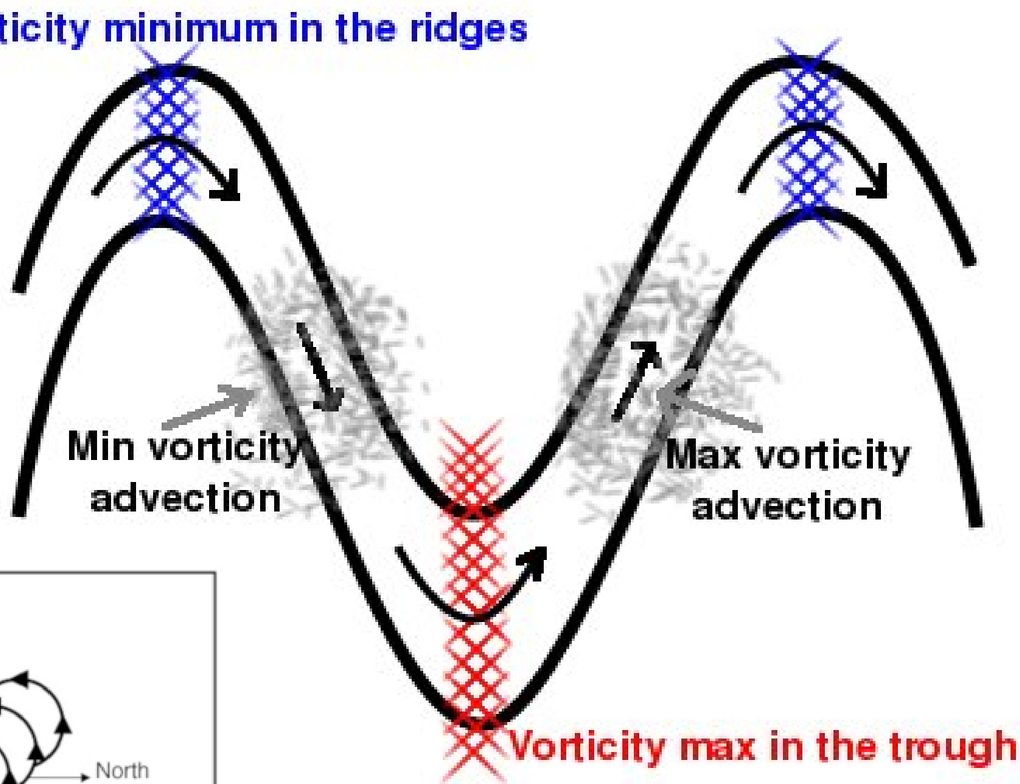
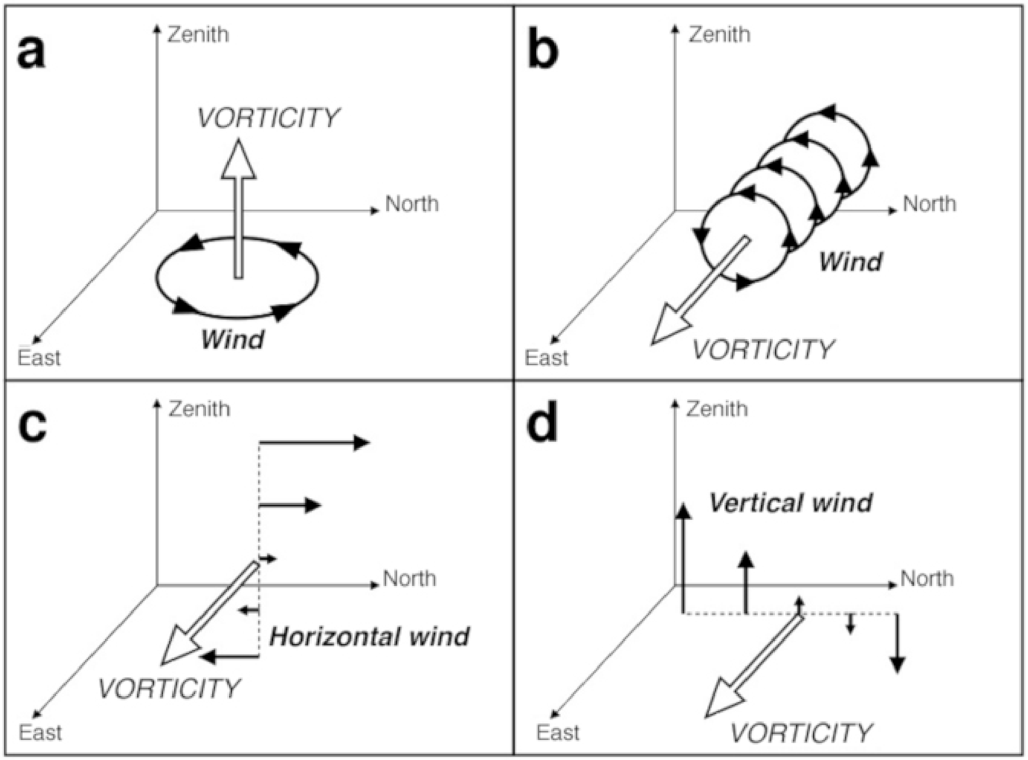
Dynamics of the Atmosphere and the Ocean

Lecture 8

Darek Baranowski

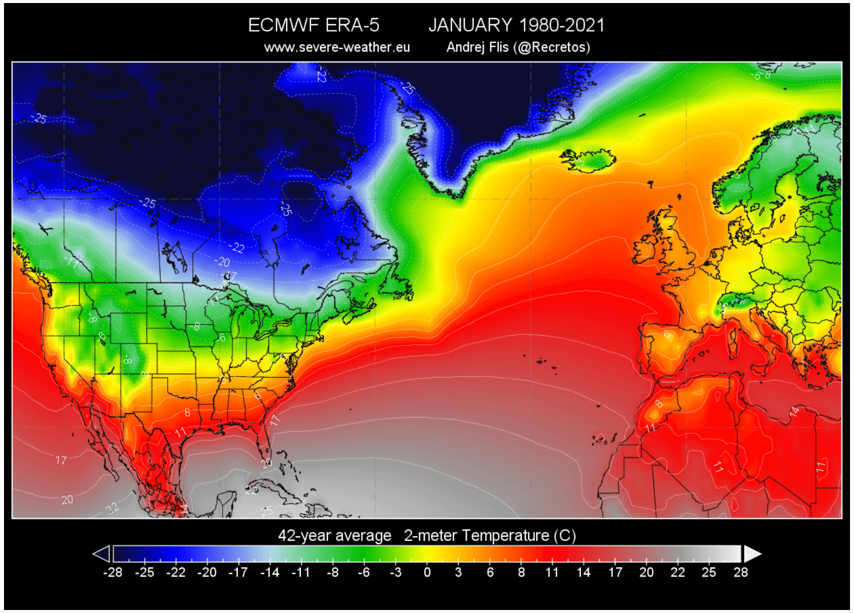
2025-2026 Fall





THE VORTICITY IN THE ATMOSPHERE AND OCEAN

GULF STREAM



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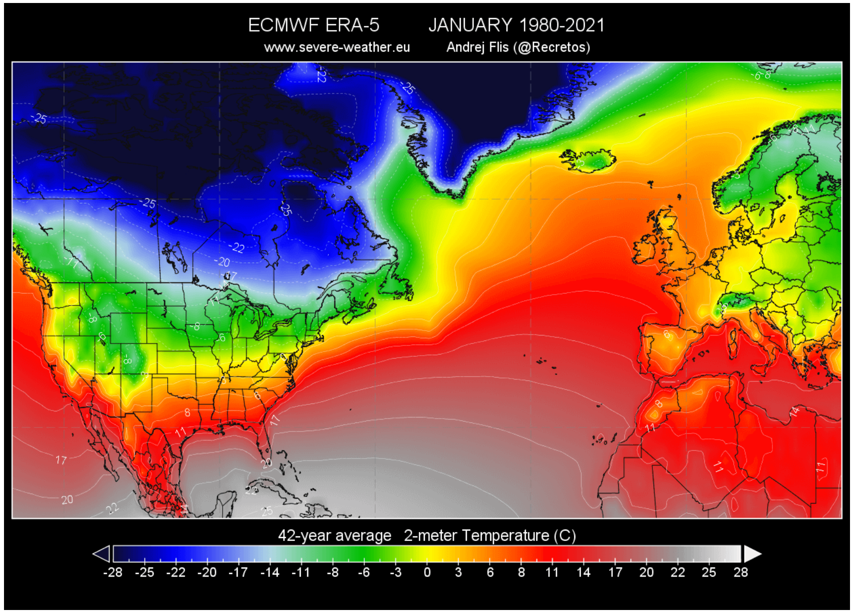

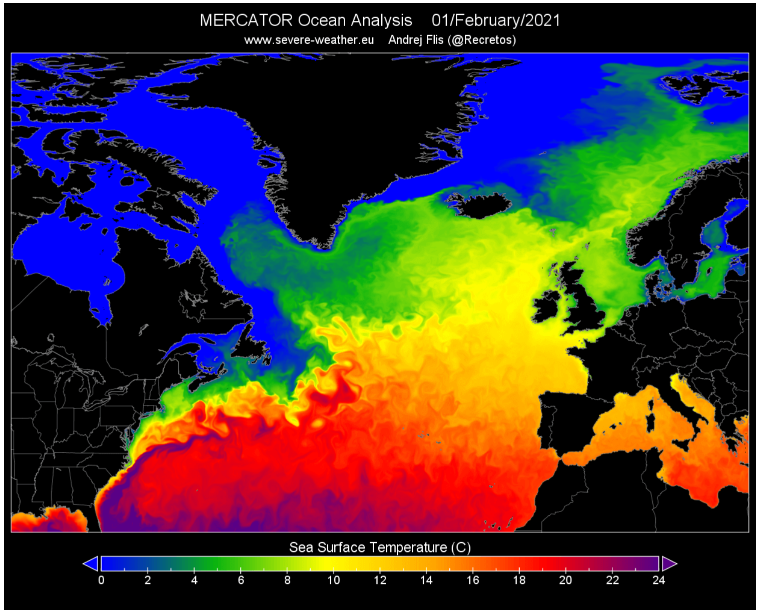

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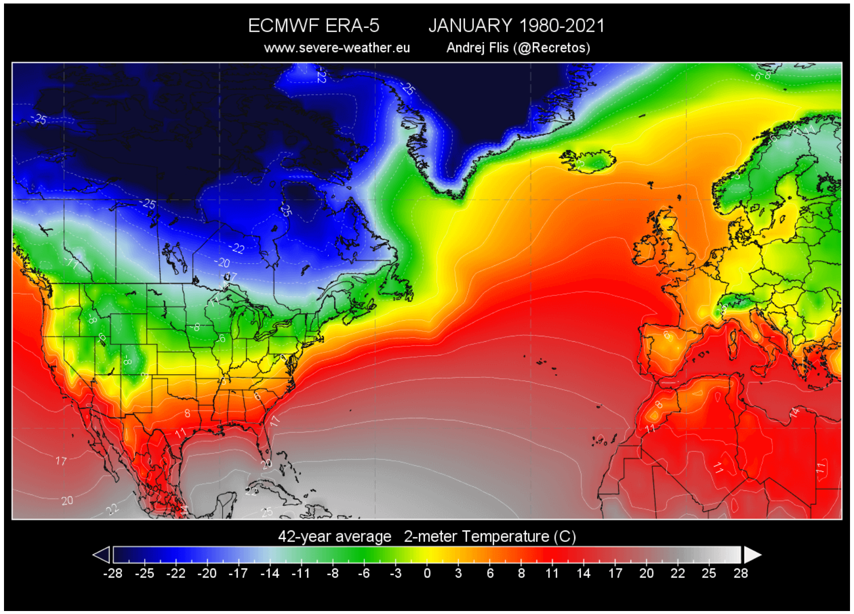

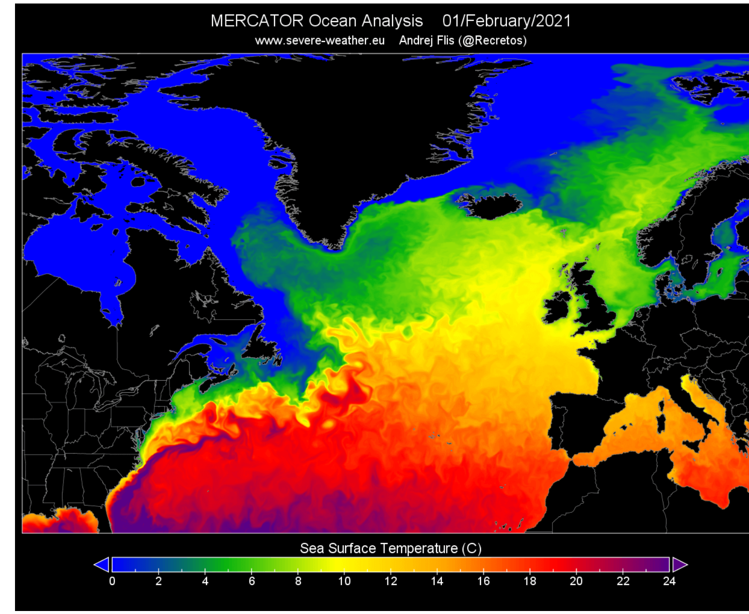

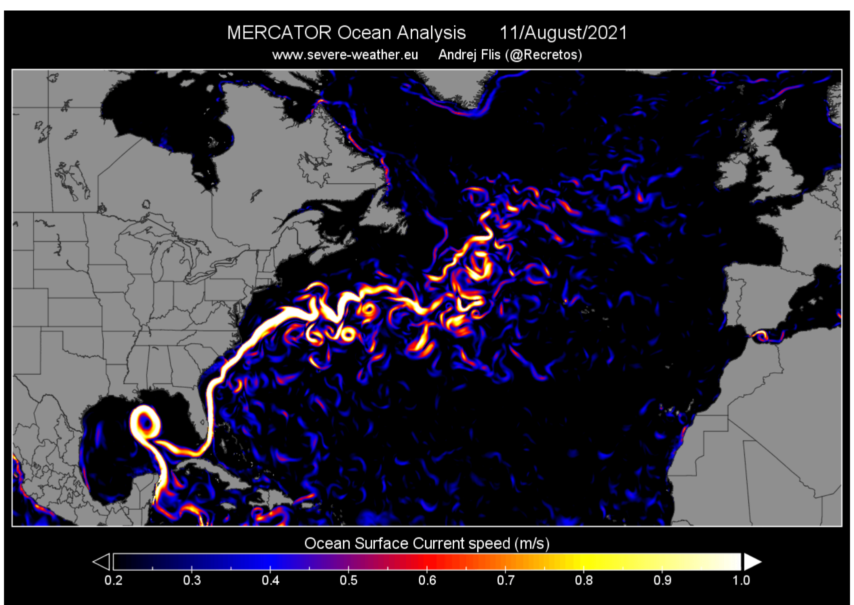

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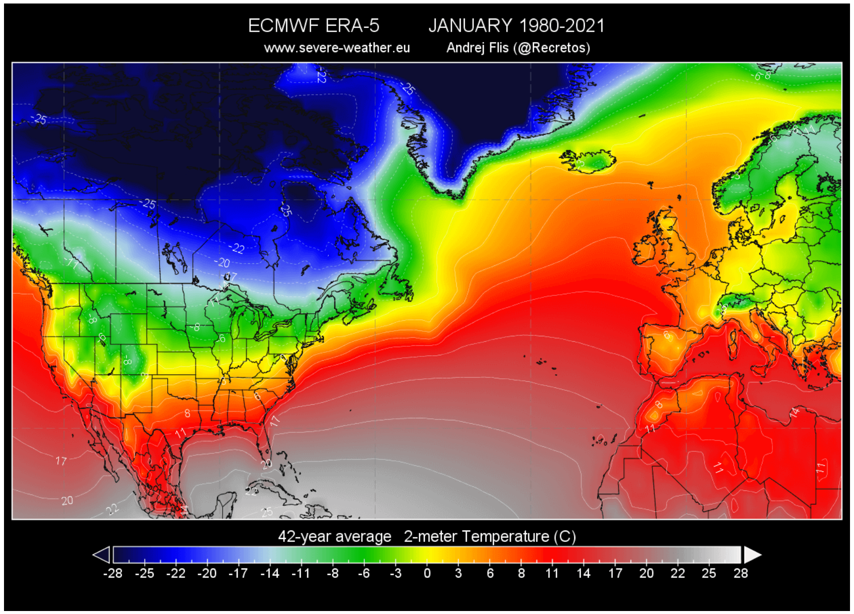
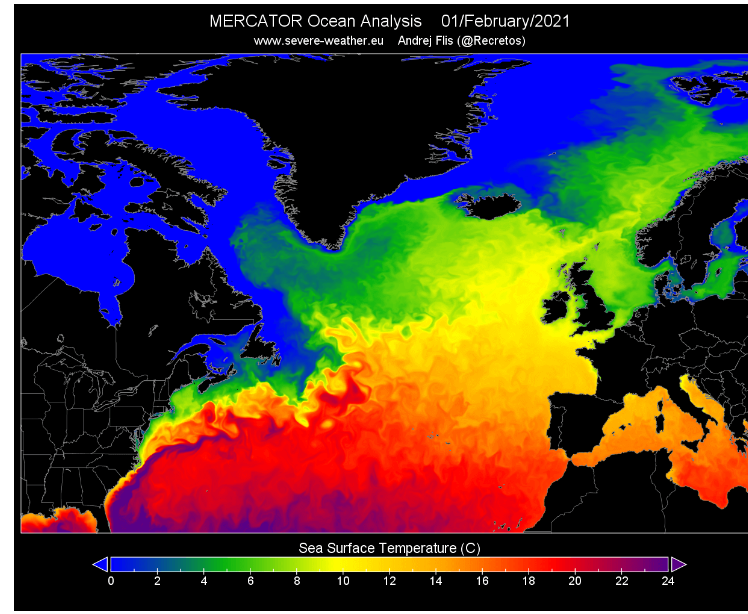
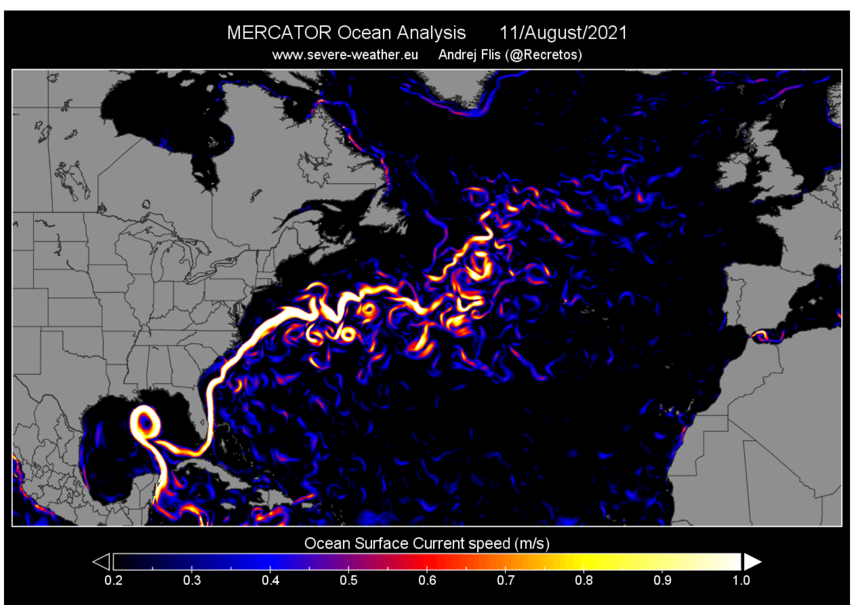
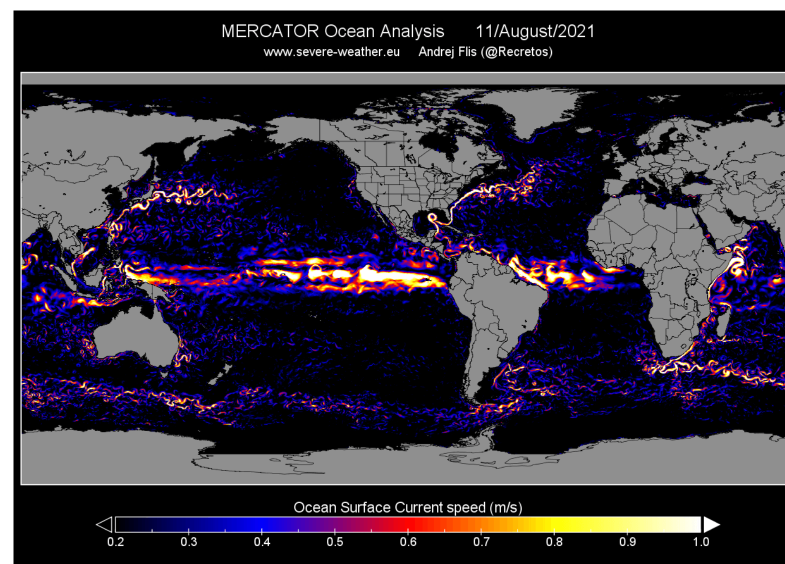


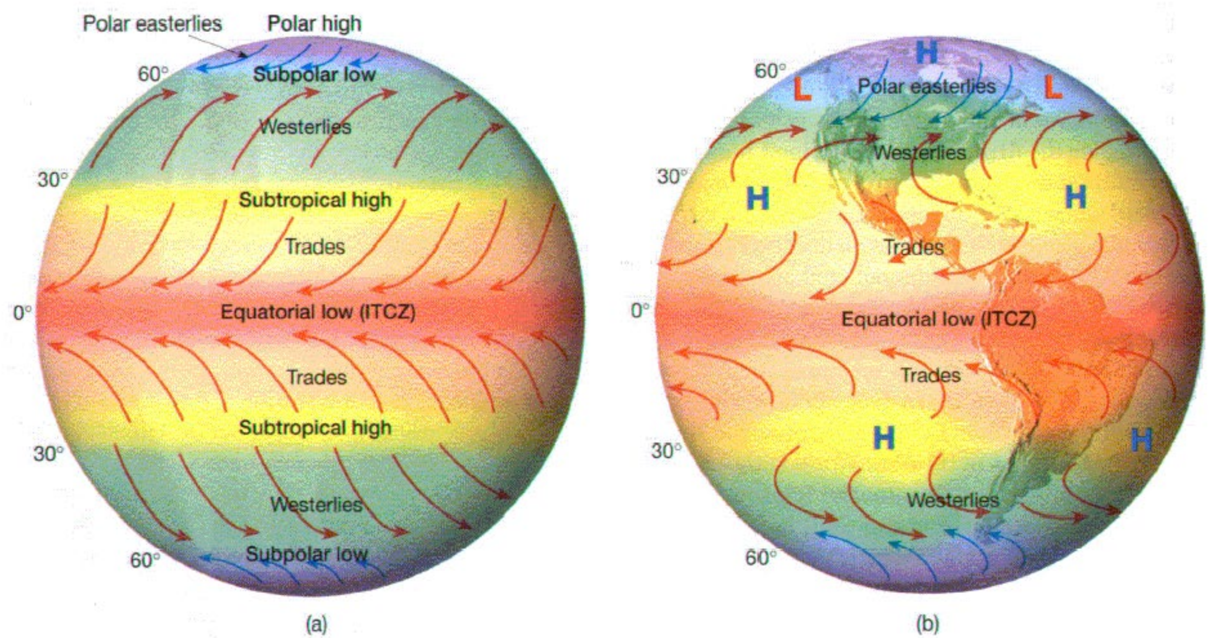


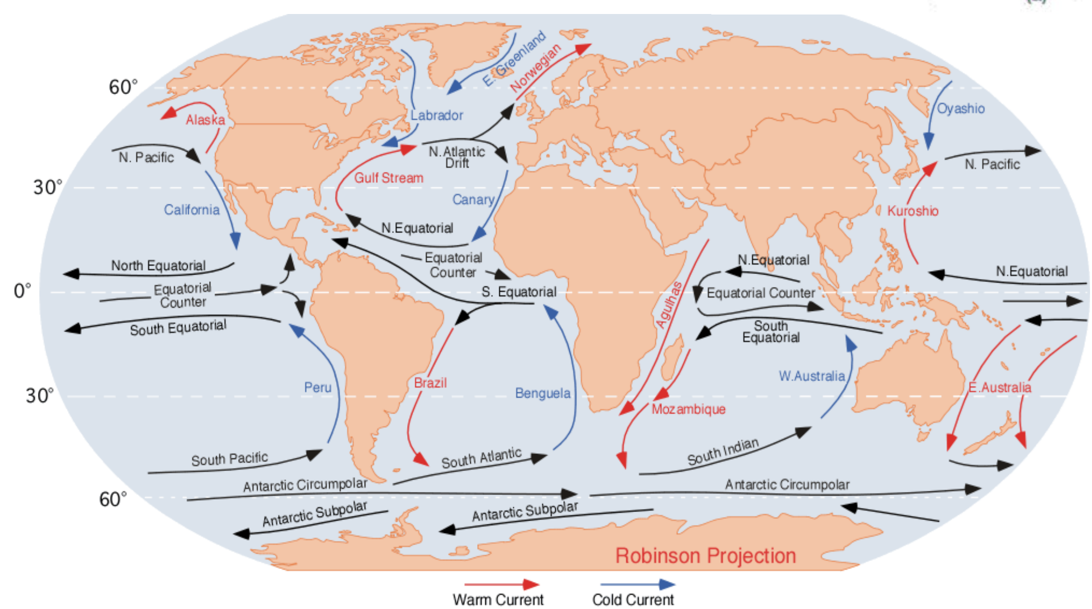




THE VORTICITY IN THE ATMOSPHERE AND OCEAN

GLOBAL (BASIN-WIDE) CIRCULATION





THE VORTICITY EQUATION

We will use the equations of motion to derive an equation for the time rate of change of vorticity without limiting the validity to adiabatic motion.

For motions of synoptic scale, the vorticity equation can be derived using the quasi-geostrophic (prognostic) approximate horizontal momentum equations. We differentiate the zonal component equation with respect to y and the meridional component equation with respect to x:

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-fv=-\frac{1}{\rho}\frac{\partial p}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+fu=-\frac{1}{\rho}\frac{\partial p}{\partial y}\right)$$

Subtracting the upper equation from the lower one and recalling that

$$\zeta = \partial v/\partial x - \partial u/\partial y,$$

we obtain the vorticity equation

The vorticity equation:

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + w\frac{\partial \zeta}{\partial z} + (\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) =$$
$$+\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + v\frac{df}{dy} = \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

may me rewritten using the fact that the Coriolis parameter depends only on y so that Df/Dt =v(df/dy).

It takes the form:

$$\frac{D}{Dt}(\zeta + f) = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$
$$-\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

The above states that the rate of change of the absolute vorticity following the motion is given by the sum of the three terms on the right, called the divergence term, the tilting or twisting term, and the solenoidal term, respectively.

$$\frac{D}{Dt}(\zeta + f) = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

Divergence term:

$$-(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

states that the concentration or dilution of vorticity by the divergence field is the fluid analog of the change in angular velocity resulting from a change in the moment of inertia of a solid body when angular momentum is conserved.

If the horizontal flow is divergent, the area enclosed by a chain of fluid parcels will increase with time and if circulation is to be conserved, the average absolute vorticity of the enclosed fluid must decrease (i.e., the vorticity will be diluted).

If, however, the flow is convergent, the area enclosed by a chain of fluid parcels will decrease with time and the vorticity will be concentrated.

This mechanism for changing vorticity following the motion is very important in synoptic-scale disturbances.

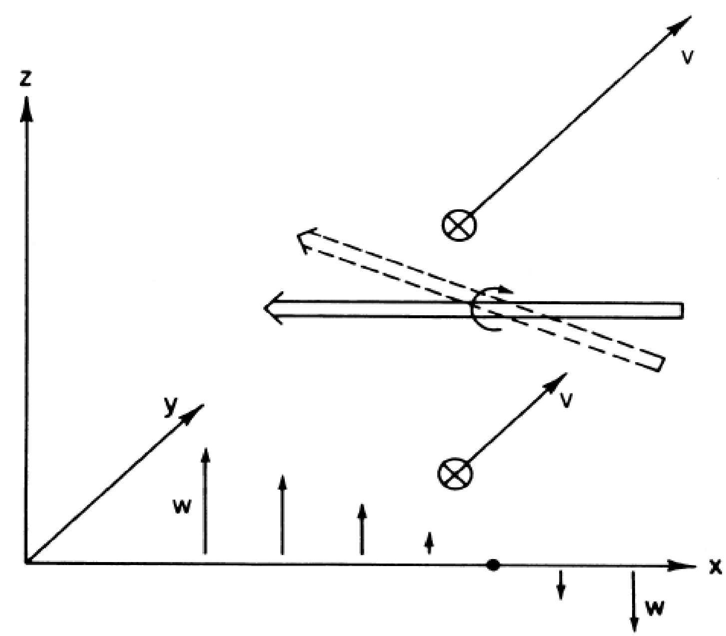
$$\frac{D}{Dt}(\zeta + f) = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

Twisting/tilting term:

$$-\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right)$$

represents vertical vorticity generated by the tilting of horizontally oriented components of vorticity into the vertical by a nonuniform vertical motion field.
Figure shows a region where the y component of velocity is increasing with height so that there is a component of shear vorticity oriented in the negative x direction as indicated by the double arrow.
If at the same time there is a vertical motion field in which w decreases with increasing x, advection by the vertical motion will tend to tilt the vorticity vector initially oriented parallel to x so that it has a component in the vertical. Thus, if $\partial v/\partial z > 0$ and $\partial w/\partial x < 0$, there will be a generation of positive vertical vorticity

Vorticity generation by the tilting of a horizontal vorticity vector (double arrow).



$$\frac{D}{Dt}(\zeta + f) = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

Tornadogenesis – example action of twisting/tilting