

# Planetary boundary layer and atmospheric turbulence.

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Lecture 02



## Equations for incompressible flow in Boussinesq approximation

$$\frac{\partial \mathbf{u}}{\partial t} + \underbrace{(\mathbf{u} \cdot \nabla) \mathbf{u}}_{inertia} = -\frac{1}{\rho_0} \nabla p + \underbrace{\nu \nabla^2 \mathbf{u}}_{friction} + \left[ \underbrace{b \hat{\mathbf{z}}}_{buoyancy} - \underbrace{f \hat{\mathbf{z}} \times \mathbf{u}}_{Coriolis} \right], \quad (1.2) \quad | \quad \nabla$$

$$\left[ \frac{\partial b}{\partial t} + \underbrace{(\mathbf{u} \cdot \nabla) b}_{advection} = \underbrace{\kappa \nabla^2 b}_{diffusion} \right], \quad (1.3)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (1.4)$$

A final remark about the only term that never appeared explicitly in the nondimensional numbers presented: the pressure force. Pressure can be formally eliminated from the equations. This is a consequence of the Boussinesq approximation. We simply need to take the divergence of the momentum equation in (1.2) and note that  $\nabla \cdot \mathbf{u}_t = 0$  because of incompressibility. This yields the relation,

$$\nabla^2 p = \rho_0 \nabla \cdot \left[ -(\mathbf{u} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \mathbf{u} + b \hat{\mathbf{z}} - f \hat{\mathbf{z}} \times \mathbf{u} \right]. \quad (1.14)$$

Since there are no time derivatives in (1.14), pressure is a purely diagnostic field, which is wholly slaved to  $\mathbf{u}$ . It can be calculated from (1.14) and then substituted for the pressure gradient force in the momentum equations. Its role is to maintain incompressibility under the action of all other forces. Therefore it would be redundant to introduce nondimensional parameters involving pressure, because those parameters could be expressed as combinations of the other parameters already discussed.

## Averaged equations.

Let us repeat the averaging procedure for the full Boussinesq equations. We start with the momentum equations,

$$\frac{\partial \bar{\mathbf{u}}}{\partial t} + (\bar{\mathbf{u}} \cdot \nabla) \bar{\mathbf{u}} - [\bar{b} \hat{\mathbf{z}} - f \hat{\mathbf{z}} \times \bar{\mathbf{u}}] = -\frac{1}{\rho_0} \nabla \bar{p} + \nu \nabla^2 \bar{\mathbf{u}} - \overline{(\mathbf{u}' \cdot \nabla) \mathbf{u}'}, \quad (2.28)$$

The extra term on the right hand side represent the effect of eddy motions on the mean flow. If the average operator is a time average over some time  $T$ , then eddy motions are those motions with time scales shorter than  $T$ . If the average operator is a spatial average over some scale  $L$ , then eddy motions are those motions with spatial scales shorter than  $L$ . If the average operator is an ensemble mean, then the eddy motions are those motions that change in every realizations, regardless of their scale, i.e. they represent the unpredictable or turbulent part of the flow.

Using the continuity equation,

$$\nabla \cdot \mathbf{u} = 0 \quad \Rightarrow \quad \nabla \cdot \bar{\mathbf{u}} = 0 \quad \text{and} \quad \nabla \cdot \mathbf{u}' = 0, \quad (2.29)$$

we can rewrite the averaged momentum equation as,

$$\frac{\partial \bar{\mathbf{u}}}{\partial t} + (\bar{\mathbf{u}} \cdot \nabla) \bar{\mathbf{u}} - [\bar{b} \hat{\mathbf{z}} - f \hat{\mathbf{z}} \times \bar{\mathbf{u}}] = -\frac{1}{\rho_0} \nabla \cdot [\bar{p} \mathbf{I} - \rho_0 \nu \nabla \bar{\mathbf{u}} + \rho_0 \overline{\mathbf{u}' \mathbf{u}'}]. \quad (2.30)$$

$\mathbf{I}$  is the unit matrix. These are the so-called **Reynolds momentum equation** and the eddy flux  $\rho_0 \overline{\mathbf{u}' \mathbf{u}'}$  represent the **Reynolds stress tensor** due to fluctuations in velocity field.

Reynolds-stress tensor.

## Równanie na średni wypór

We can similarly decompose the buoyancy equation into a mean and a fluctuating component,  $b = \bar{b} + b'$ , and write an equation for the mean component by substituting back into the buoyancy equation,

$$\frac{\partial \bar{b}}{\partial t} + (\bar{\mathbf{u}} \cdot \nabla) \bar{b} = -\nabla \cdot \left[ -\kappa \nabla \bar{b} + \overline{\mathbf{u}'b'} \right]. \quad (2.31)$$

The eddy term  $\overline{\mathbf{u}'b'}$  represent the **Reynolds eddy flux** of buoyancy.

The problem of turbulence might be considered to find a representation of such Reynolds stress and flux terms in terms of mean flow quantities. However, it is not at all clear that any reasonable general solution (or parameterization) even exists, short of computing the terms explicitly.

Reynolds eddy flux



Closure problem

## Najprostsze domknięcie: K-teoria (ang. K-theory):

The simplest closure for the Reynolds stress terms is one which relates  $\overline{\mathbf{u}'\mathbf{u}'}$  to the mean flow, by assuming a relation of the form,

$$\overline{\mathbf{u}'\mathbf{u}'} = -\nu_T \nabla \bar{\mathbf{u}}, \quad (2.32)$$

where  $\nu_T$  is the **eddy viscosity**. With such a closure the Reynolds stress term takes the same form as the mean viscosity term, but with a different viscosity. In essence, this closure states that turbulent eddies are similar to molecular motions that constantly act to redistribute and homogenize momentum. Similarly, for the tracer flux term we can define an **eddy diffusivity**

$$\overline{\mathbf{u}'b'} = -\kappa_T \nabla \bar{b}. \quad (2.33)$$

This eddy viscosity/diffusivity closure is the most commonly used in ocean modeling and interpretation of observations. At the crudest level  $\kappa_T$  and  $\nu_T$  are assumed to be constants; in more sophisticated models they are functions of the large scale flow. However, an eddy viscosity/diffusivity closure is rarely appropriate.

Scalar diffusivities are often denoted with  $\kappa$ , hence  $\kappa_T$  is the eddy diffusivity

## Turbulent Kinetic Energy TKE

1. What is the energy source for the turbulence, i.e. what aspects of the large-scale flow/buoyancy field lead to generation of turbulence?
2. Where does the turbulent energy go? Does it feedback on the large-scale, or is it "lost" to molecular processes?
3. Once generated, how is turbulent energy redistributed both spatially, and between different components of the turbulent flow?

To examine the turbulent energy budgets, we make a separation of fields into a large scale component and a small-scale component:

small scale does not mean small values !!!

$$u = \bar{U} + u' \quad (3.1)$$

where

$$\int_V u dV = \bar{U} ; \int_V u' dV = 0 \quad (3.2)$$

where  $V$  is the volume over which the spatial averaging takes place. The turbulent flow is arbitrarily defined to be any flow below the averaging scale. Of course this might include waves as well as turbulence, and here we will not be able to distinguish between the two.

# Kinetic Energy Budgets

$$KE = \frac{1}{2} \mathbf{u} \cdot \mathbf{u} \quad (3.3)$$

$$\text{KE of mean or large-scale flow} = KE_{mean} = \frac{1}{2} \bar{\mathbf{U}} \cdot \bar{\mathbf{U}} \quad (3.4)$$

$$\text{KE of turbulent or fluctuating flow} = KE_{turb} = \frac{1}{2} \overline{\mathbf{u}' \cdot \mathbf{u}'} \quad (3.5)$$

Kinetic energy of large-scale motion. Component x.

We begin with the Boussinesq equations, to derive equations for the evolution of  $KE_{mean}$ . Consider the portion due to each velocity component separately. In the x-direction, multiply the evolution equation for  $\bar{U}$  by  $\bar{U}$ :

$$\frac{\partial}{\partial t} \left( \frac{\bar{U}^2}{2} \right) + \bar{U} \bar{\mathbf{U}} \cdot \nabla \bar{U} + \overline{\bar{U} \mathbf{u}' \cdot \nabla \mathbf{u}'} = - \frac{\bar{U}}{\rho_0} \frac{\partial \bar{P}}{\partial x} + \nu \bar{U} \nabla^2 \bar{U} + f \bar{U} \bar{V} \quad (3.6)$$

$$\begin{aligned} \text{Now } \bar{U}(\bar{\mathbf{U}} \cdot \nabla \bar{U}) &= \bar{\mathbf{U}} \cdot \nabla (\bar{U}^2 / 2); \\ \overline{\bar{U}(\mathbf{u}' \cdot \nabla \mathbf{u}')} &= \nabla \cdot (\overline{\mathbf{u}' \mathbf{u}' U}) - \overline{\mathbf{u}' \mathbf{u}'} \cdot \nabla \bar{U} \\ \text{and } \bar{U} \nabla^2 \bar{U} &= \nabla^2 (\bar{U}^2 / 2) - \nabla \bar{U} \cdot \nabla \bar{U}. \end{aligned}$$



$$\left(\frac{\partial}{\partial t} + \bar{\mathbf{U}} \cdot \nabla\right) \frac{\bar{U}^2}{2} = -\frac{1}{\rho_0} \frac{\partial}{\partial x} (\overline{PU}) + \nu \nabla^2 \left(\frac{\bar{U}^2}{2}\right) - \nabla \cdot (\overline{\mathbf{u}'u'U}) - \nu \nabla \bar{U} \cdot \nabla \bar{U} + \overline{\mathbf{u}'u'} \cdot \nabla \bar{U} + f\bar{U}\bar{V} + \frac{P}{\rho_0} \frac{\partial \bar{U}}{\partial x} \quad (3.7)$$

The first three terms on the right hand side describe redistribution of mean KE within the volume:

$-\frac{1}{\rho_0} \frac{\partial}{\partial x} (\overline{PU})$ : pressure work

$\nu \nabla^2 \left(\frac{\bar{U}^2}{2}\right)$ : transport by viscous stresses

$-\nabla \cdot (\overline{\mathbf{u}'u'U})$ : transport by Reynolds stresses.

When integrated over a volume with no flux of KE in or out, these terms are zero.

The 4th and 5th terms represent net sources/sinks of mean KE:

$-\nu \nabla \bar{U} \cdot \nabla \bar{U}$ : loss of KE to dissipation;

$\overline{\mathbf{u}'u'} \cdot \nabla \bar{U}$ : transfer of mean KE to the fluctuating/turbulent part of the flow.

The 6th and 7th terms represent transfer of kinetic energy from the  $\bar{U}$ -component of the flow to the  $\bar{V}$ - and  $\bar{W}$ - components.



Kinetic energy of large-scale motion. Components y and z.

We can write down similar equations for the time-evolution of  $\bar{V}^2$  and  $\bar{W}^2$ :

$$\left(\frac{\partial}{\partial t} + \bar{\mathbf{U}} \cdot \nabla\right) \frac{\bar{V}^2}{2} = -\frac{1}{\rho_0} \frac{\partial}{\partial y} (\bar{P}\bar{V}) + \nu \nabla^2 \left(\frac{\bar{V}^2}{2}\right) - \nabla \cdot (\bar{\mathbf{u}}'v'\bar{V}) - \nu \nabla \bar{V} \cdot \nabla \bar{V} + \bar{\mathbf{u}}'v' \cdot \nabla \bar{V} - f\bar{U}\bar{V} + \frac{\bar{P}}{\rho_0} \frac{\partial \bar{V}}{\partial y} \quad (3.8)$$

$$\left(\frac{\partial}{\partial t} + \bar{\mathbf{U}} \cdot \nabla\right) \frac{\bar{W}^2}{2} = -\frac{1}{\rho_0} \frac{\partial}{\partial z} (\bar{P}\bar{W}) + \nu \nabla^2 \left(\frac{\bar{W}^2}{2}\right) - \nabla \cdot (\bar{\mathbf{u}}'w'\bar{W}) - \nu \nabla \bar{W} \cdot \nabla \bar{W} + \bar{\mathbf{u}}'w' \cdot \nabla \bar{W} + \bar{W}\bar{b} + \frac{\bar{P}}{\rho_0} \frac{\partial \bar{W}}{\partial z} \quad (3.9)$$

The  $\bar{V}^2$  equation contains terms analogous to the  $\bar{U}^2$  equation, while the  $\bar{W}^2$  equation lacks the coriolis term (since we have assumed Coriolis is aligned with the vertical), but includes a buoyancy term, through which large scale potential energy is converted to kinetic energy.

## Energia kinetyczna przepływu średniego.

If we sum these three equations, to obtain the evolution equation for  $1/2\overline{\mathbf{U}} \cdot \overline{\mathbf{U}}$ , and rewrite it in Einstein notation, we have

$$\left(\frac{\partial}{\partial t} + \overline{U}_j \frac{\partial}{\partial x_j}\right) \frac{\overline{U}_i^2}{2} = \frac{\partial}{\partial x_j} \left( -\frac{\overline{P}}{\rho_0} \overline{U}_j \delta_{ij} + \nu \frac{\partial \overline{U}_i^2}{\partial x_j} - \overline{u'_j u'_i U_i} \right) - \nu \left( \frac{\partial \overline{U}_i}{\partial x_j} \right)^2 + \overline{u'_j u'_i} \frac{\partial \overline{U}_i}{\partial x_j} + \overline{Wb} \quad (3.10)$$

where the first three terms on the right hand side are once again the transport terms: **pressure work**, **transport by viscous stresses** and **transport by Reynolds stresses**. The 4th term is again the **dissipation**, and the 5th term represents the transfer of kinetic energy between the mean flow and the turbulent fluctuating flow. This term is known as the **Shear production term**, since the shear in the mean flow (finite gradients in  $\overline{U}_i$ ) leads to production of turbulent kinetic energy. The final term on the right hand side is the **large-scale buoyancy production** term.

Note that the terms  $\overline{P}/\rho_0 \partial \overline{U}_i / \partial x_i$ , which transfer kinetic energy between the different components of the flow  $\overline{U}, \overline{V}, \overline{W}$  vanish from the equation for the total, due to the divergence relation  $\nabla \cdot \mathbf{U} = 0$ . The Coriolis term similarly does not influence the total kinetic energy, but only its transfer between  $\overline{U}$  and  $\overline{V}$  components.

## Turbulent kinetic energy. Component x.

To find the evolution equation for the x-component of the turbulent kinetic energy (TKE) multiply  $\partial u'/\partial t = \partial U/\partial t - \partial \bar{U}/\partial t$  by  $u'$  and take the spatial average:

$$\left(\frac{\partial}{\partial t} + \bar{\mathbf{U}} \cdot \nabla\right) \frac{\overline{u'^2}}{2} = -\frac{1}{\rho_0} \frac{\partial}{\partial x} \overline{u'p'} + \nu \nabla^2 \frac{\overline{u'^2}}{2} - \nabla \cdot \left( \frac{\overline{\mathbf{u}'u'^2}}{2} \right) - \nu \nabla u' \cdot \nabla u' - \overline{u'u' \cdot \nabla U} + \overline{f u' v'} + \frac{1}{\rho_0} \overline{p' \frac{\partial u'}{\partial x}} \quad (3.11)$$

Comparing with the equation for  $\bar{U}^2/2$  we see that once again, there are three transport terms: **pressure work**, **transport by viscous stresses** and **transport by Reynolds stresses**, and **loss of TKE to dissipation**. The **shear production terms** appears once again, but with the opposite sign to that in eqn 3.7 - hence this term represents no net loss of KE but a transfer between mean and turbulent components.

## Turbulent kinetic energy. Components y and z.

The analogous equations for  $\overline{v'^2}$  and  $\overline{w'^2}$  are:

$$\left(\frac{\partial}{\partial t} + \bar{\mathbf{U}} \cdot \nabla\right) \frac{\overline{v'^2}}{2} = -\frac{1}{\rho_0} \frac{\partial \overline{v'p'}}{\partial y} + \nu \nabla^2 \frac{\overline{v'^2}}{2} - \nabla \cdot \left( \frac{\overline{\mathbf{u}'v'^2}}{2} \right) - \nu \nabla v' \cdot \nabla v' - \overline{v' \mathbf{u}' \cdot \nabla \bar{V}} - \overline{f u' v'} + \frac{1}{\rho_0} p' \frac{\partial v'}{\partial y} \quad (3.12)$$

$$\left(\frac{\partial}{\partial t} + \bar{\mathbf{U}} \cdot \nabla\right) \frac{\overline{w'^2}}{2} = -\frac{1}{\rho_0} \frac{\partial \overline{w'p'}}{\partial z} + \nu \nabla^2 \frac{\overline{w'^2}}{2} - \nabla \cdot \left( \frac{\overline{\mathbf{u}'w'^2}}{2} \right) - \nu \nabla w' \cdot \nabla w' - \overline{w' \mathbf{u}' \cdot \nabla \bar{W}} + \overline{w'b'} + \frac{1}{\rho_0} p' \frac{\partial w'}{\partial z} \quad (3.13)$$

The  $\overline{w'^2}$  equation contains an additional term: the buoyant production of kinetic energy, representing conversion from potential to kinetic energy.

## Turbulent kinetic energy.

Adding the contributions due to the 3 velocity components and rewriting in Einstein notation we have

$$\left(\frac{\partial}{\partial t} + \bar{U}_j \frac{\partial}{\partial x_j}\right) \frac{\overline{u_i'^2}}{2} = \frac{\partial}{\partial x_j} \left( -\frac{1}{\rho_0} \overline{u_i' p'} \delta_{ij} + \nu \frac{\partial}{\partial x_j} \frac{\overline{u_i'^2}}{2} - \overline{u_j' u_i' u_i'} \right) - \nu \overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2} - \overline{u_j' u_i'} \frac{\partial}{\partial x_j} \bar{U}_i + \overline{b' w'}$$
(3.14)

Hence TKE is generated by (a) shear production,

$$P = -\overline{u_j' u_i'} \frac{\partial}{\partial x_j} \bar{U}_i$$
(3.15)

and (b) buoyant production

$$B = \overline{b' w'}$$
(3.16)

and lost through dissipation

$$\epsilon = \nu \overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2}$$
(3.17)

The buoyant production term may be either positive (generation of kinetic energy, loss of potential energy) or negative (loss of KE, increase in PE).

Simplifications.

**Stationary turbulence** – statistics do not depend on time (are statistically invariant under time translations  $t^* \rightarrow t+a$ ), e.g.

$$\langle u_i(\mathbf{x}, t) \rangle = f_i(\mathbf{x}), \quad \frac{\partial \langle u_i(\mathbf{x}, t) \rangle}{\partial t} = 0$$

**Homogeneous turbulence** – statistics do not depend on position where they are measured, (are statistically invariant under space translations  $\mathbf{x}_1^* \rightarrow \mathbf{x}_1+a$ ,  $\mathbf{x}_2^* \rightarrow \mathbf{x}_2+a$ ) e.g.

$$\langle u_i(\mathbf{x}, t) \rangle = g_i(t), \quad \langle u_i(\mathbf{x}_1, t) u_j(\mathbf{x}_2, t) \rangle = h_{ij}(\mathbf{x}_1 - \mathbf{x}_2, t),$$

**Homogeneous, isotropic turbulence** – statistics additionally do not depend on direction, i.e. are invariant under rotations and reflections of the coordinate system

$$\langle u_i(\mathbf{x}, t) \rangle = g_i(t), \quad \langle u_i(\mathbf{x}_1, t) u_j(\mathbf{x}_2, t) \rangle = h_{ij}(|\mathbf{x}_1 - \mathbf{x}_2|, t),$$

## Stationary turbulence

Now we see the importance of turbulence to the total energy of the system. The viscous terms in the KE of the mean flow can be quite small, so that most KE loss from the mean flow might be due to transfer to the turbulence via the shear production term. Then once in the turbulent regime the KE may either be dissipated, or converted to potential energy via the buoyancy term.

Note that the TKE equations are far from isotropic. Shear production reflects any isotropy in the mean flow, while buoyant production appears only in the  $\overline{w'^2}$  equation. The pressure interaction terms (and coriolis terms) transfer the TKE between different velocity components.

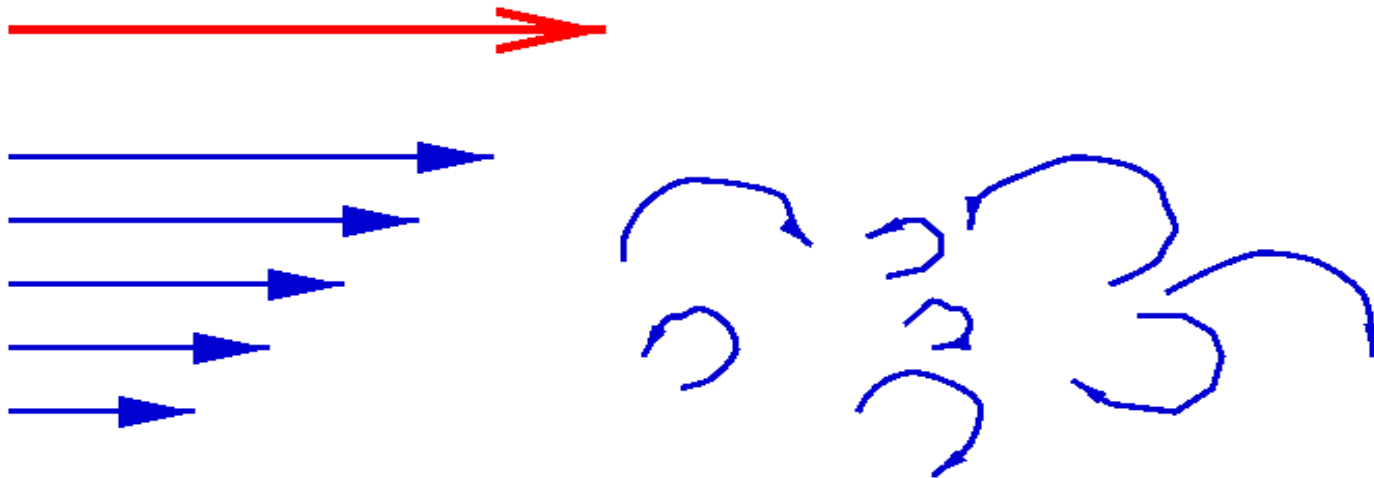
If (a) the Turbulence is stationary ( $D/Dt(KE) = 0$ ), and (b) we integrate over a volume bounded by surfaces through which there are no energy fluxes, then there is a balance between production and dissipation of TKE:

$$P + B = \epsilon \quad (3.18)$$



## Pure Shear flow

If the large-scale flow consists of a pure shear flow of the form  $(U, V, W) = (U(z), 0, 0)$  with no buoyancy forcing, then the TKE shear production term becomes  $\overline{u'w'}\partial\overline{U}/\partial z$ , and it appears only in the  $\overline{u'^2}$  equation. Hence the large-scale flow directly generates TKE only in the x-direction.  $\overline{v'^2}$  and  $\overline{w'^2}$  are then generated by transfer of TKE from the x-direction via the pressure interaction terms.

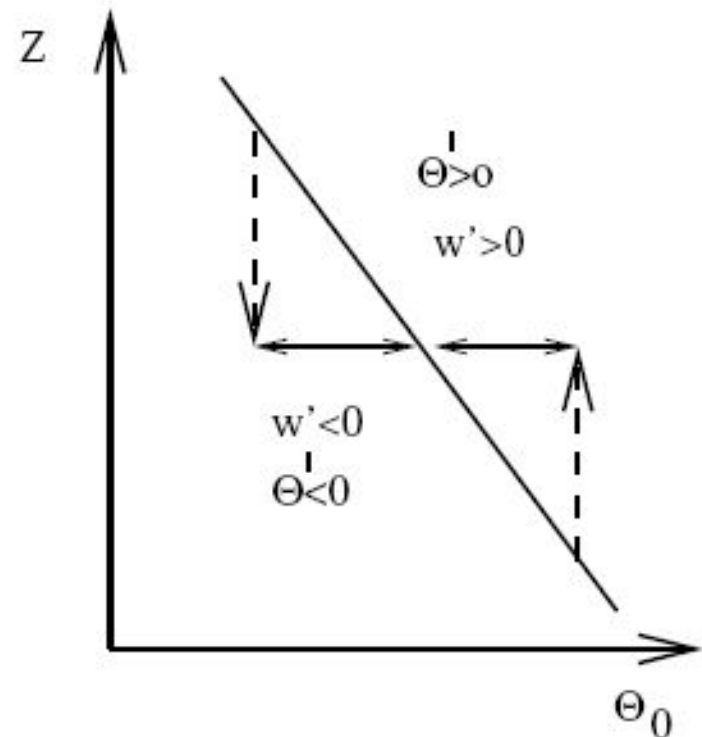


## Pure convective flow

If there is no large-scale flow, and turbulence is generated entirely through buoyancy forcing,  $P = 0$ . The source of TKE is  $\overline{w'b'}$ , and TKE is directly generated only in the z-direction. Again,  $\overline{u'^2}$  and  $\overline{v'^2}$  are then generated by transfer of TKE via the pressure interaction terms.

**BPL** - produkcja lub ubytek wskutek sił wyporu

$$BPL = \overline{(w'\Theta')}\left(\frac{g}{\theta_0}\right)$$



## Flux Richardson number

Obviously an important parameter is the ratio between shear and buoyancy production of TKE, known as the flux Richardson number:

$$R_f = \frac{B}{P} = \frac{\overline{w'b'}}{\overline{u'w'}\partial\bar{U}/\partial z} \quad (3.19)$$

If  $\partial U/\partial z > 0$ , then  $\overline{u'w'} < 0$  if the flux of momentum is downgradient (positive eddy viscosity). Hence we expect  $\overline{u'w'}\partial\bar{U}/\partial z < 0$ .  $R_f < 0$  therefore if  $\overline{w'b'} > 0$  (convective instability, buoyancy generating TKE), and  $R_f > 0$  if  $\overline{w'b'} < 0$  (stable stratification, loss of TKE to PE).

### 3.3 Tracer variance equation

As for the kinetic energy components, we can derive a time evolution equation for  $\overline{T'^2}$  (where  $T$  could be any conserved tracer) by multiplying the equation for  $T'$  through by  $T'$ :

$$\left(\frac{\partial}{\partial t} + \overline{\mathbf{U}} \cdot \nabla\right) \frac{\overline{T'^2}}{2} = \kappa \nabla^2 \frac{\overline{T'^2}}{2} - \nabla \cdot \overline{\mathbf{u}' T'^2 / 2} - \overline{T' \mathbf{u}' \cdot \nabla T} - \kappa \overline{\nabla T' \cdot \nabla T'} \quad (3.27)$$

Like the kinetic energy equation, the first two terms on the right hand side are transport terms (transport by viscous stresses and transport by Reynolds stresses), while tracer variance is produced by the term

$$P_T = -\overline{T' \mathbf{u}' \cdot \nabla T} \quad (3.28)$$

and dissipated by the term

$$\epsilon_T = \kappa \overline{\nabla T' \cdot \nabla T'} \quad (3.29)$$

If there is a balance between production and dissipation of tracer variance (implying stationarity and homogeneity) then

$$P_T = \epsilon_T \quad (3.30)$$

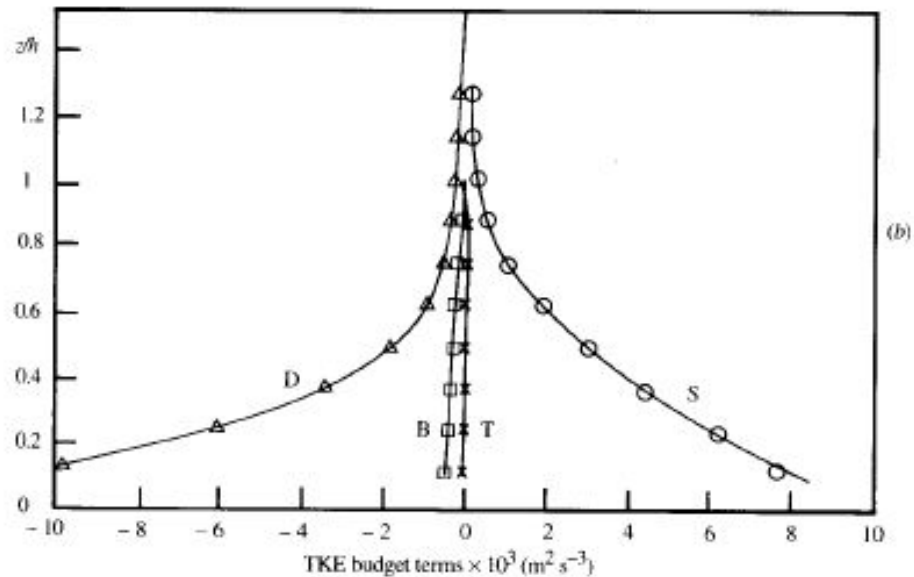
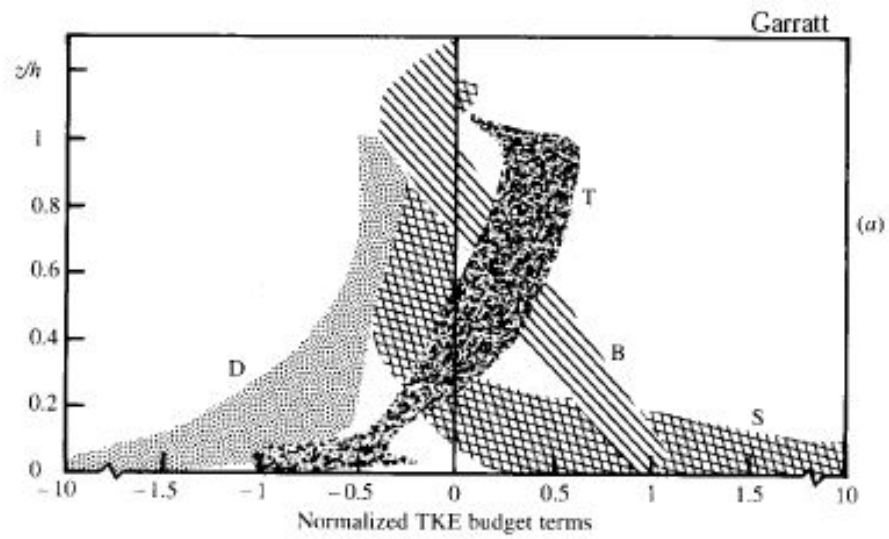


Fig. 2.4 Terms in the TKE equation (2.74b) as a function of height, normalized in the case of the clear daytime ABL (a) through division by  $w_*^3/h$ ; actual terms are shown in (b) for the clear night-time ABL. Profiles in (a) are based on observations and model simulations as described in Stull (1988; Figure 5.4), and in (b) are from Lenschow *et al.* (1988) based on one aircraft flight. In both, B is the buoyancy term, D is dissipation, S is shear generation and T is the transport term. Reprinted by permission of Kluwer Academic Publishers.



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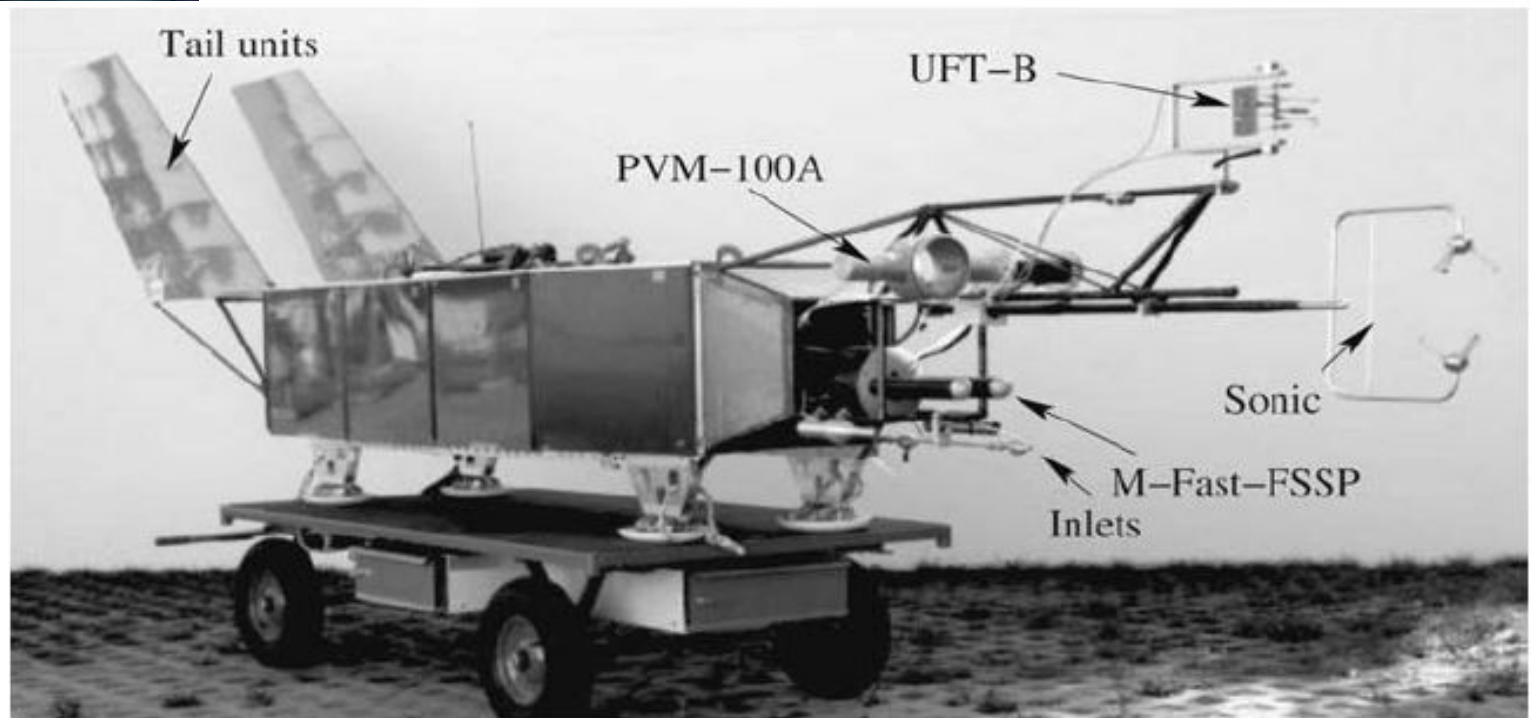


FIG. 1. The turbulence payload ACTOS with sonic, UFT-B, PVM-100A, and M-Fast-FSSP. Also shown are the inlets for humidity and aerosol particle measurements.

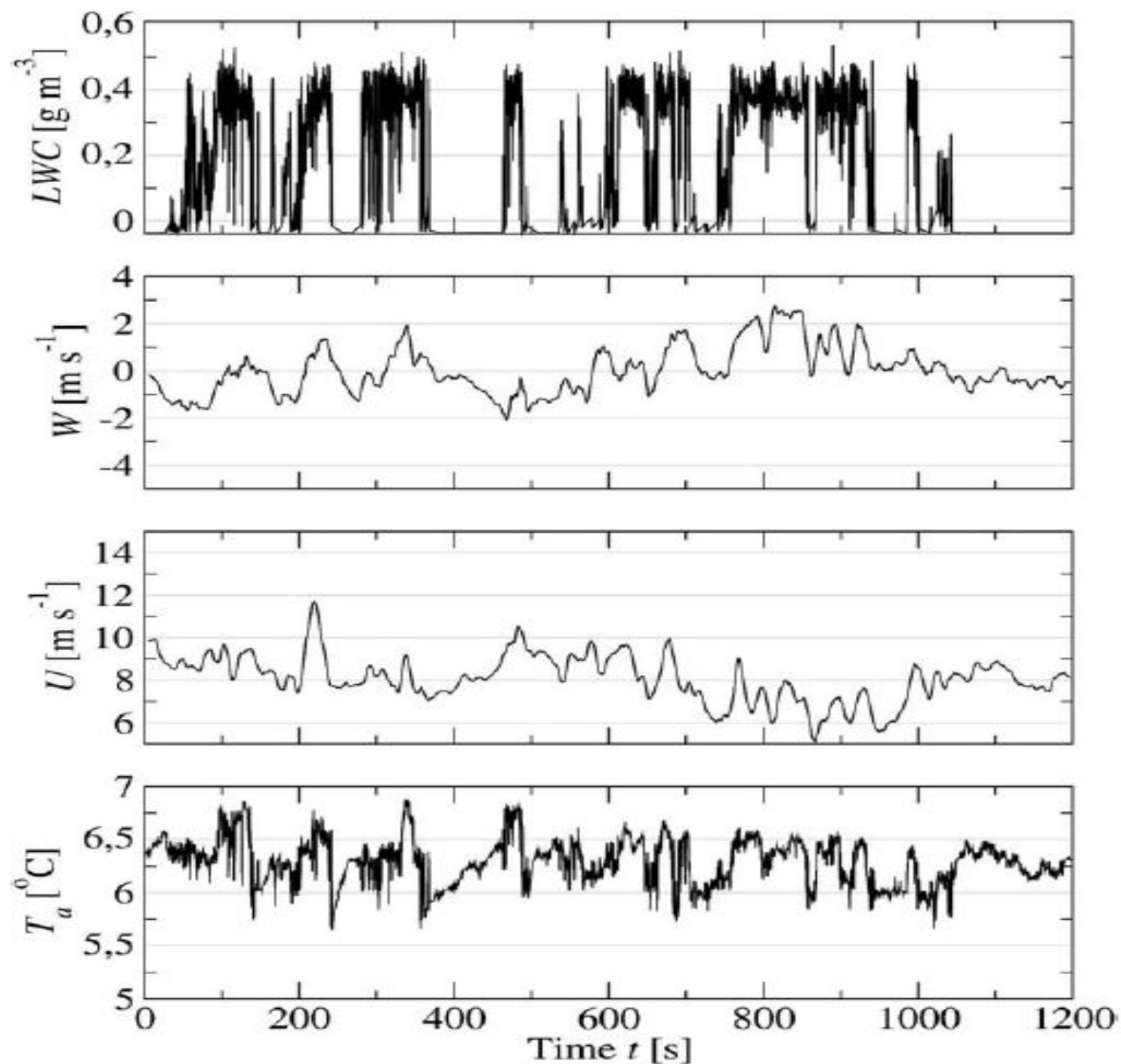


FIG. 3. Time series of LWC, vertical wind velocity  $W$ , horizontal wind velocity  $U$ , and temperature  $T_a$  as measured with ACTOS at a height of around 760 m AGL on 21 May 2003 during the BBC2 campaign.



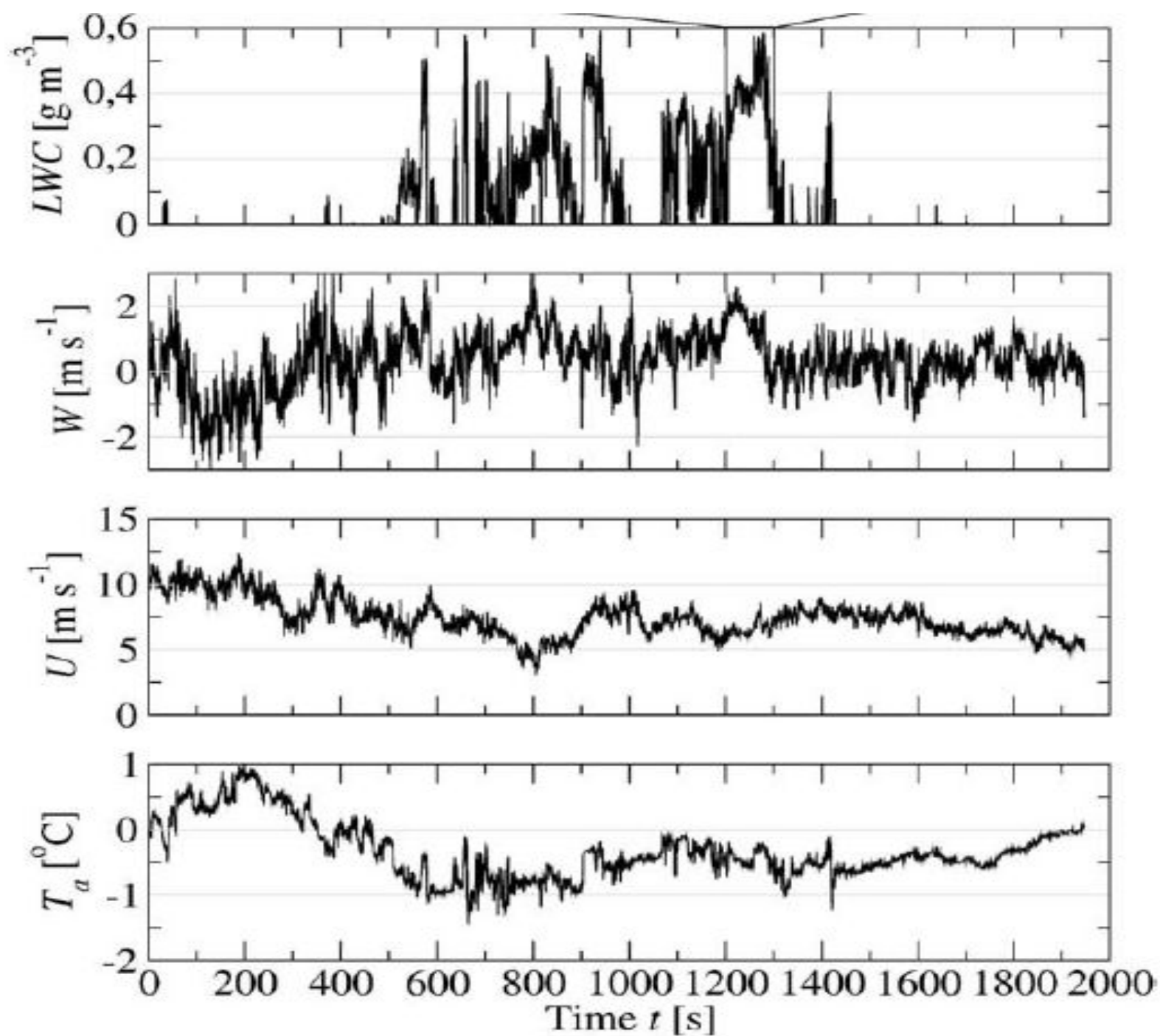


FIG. 4. Time series of LWC, vertical wind velocity  $W$ , horizontal wind velocity  $U$ , and temperature  $T_a$  as measured with ACTOS at a height of around 1540 m AGL on 16 May 2004 during the INSPECTRO campaign.

Probability that a random variable  $\mathbf{U}$  is contained within  $\mathbf{V}$  and  $\mathbf{V}+d\mathbf{V}$ :

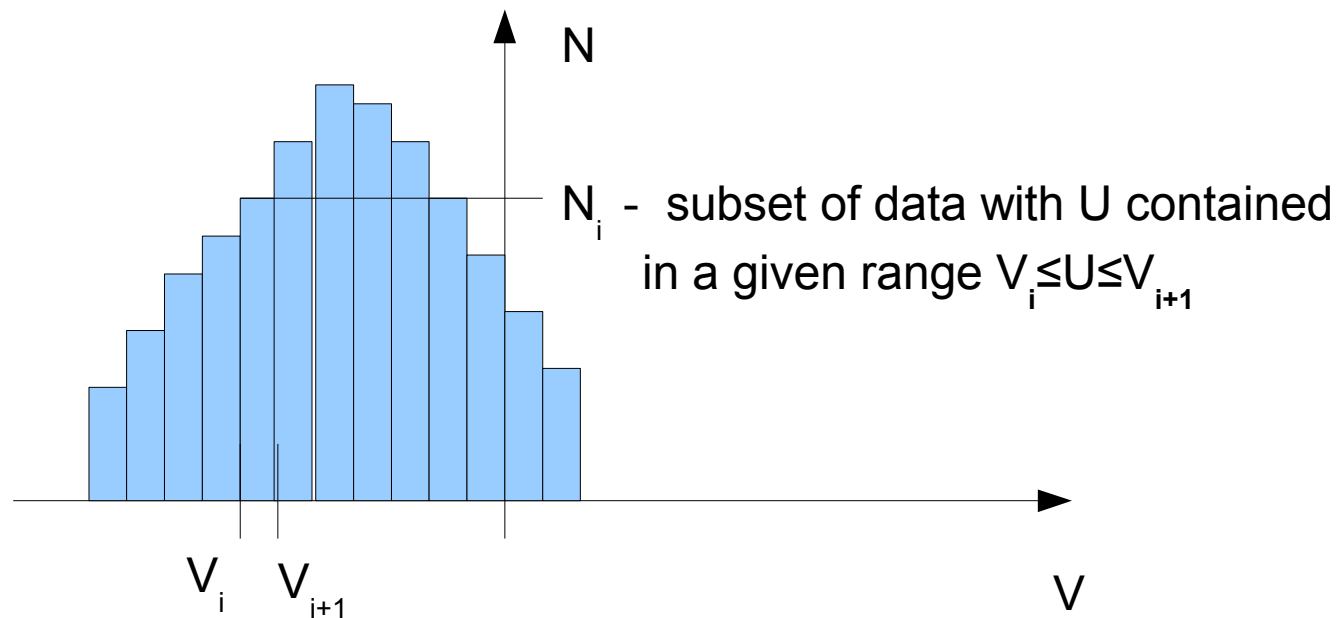
$$P(\mathbf{V} \leq \mathbf{U} \leq \mathbf{V} + d\mathbf{V}) = f(\mathbf{V}) d\mathbf{V}$$

**PDF** of velocity at point  $\mathbf{x}$  and at time  $t$ :  $f(\mathbf{V}; \mathbf{x}, t)$

Ensemble average:

$$\langle G(\mathbf{U}(\mathbf{x}, t)) \rangle = \int G(\mathbf{v}) f(\mathbf{v}; \mathbf{x}, t) d\mathbf{v}$$

Calculating PDF from a set of data (box method):



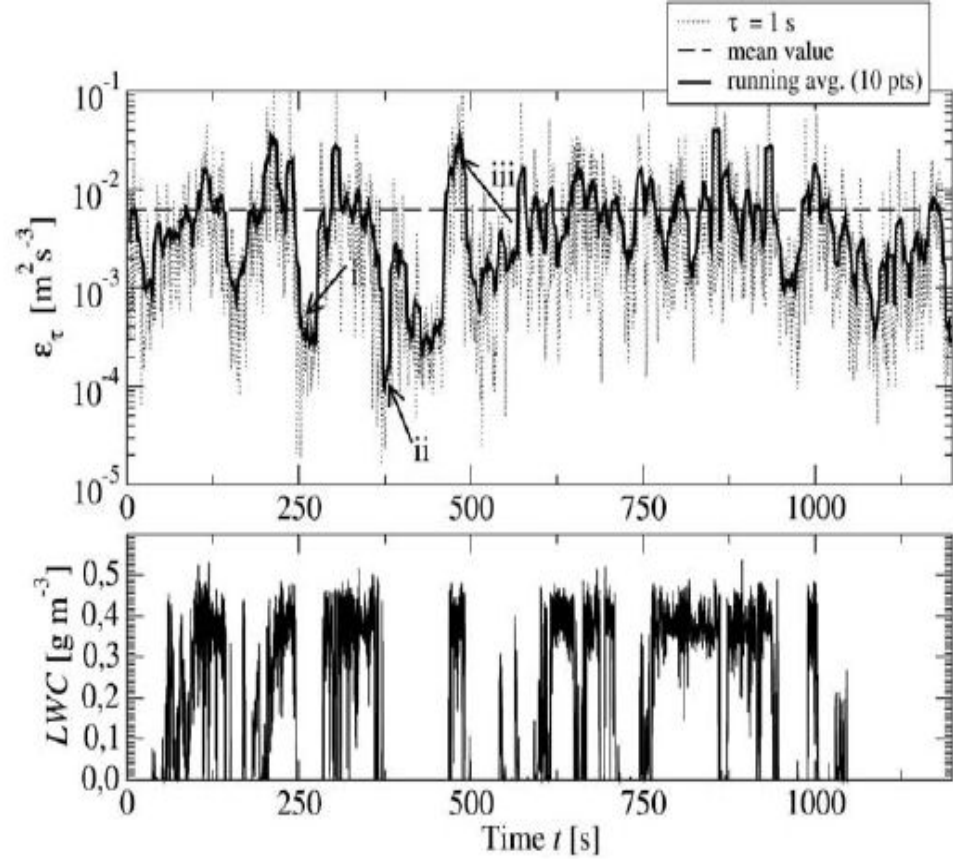


FIG. 8. (top) Time series of local energy dissipation rate  $\epsilon_\tau$  and (bottom) LWC of BBC2 data. The integration time  $\tau$  for  $\epsilon_\tau$  is 1 s; a running average over 10 points is included.

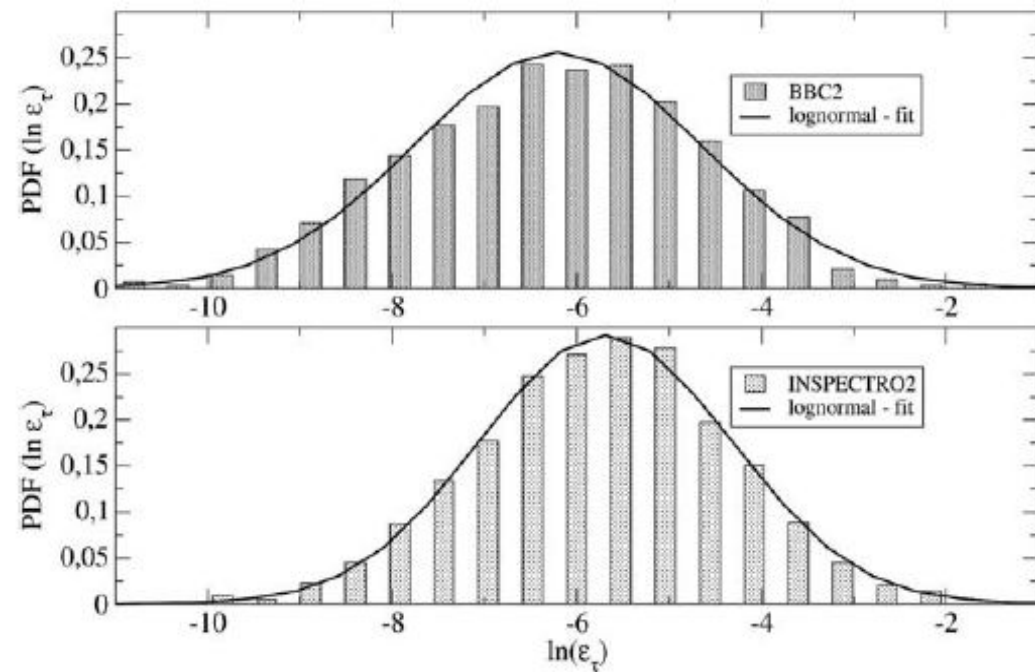



FIG. 11. PDF of natural logarithm of local energy dissipation rates  $\epsilon_\tau$ . A Gauss fit is included for reference.

An aerial photograph taken from an aircraft, showing a vast, textured sea of stratocumulus clouds below. The sun is low on the horizon to the left, creating a bright glow and lens flare. The sky above is a clear, pale blue. The right edge of the image shows the dark silhouette of an aircraft wing and part of a propeller.

**POST** (Physics of Stratocumulus Top)



# POST – Physics of Stratocumulus Top, California, 2008

aerosol (CCN)



microphysics

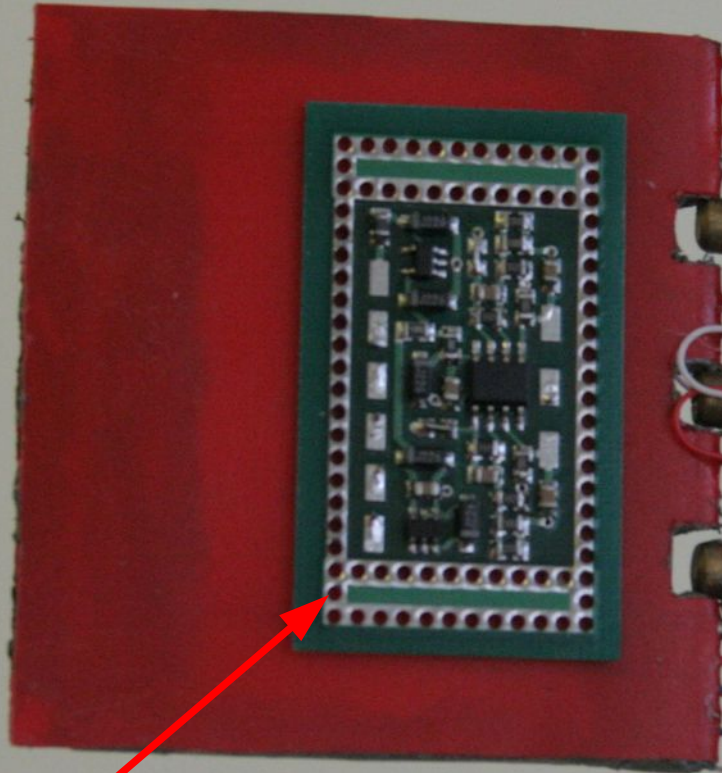


temperature,  
humidity,  
liquid water,  
turbulence,

droplet counting



UFT-F 2



electronics/amplifiers  
(here exposed for  
demonstration only)

protective rod

sensing wires,  
 $\text{\O} 2.5\mu\text{m}$

