CLOUD PHYSICS - tutorial 4

Condensational growth of a cloud droplet

The condensational growth of a cloud droplet, governed by the diffusion of water vapor, is described by:

$$r\frac{dr}{dt} = \frac{S - S_{eq}}{F_d + F_k}$$

where

$$F_d = \frac{\rho_l R_v T}{D e_s(T)}, \quad F_k = \left(\frac{L}{R_v T} - 1\right)\frac{\rho_l L}{KT}$$

D is the diffusion coefficient of water vapor in air. It depends on temperature and pressure: $D = 2.11 \cdot 10^{-5} \left(\frac{T}{T_0}\right)^{1.94} \left(\frac{p_0}{p}\right)\ \mathrm{m^2 s^{-1}}$, where $p_0 = 101\,325\,\mathrm{Pa}$.
K is the thermal conductivity coefficient: $K = 4.1868{\cdot}10^{-3}[5.69{+}0.017(T{-}T_0)]\, W m^{-1} K^{-1}$.

$$S_{eq} = 1 \quad OR \quad S_{eq} = \frac{r^3 - r_d^3}{r^3 - (1-\kappa)r_d^3} exp(\frac{A}{r}) \approx 1 + \frac{A}{r} - \frac{\kappa r_d^3}{r^3} \quad \text{where} \quad A(T) = \frac{2\sigma}{\rho_l R_v T}$$

Assume that $\sigma = 75.64 \cdot 10^{-3} N m^{-1}$.

1. Show how $\xi = 1/(F_d + F_k)$ depends on temperature, T, and pressure, p.

2. Determine the droplet radius growth rate, dr/dt, as a function of r for various supersaturation values, S= 0.1, 0.3, 0.5%, and for the equilibrium saturation, S_{eq}, expressed either via the κ-Köhler relation (assuming $\kappa = 0.67$, $r_d = 0.01\mu m$), or by setting $S_{eq} = 1$.

   How does it depend on temperature and pressure?

3. Calculate $r(t)$. How does it depend on saturation, temperature, and pressure?

4. Corrections (see *A Short Course in Cloud Physics* by R.R. Rogers and M.K. Yau). In the condensational growth equation, the diffusion and thermal conductivity coefficients must be replaced by D' and K', as shown below.

$$D' = D\frac{r}{r + l_\beta}, \quad l_\beta = \frac{D}{\beta}\left(\frac{2\pi}{R_v T}\right)^{1/2}, \quad \beta = 0.04$$

$$K' = K\frac{r}{r + l_\alpha}, \quad l_\alpha = \left(\frac{K}{\alpha p}\right)\frac{(2\pi R_d T)^{1/2}}{c_v + R_d/2}, \quad \alpha = 1$$

   where l_α and l_β are the length scales.

   Show how these length scales depend on temperature and pressure.

   Demonstrate how these corrections affect the condensational growth of cloud droplets. Accordingly, demonstrate how the ratio ξ/ξ_{corr} (where $\xi = 1/(F_d + F_k)$) varies as a function of temperature and pressure.