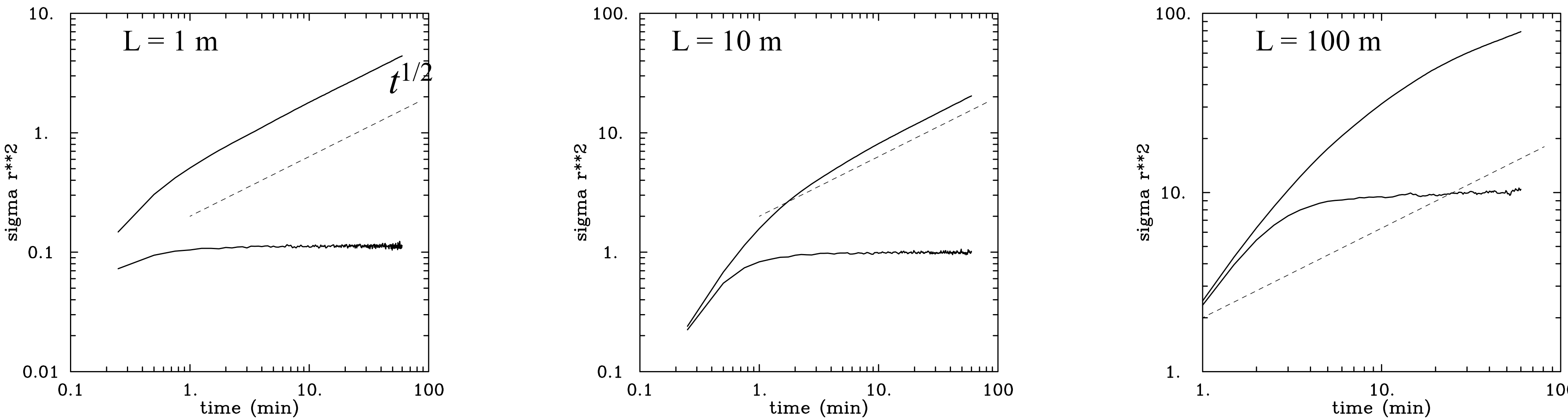


L = 10 m; ε = 10 cm² s⁻³
all droplets
t^1/2
only droplets inside –L/2 and L/2
sigma r**2
time (min)
100.
10.
1.
0.1
0.1
1.
10.
100

TKE dissipation of 1000 $cm^2 s^{-3}$



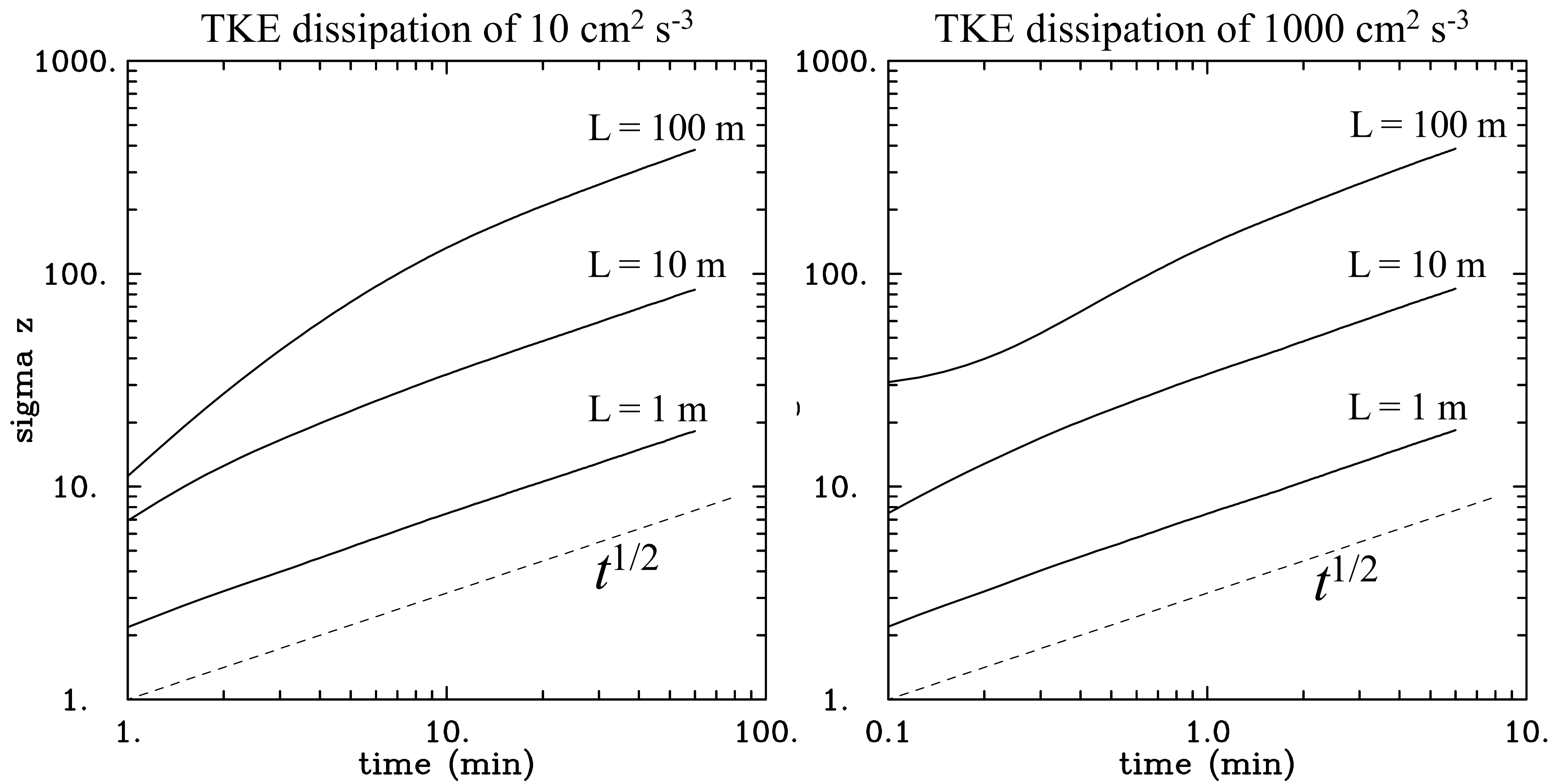
TKE dissipation of 10 $cm^2 s^{-3}$



So where does the $t^{1/2}$ scaling come from?

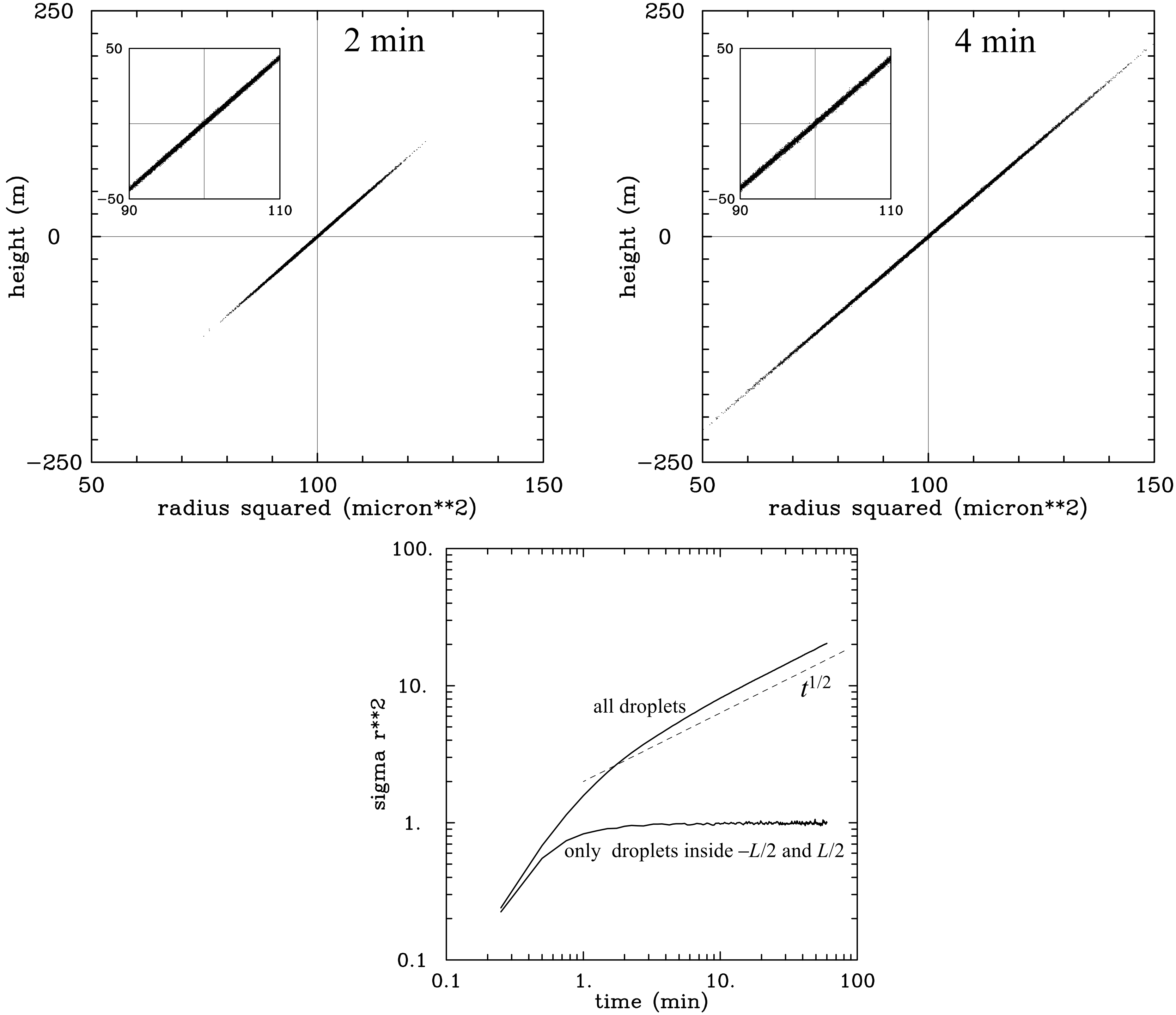
So where does the $t^{1/2}$ scaling come from?

The dispersion of droplets in turbulence is the same as in a random walk model (e.g., Brownian motion). The droplet displacement standard deviation in such a case continuously increases with the $t^{1/2}$ scaling (e.g., Einstein 1905; Smoluchowski, 1907).

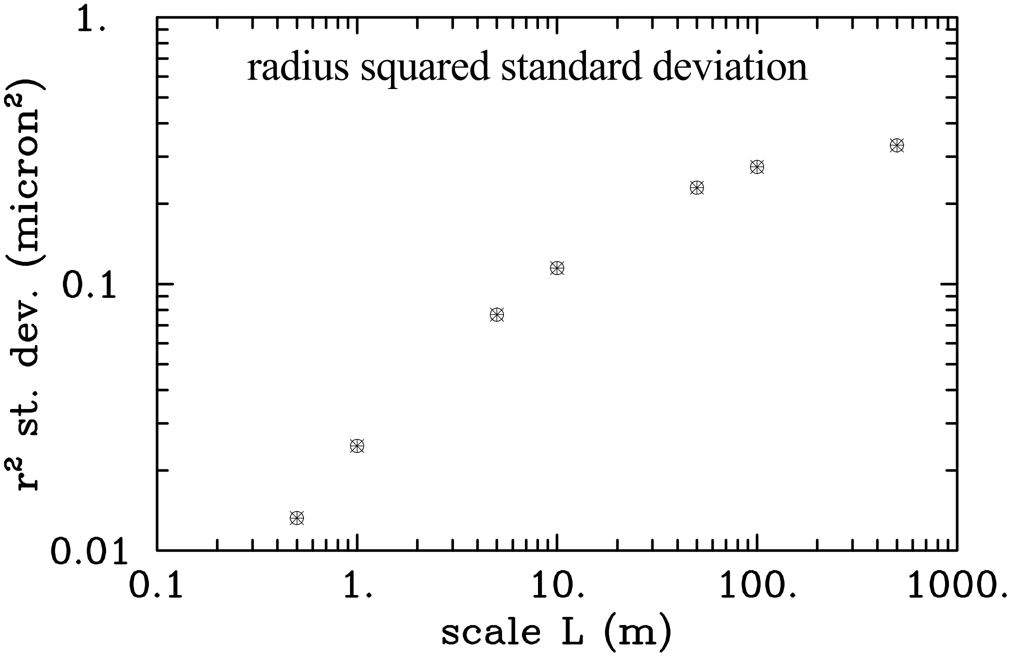


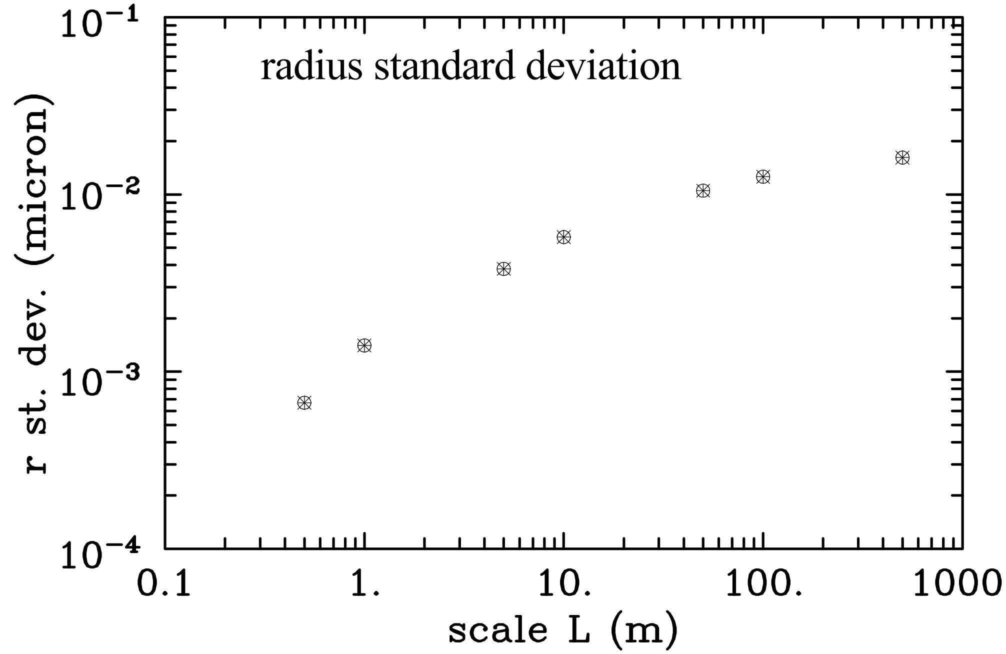
standard deviation of the droplet height

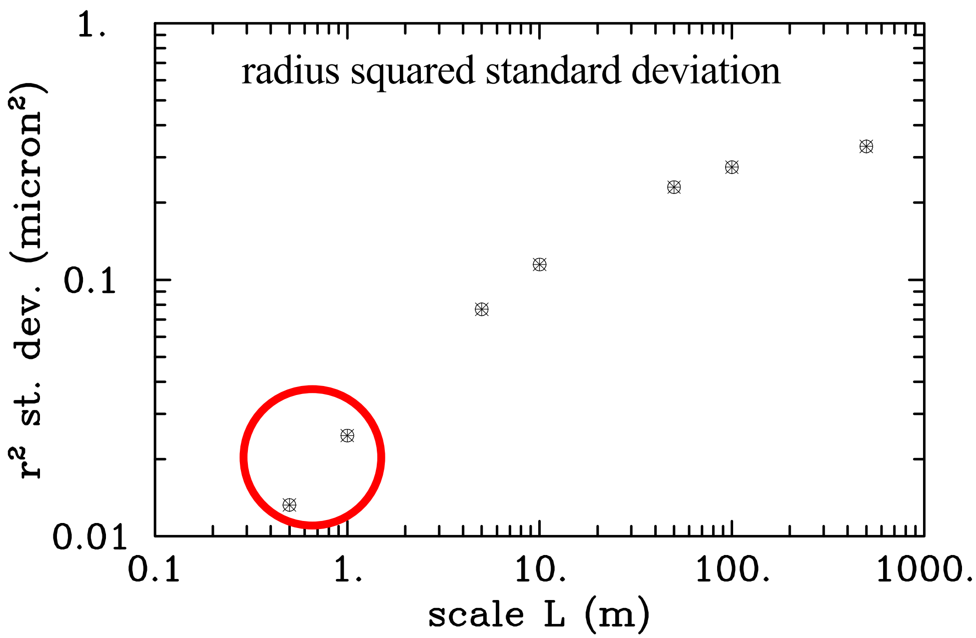
TKE dissipation: 10 cm^2 s^{-3} *L*: 100 m

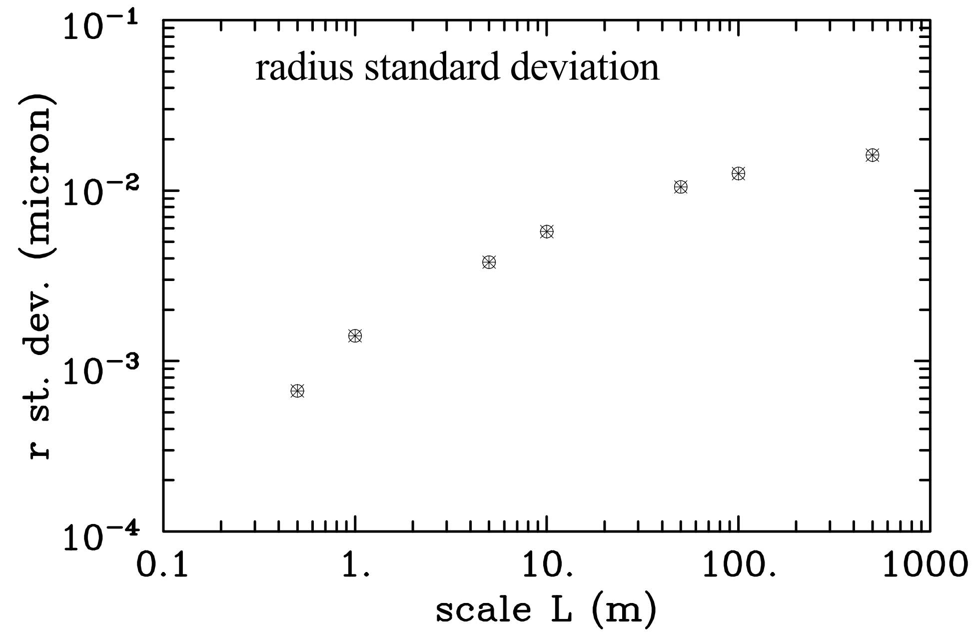


One can derive the true impact of eddy hoping by removing the mean vertical gradient of the droplet radius









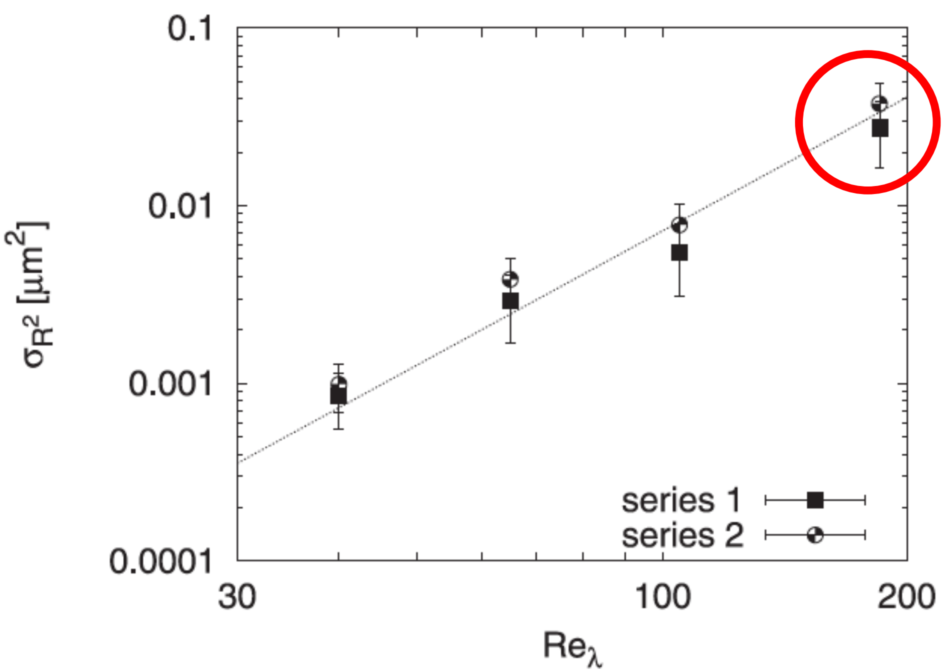


FIG. 3. Log–log plot of the spreading of droplet size distribution $\sigma_{R^2}(T_L)$ for the square radius R^2, measured after one large-scale eddy turnover time T_L as a function of the Reynolds number Re_λ.

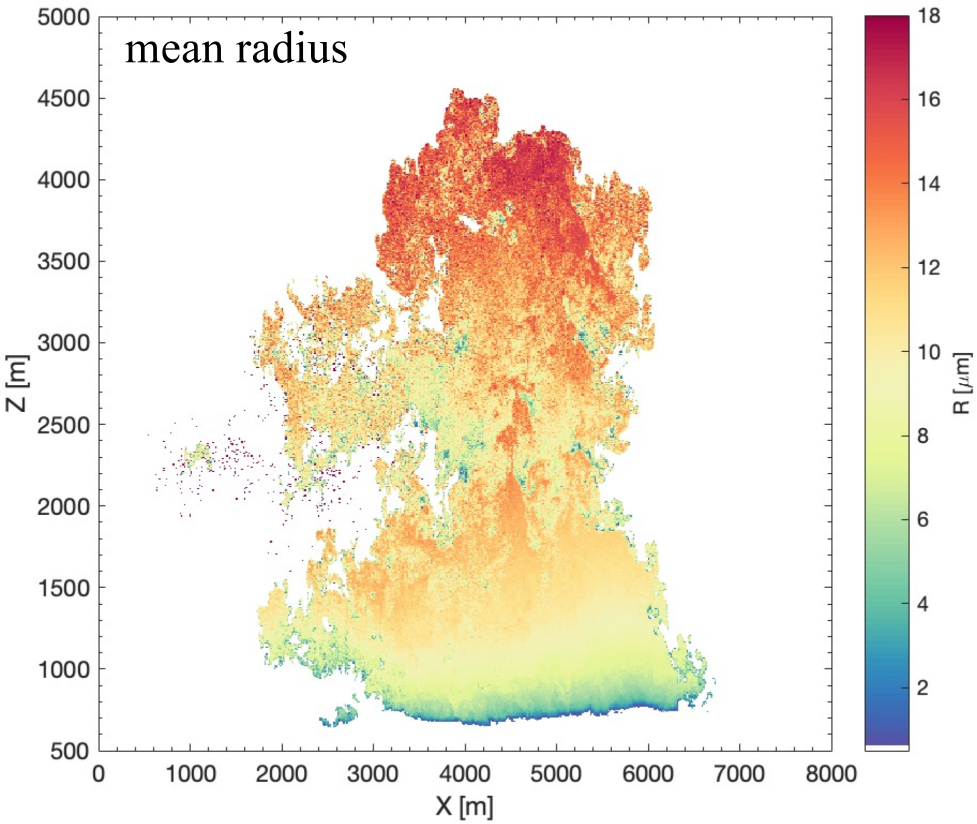
Lanotte et al. *JAS* 2009

Where do we go from here?

The way forward:

Realistic high-resolution cloud simulations (grid length of ~10 m and higher) with *Lagrangian particle-based microphysics* (super-droplet method, SDM, Shima et al. *QJ* 2009 and others) that includes the impact of both resolved and subgrid-scale turbulence (the latter included through the stochastic modeling) on diffusional and collisional growth of cloud droplets (e.g., Chandrakar et al.)…

Recent ASD project lead by MMM's Kamal Kant Chandrakar: Simulations of an isolated cumulus congestus (CAMP2Ex)



Domain size: 12 km x 12 km x 11 km (1600 pt x 1600 pt x 1465 pt)

Grid spacings: dx = dy = dz = 7.5 m

Super-particles: 64 per grid box (non precipitating) and 62 (precipitating)

Aerosols: bimodal lognormal distribution of $(NH_4)_2SO_4$; 639.7 cm^{-3}; vertically varying

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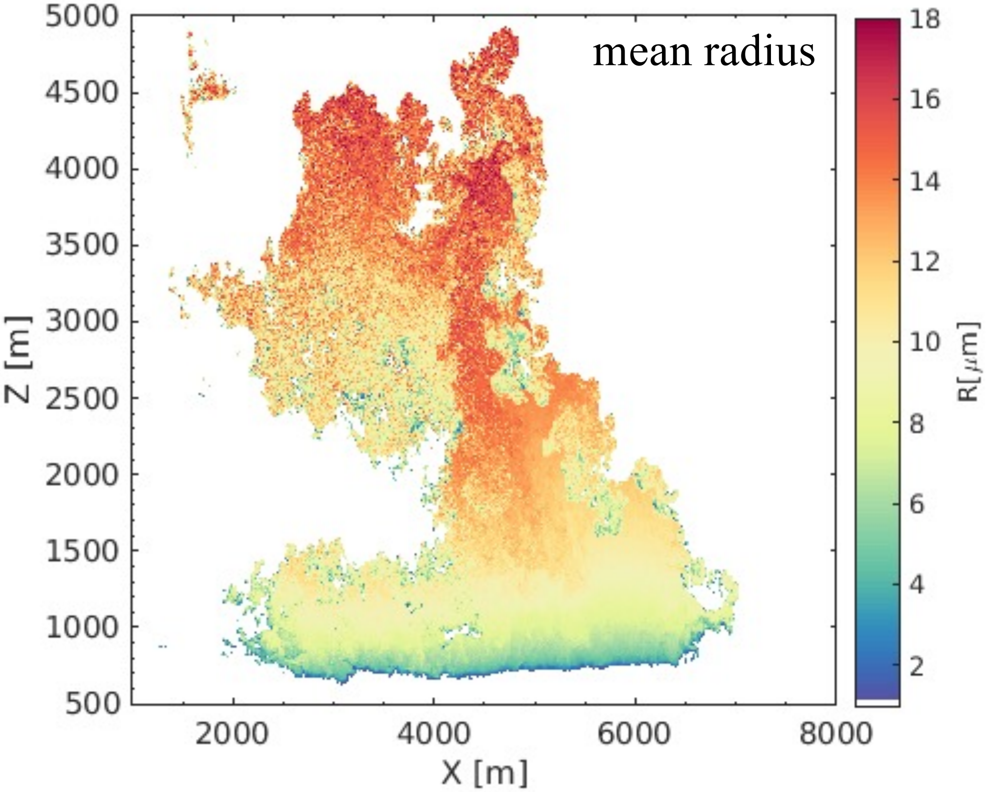

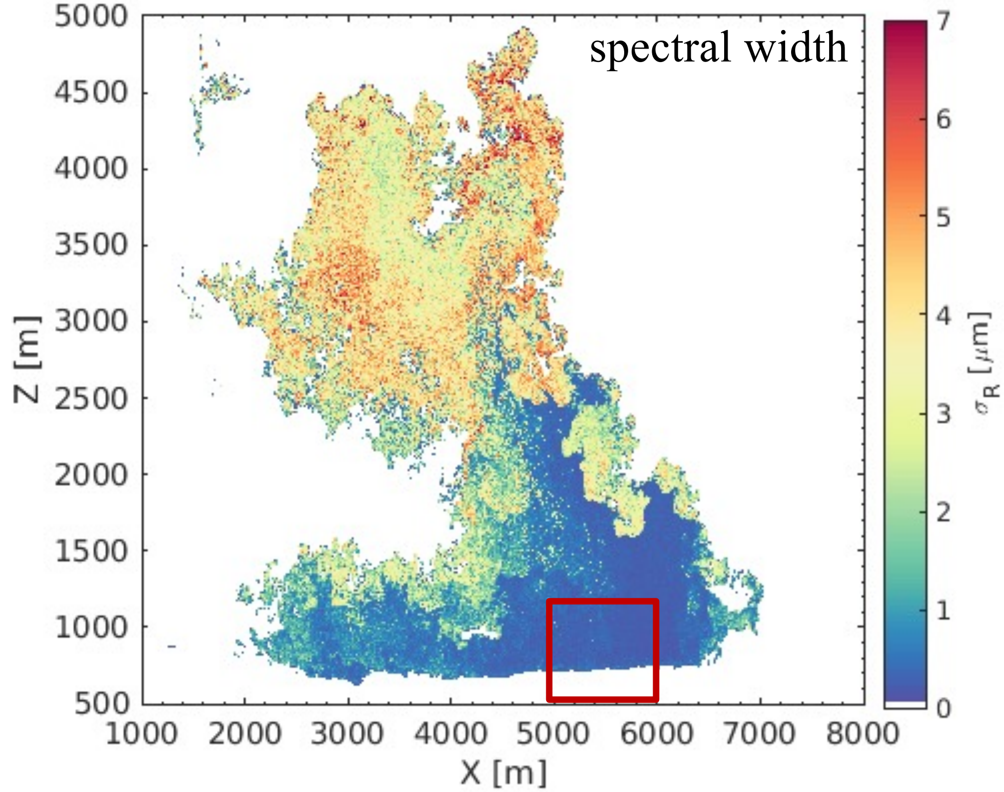

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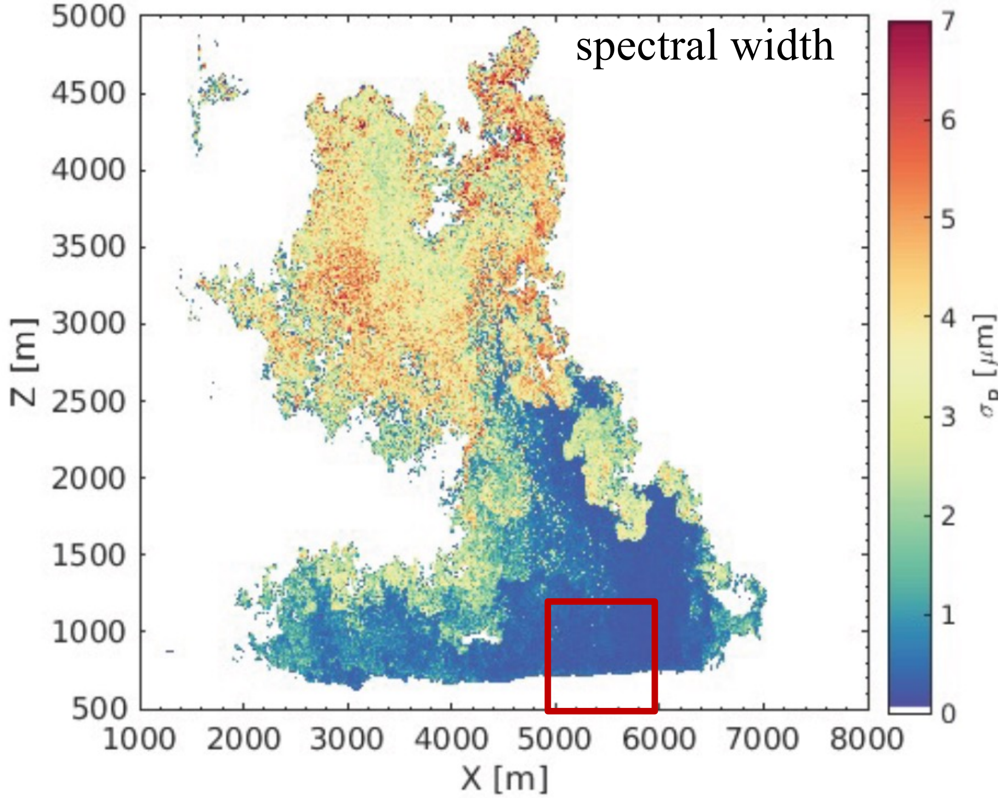

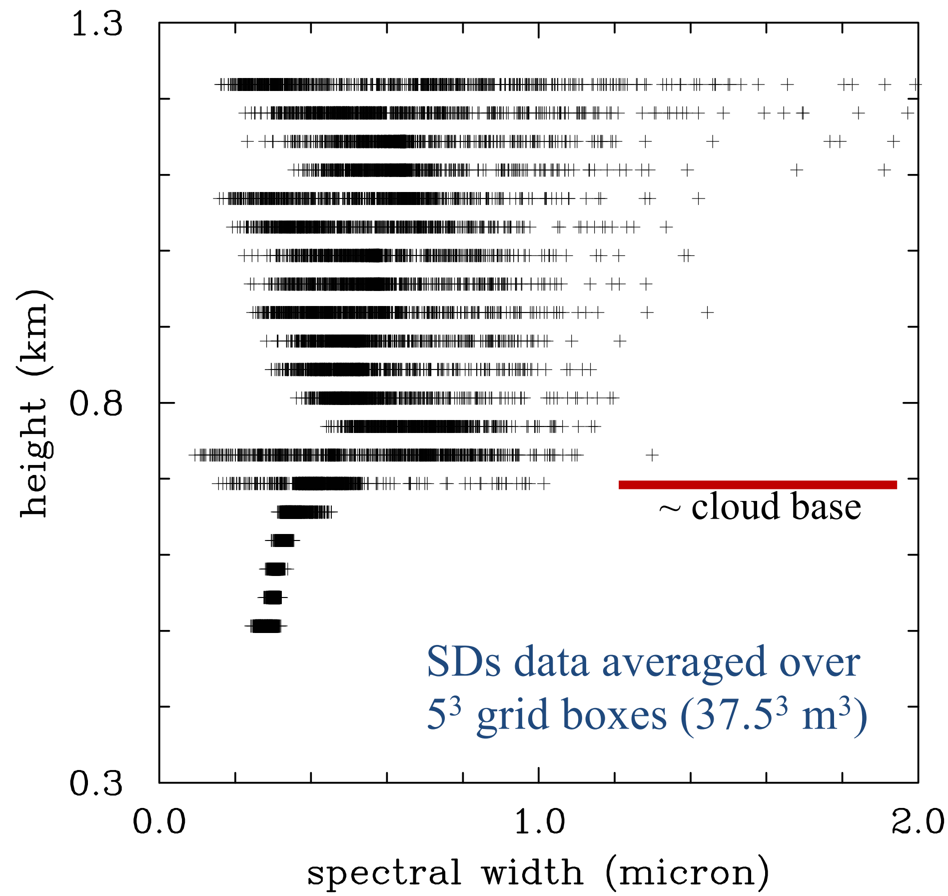

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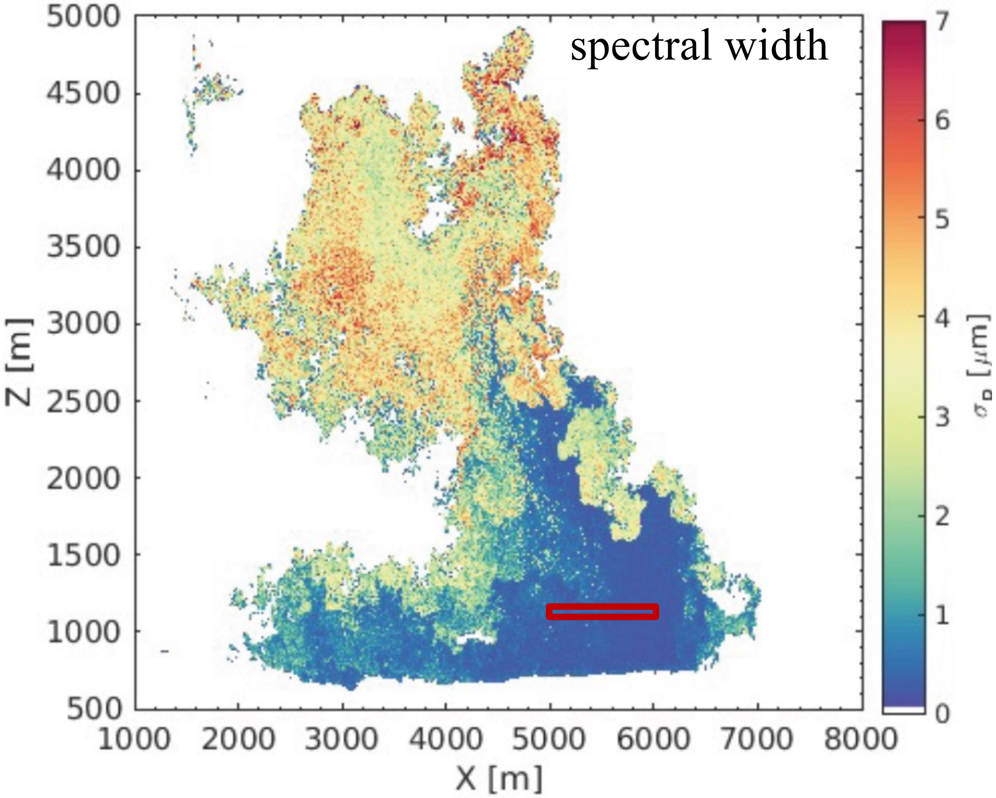

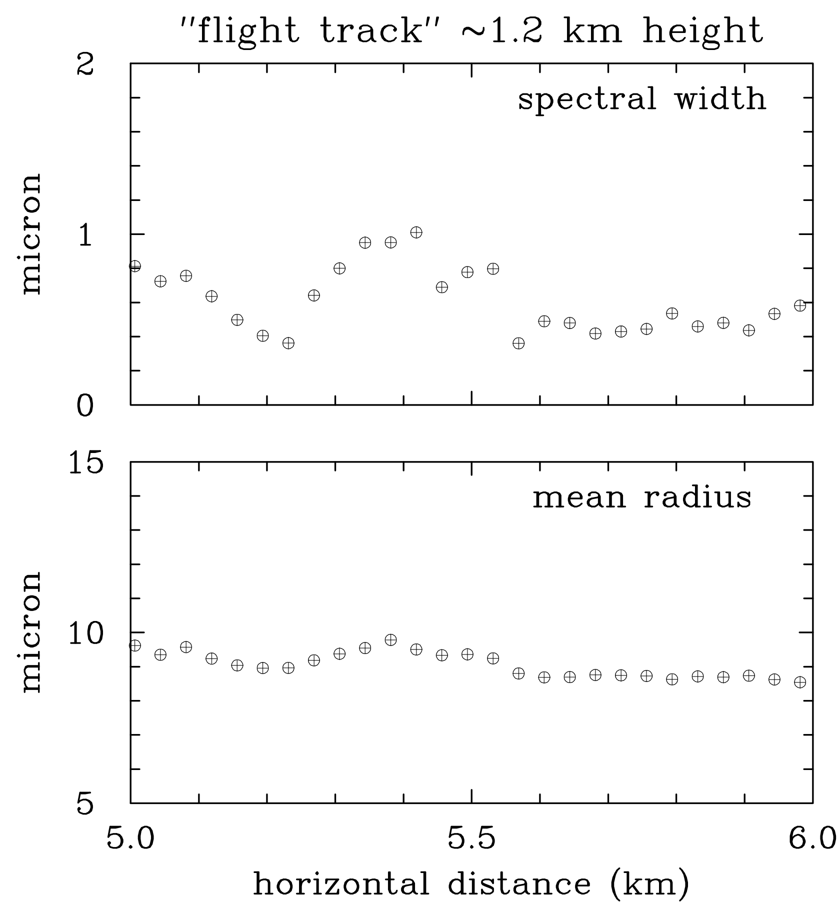

SDs data averaged over 5^3 grid boxes (37.5^3 m^3)

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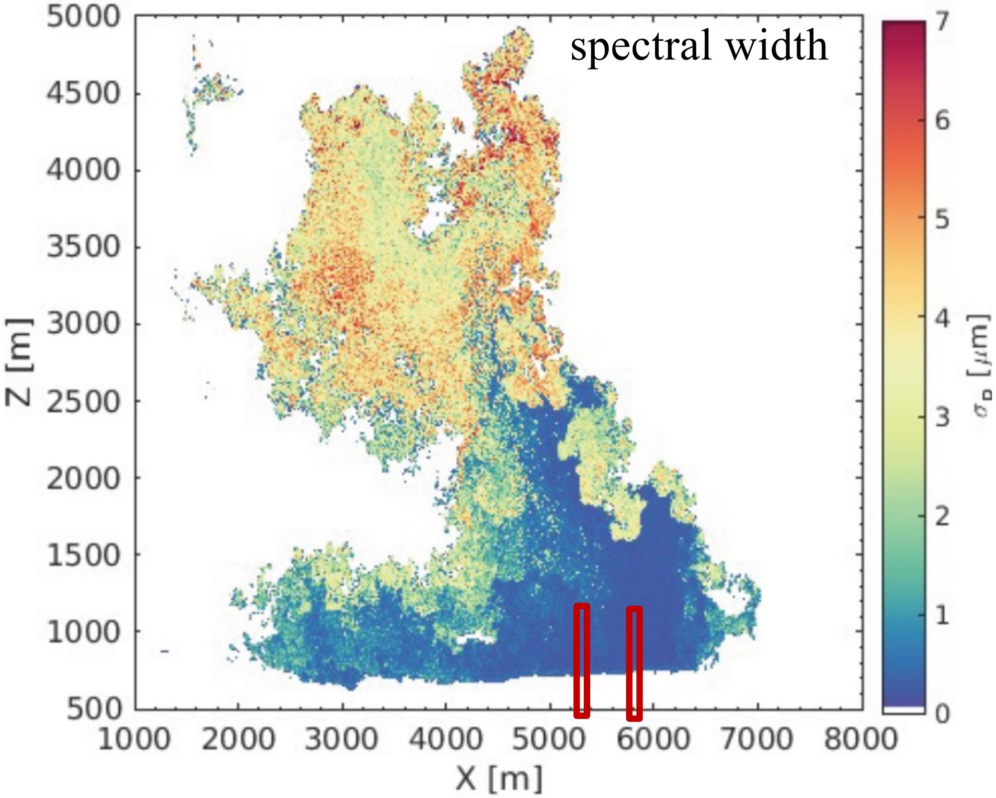

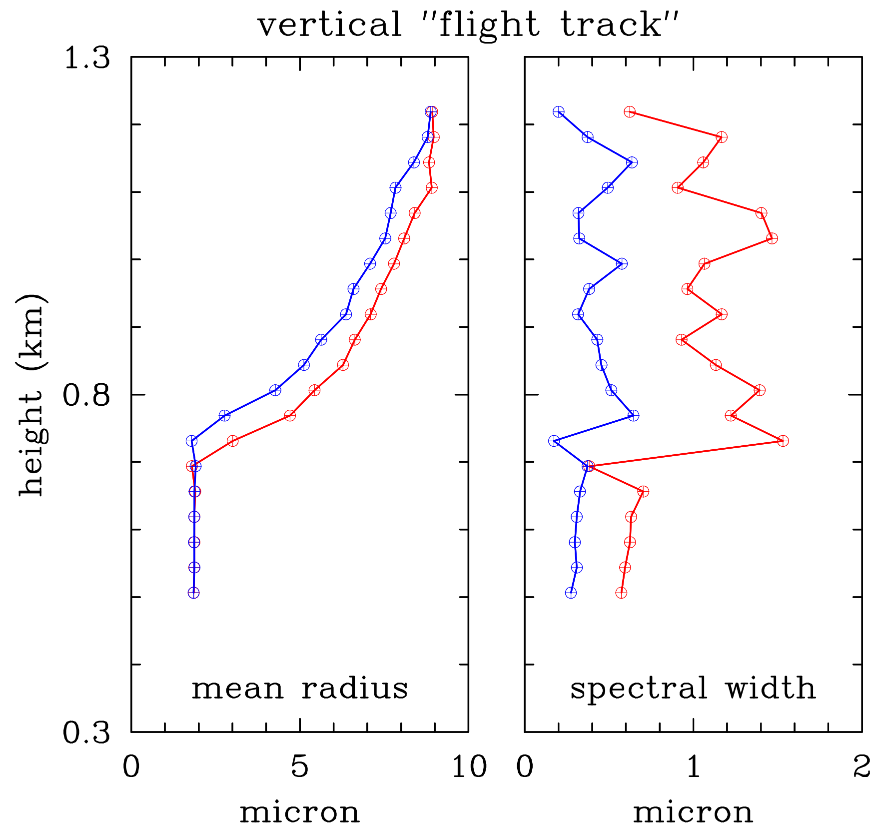

SDs data averaged over
5^3 grid boxes (37.5^3 m^3)

Summary:

Understanding droplet spectral broadening due to diffusional growth in turbulent environments attracted significant attention of turbulence community in recent decades. DNS, scaled-up DNS, and stochastic models have been used to show the impact of turbulence in undiluted cloudy volumes on the adiabatic droplet spectral width.

Theoretical analysis suggests that the droplet area (radius squared) standard deviation should continuously increase in time with the $t^{1/2}$ scaling. This has been confirmed in several investigations of droplet population growth in isotropic homogeneous turbulence. It follows that the droplet spectra in undiluted volumes should continuously broaden above the cloud base as the updraft carries droplets upwards.

However, those idealized studies feature a fundamental flaw of periodic domains that allow droplets to circulate in the vertical. If droplet vertical spread is taken into account, the spectral width saturates. The $t^{1/2}$ scaling simply comes from the droplet position standard deviation as in the random walk model.

Analysis of realistic high-resolution cloud simulations (grid length of ~10 m) applying Lagrangian particle-based microphysics helps further quantify the impact of turbulence (resolved + subgrid-scale) on droplet spectral evolution in simulated clouds.