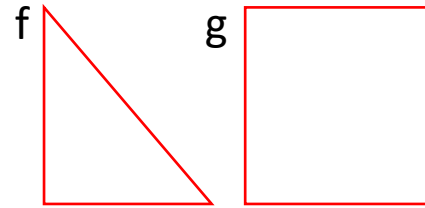
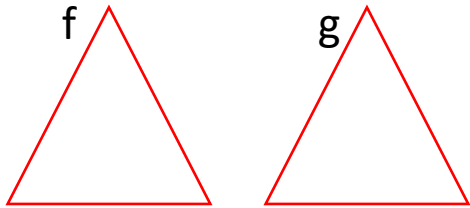


WSTĘP DO OPTYKI FOURIEROWSKIEJ

dr hab. Rafał Kasztelanic

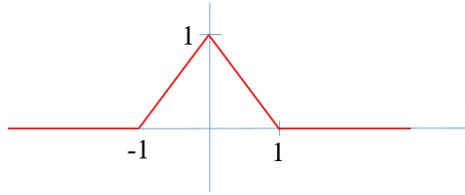
WYKŁAD 2

Spot



Spot

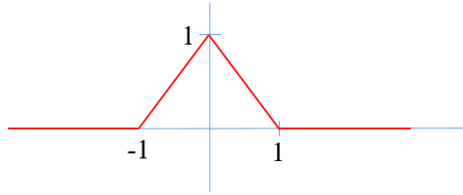
$$\Lambda(x) = \begin{cases} 1 - |x| & |x| < 1 \\ 0 & |x| \geq 1 \end{cases}$$



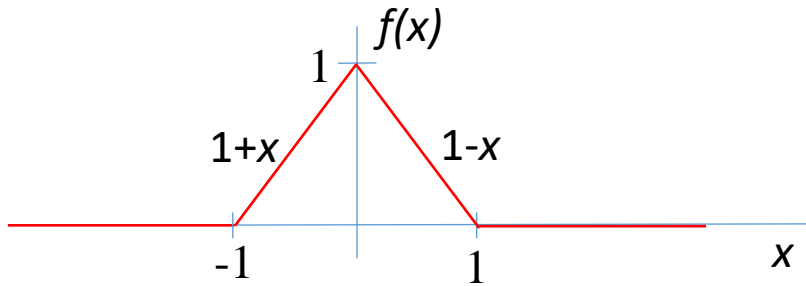
$$h(x) = \int_{-\infty}^{\infty} f(x-x')g(x')dx'$$

Spot

$$\Lambda(x) = \begin{cases} 1 - |x| & |x| < 1 \\ 0 & |x| \geq 1 \end{cases}$$

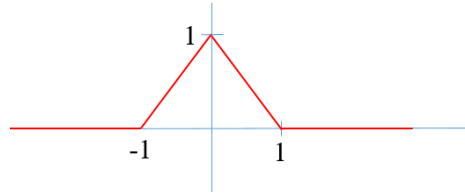


$$h(x) = \int_{-\infty}^{\infty} f(x-x')g(x')dx'$$

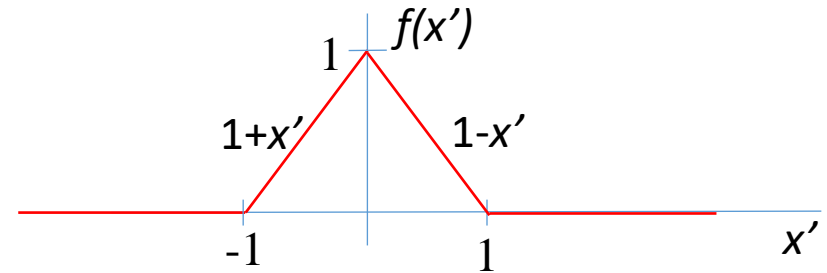
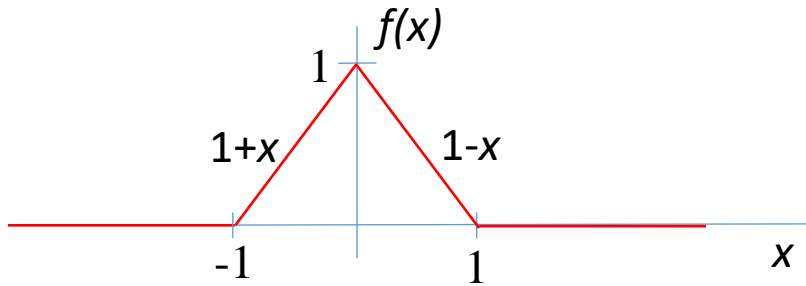


Spot

$$\Lambda(x) = \begin{cases} 1 - |x| & |x| < 1 \\ 0 & |x| \geq 1 \end{cases}$$

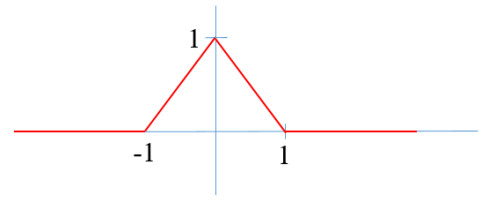


$$h(x) = \int_{-\infty}^{\infty} f(x-x')g(x')dx'$$

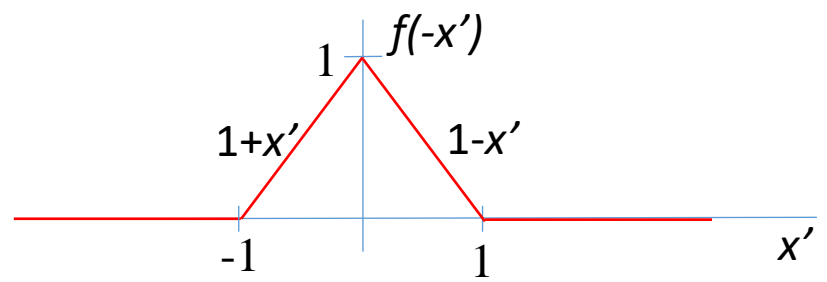
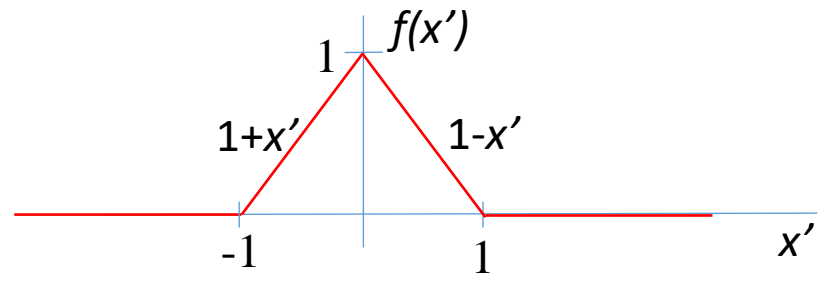
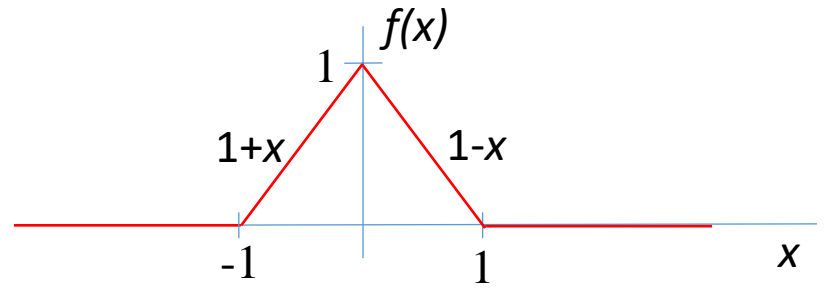


Plot

$$\Lambda(x) = \begin{cases} 1 - |x| & |x| < 1 \\ 0 & |x| \geq 1 \end{cases}$$

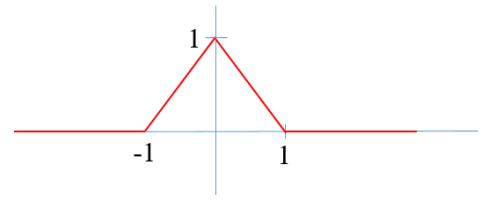


$$h(x) = \int_{-\infty}^{\infty} f(x-x')g(x')dx'$$

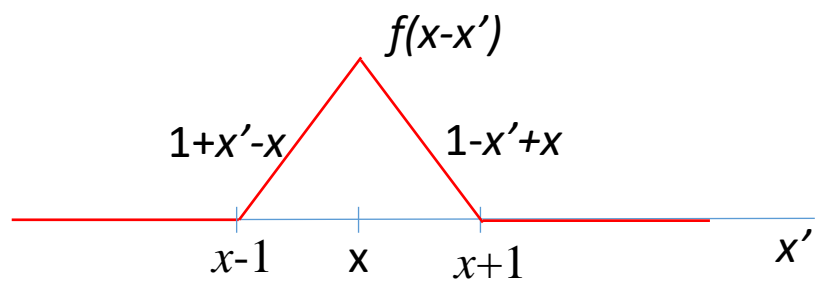
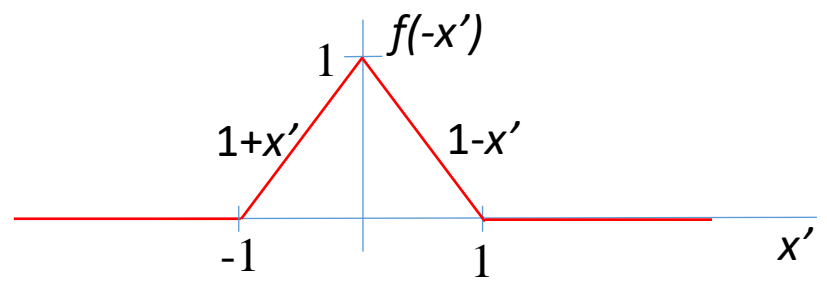
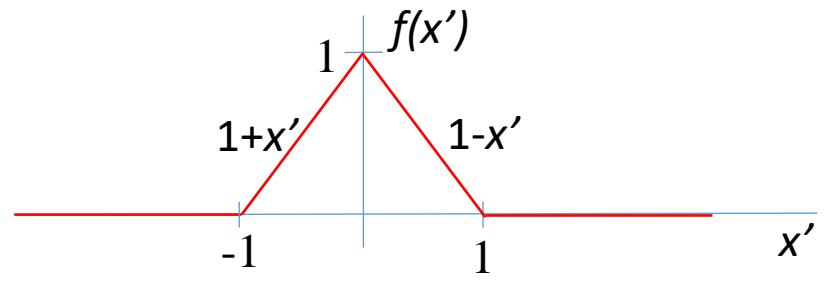
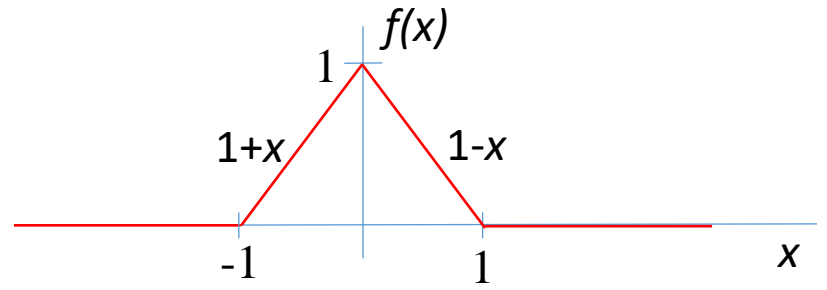


Plot

$$\Lambda(x) = \begin{cases} 1 - |x| & |x| < 1 \\ 0 & |x| \geq 1 \end{cases}$$

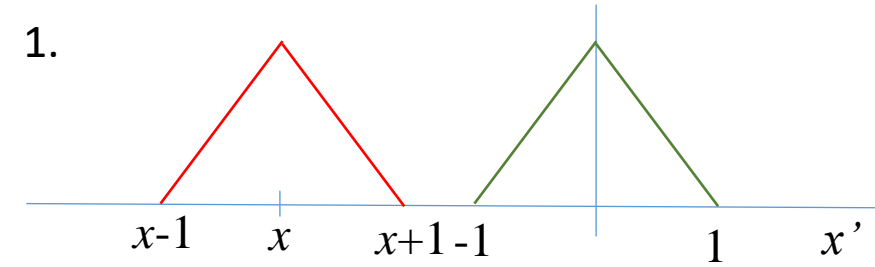


$$h(x) = \int_{-\infty}^{\infty} f(x-x')g(x')dx'$$



Spot

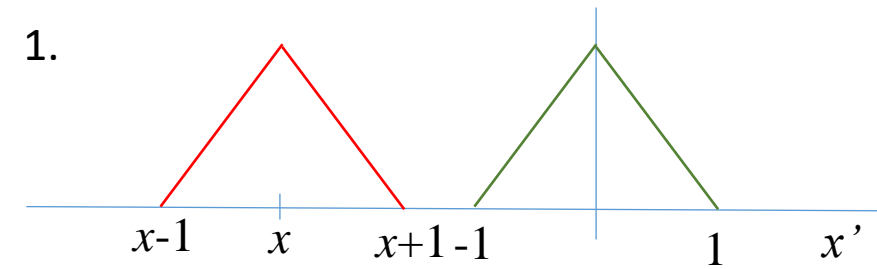
1.



$$x + 1 < -1 \rightarrow x < -2$$

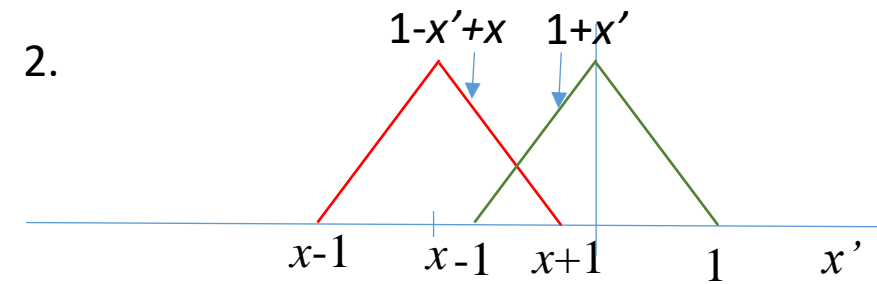
$$h(x) = 0$$

Split



$$x + 1 < -1 \rightarrow x < -2$$

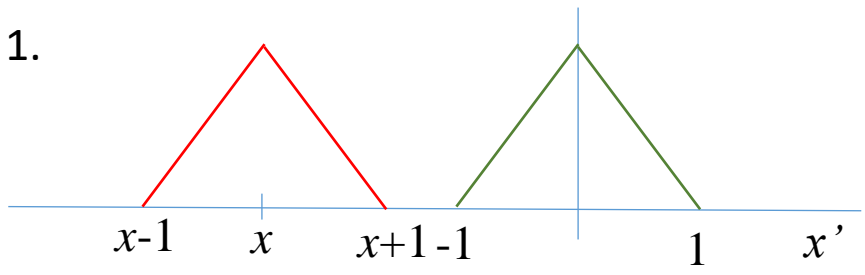
$$h(x) = 0$$



$$-1 < x + 1 < 0 \rightarrow -2 < x < -1$$

Spot

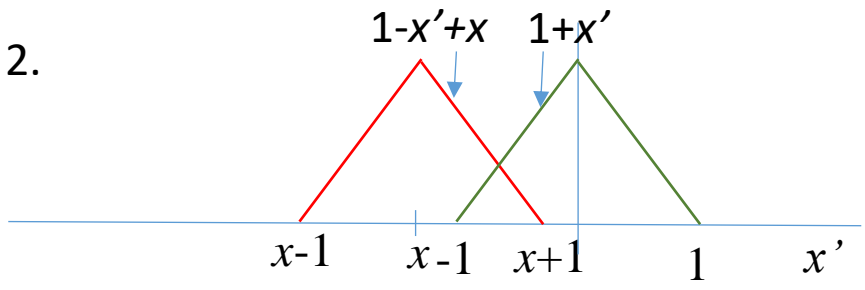
1.



$$x + 1 < -1 \rightarrow x < -2$$

$$h(x) = 0$$

2.



$$-1 < x + 1 < 0 \rightarrow -2 < x < -1$$

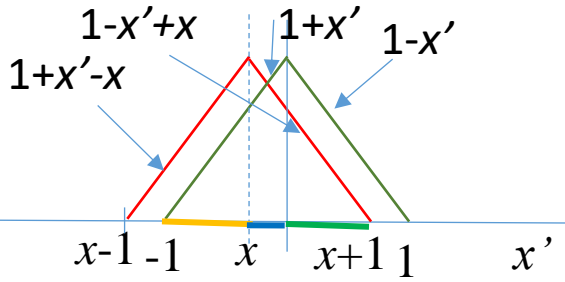
$$h(x) = \int_{-1}^{x+1} (1 + x')(1 - x' + x) dx' =$$

$$= \int_{-1}^{x+1} (1 + x - x'^2 + xx') dx' = (1 + x) \int_{-1}^{x+1} dx' + x \int_{-1}^{x+1} x' dx' - \int_{-1}^{x+1} x'^2 dx' =$$

$$\underbrace{-(x + 1)(x + 2)}_{-\frac{x}{2}((x + 1)^2 - 1)} \quad \underbrace{-\frac{1}{3} - \frac{(x + 1)^3}{3}}$$

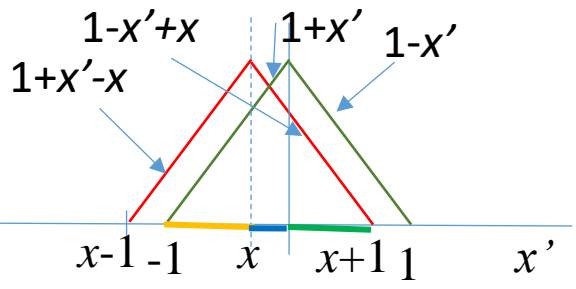
$$h(x) = \frac{1}{6}(x + 2)^3$$

3.



$$0 < x + 1 < 1 \rightarrow -1 < x < 0$$

3.



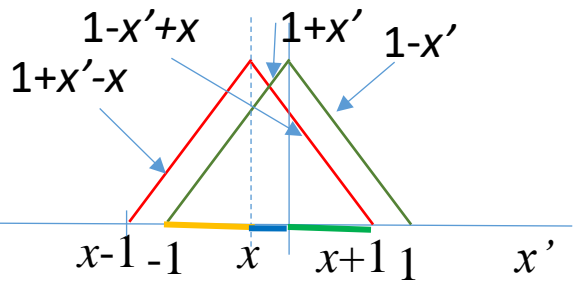
$$0 < x + 1 < 1 \rightarrow -1 < x < 0$$

$$h(x) = \underbrace{\int_{-1}^x (1+x')(1+x'-x) dx'}_{\frac{1}{6}(-x^3 + 3x + 2)} + \underbrace{\int_x^0 (1+x')(1-x'+x) dx'}_{-\frac{1}{6}x(x^2 + 6x + 6)} + \underbrace{\int_0^{x+1} (1-x')(1-x'+x) dx'}_{\frac{1}{6}x(-x^3 + 3x + 2)}$$

$$h(x) = \frac{1}{3}(-x^3 + 3x + 2) - \frac{1}{6}x(x^2 + 6x + 6)$$

Spot

3.



$$0 < x + 1 < 1 \rightarrow -1 < x < 0$$

$$h(x) = \int_{-1}^x (1+x')(1+x'-x) dx' + \int_x^0 (1+x')(1-x'+x) dx' + \int_0^{x+1} (1-x')(1-x'+x) dx'$$

$$\frac{1}{6}(-x^3 + 3x + 2) \quad -\frac{1}{6}x(x^2 + 6x + 6) \quad \frac{1}{6}x(-x^3 + 3x + 2)$$

$$h(x) = \frac{1}{3}(-x^3 + 3x + 2) - \frac{1}{6}x(x^2 + 6x + 6)$$

4. $-1 < x - 1 < 0 \rightarrow 0 < x < 1$

Symetrycznie do 3

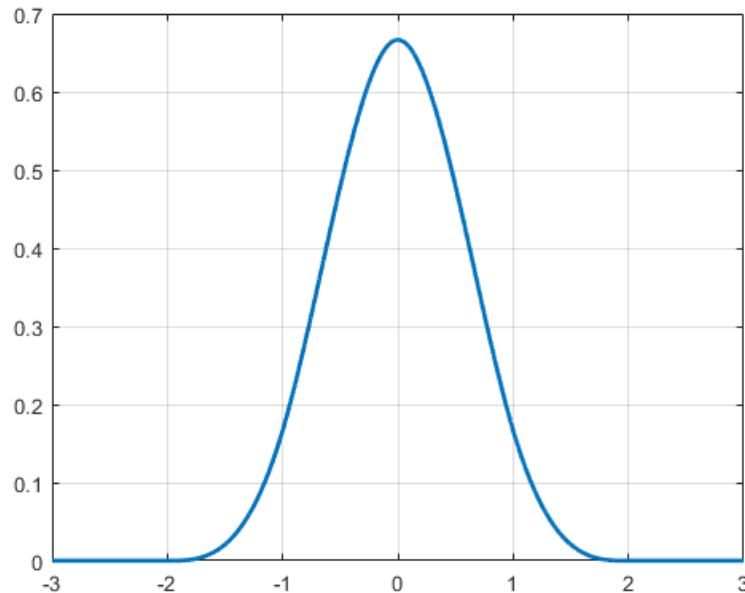
5. $0 < x - 1 < 1 \rightarrow 1 < x < 2$

Symetrycznie do 2

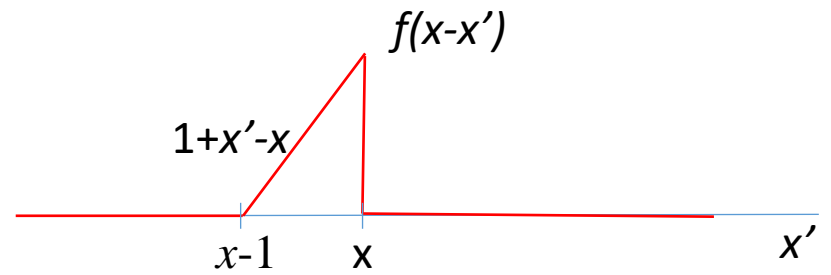
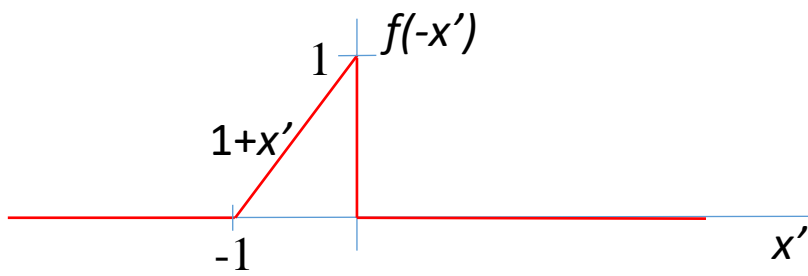
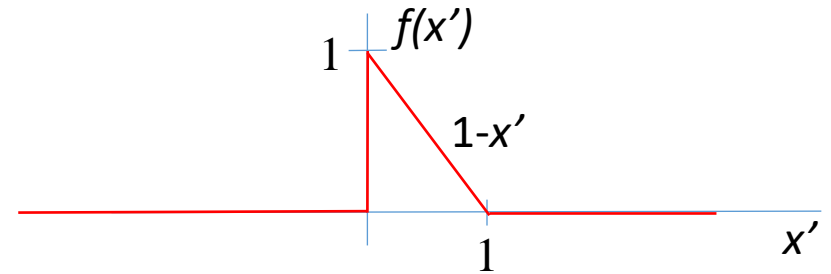
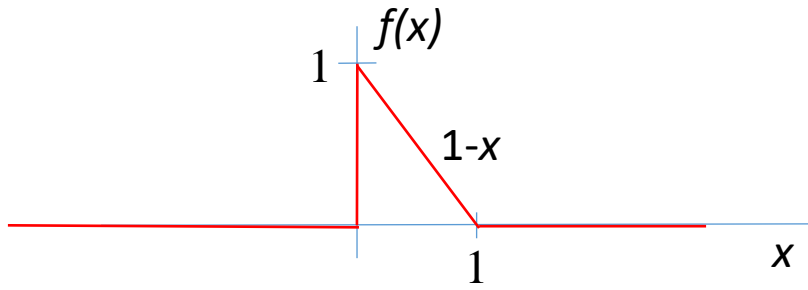
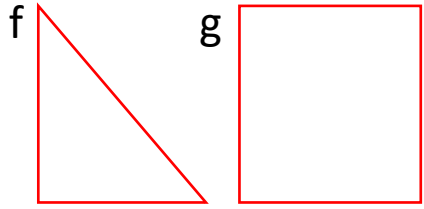
6. $1 < x - 1 \rightarrow 2 < x$

Symetrycznie do 1

$$h(x) = \begin{cases} 0 & x < -2 \\ \frac{1}{6}(x+2)^3 & -2 < x < -1 \\ \frac{1}{3}(-x^3 + 3x + 2) - \frac{1}{6}x(x^2 + 6x + 6) & -1 < x < 0 \\ \frac{1}{3}(x^3 - 3x + 2) + \frac{1}{6}x(x^2 - 6x + 6) & 0 < x < 1 \\ -\frac{1}{6}(x-2)^3 & 1 < x < 2 \\ 0 & 2 < x \end{cases}$$

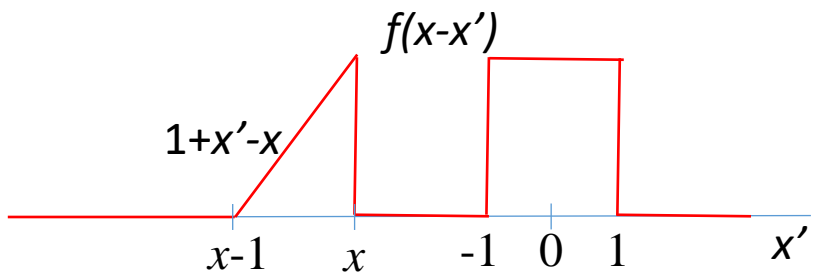


Plot



Spot

1.

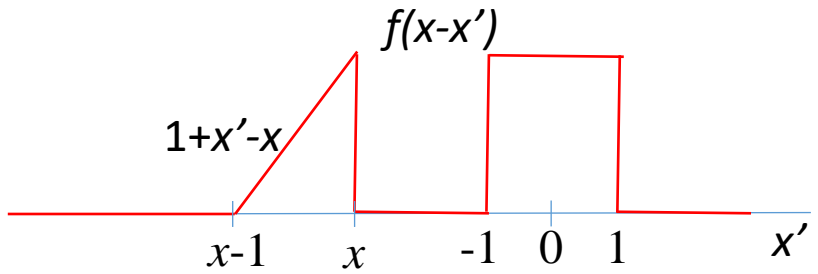


$$x < -1$$

$$h(x) = 0$$

Splot

1.



$$x < -1$$

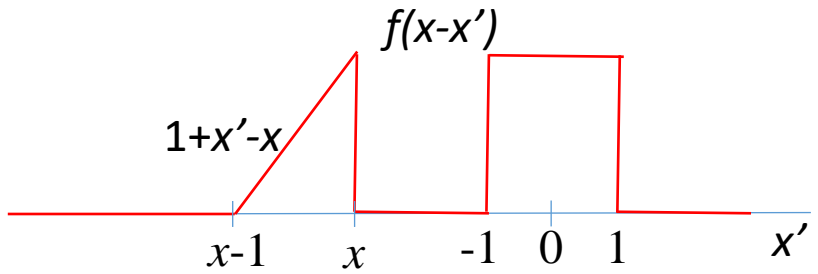
$$h(x) = 0$$

2. $-1 < x < 0$

$$\begin{aligned} h(x) &= \int_{-1}^x (1 + x' - x) dx' = (1 - x) \int_{-1}^x dx' + \int_{-1}^x x' dx' = \\ &= (1 - x)(x + 1) + \frac{x^2}{2} + \frac{1}{2} = \frac{1}{2}(1 - x^2) \end{aligned}$$

Spot

1.



$$x < -1$$

$$h(x) = 0$$

2. $-1 < x < 0$

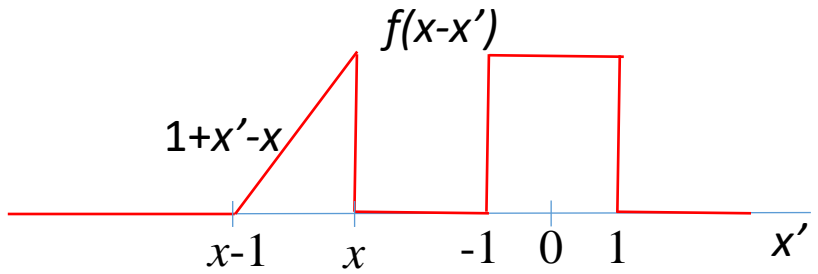
$$\begin{aligned} h(x) &= \int_{-1}^x (1 + x' - x) dx' = (1 - x) \int_{-1}^x dx' + \int_{-1}^x x' dx' = \\ &= (1 - x)(x + 1) + \frac{x^2}{2} + \frac{1}{2} = \frac{1}{2}(1 - x^2) \end{aligned}$$

3. $0 < x < 1$

$$h(x) = \frac{1}{2}$$

Spot

1.



$$x < -1$$

$$h(x) = 0$$

2. $-1 < x < 0$

$$h(x) = \int_{-1}^x (1 + x' - x) dx' = (1 - x) \int_{-1}^x dx' + \int_{-1}^x x' dx' =$$
$$= (1 - x)(x + 1) + \frac{x^2}{2} + \frac{1}{2} = \frac{1}{2}(1 - x^2)$$

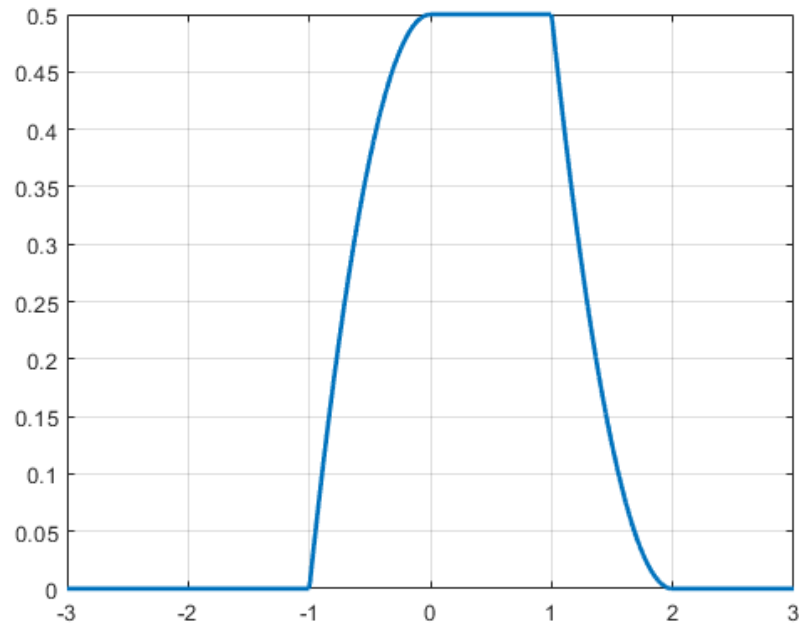
3. $0 < x < 1$

$$h(x) = \frac{1}{2}$$

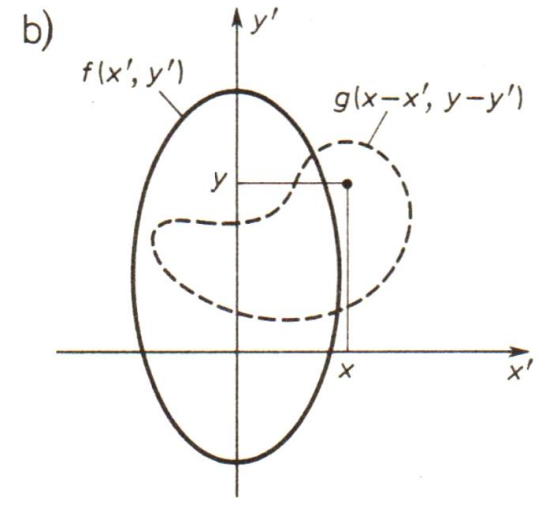
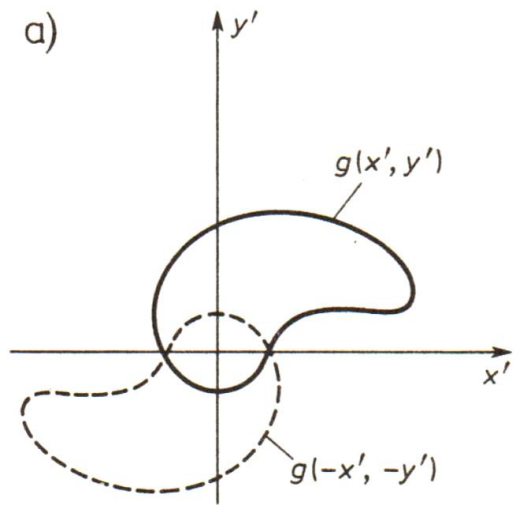
4. $0 < x - 1 < 1 \rightarrow 1 < x < 2$

$$h(x) = \int_{x-1}^1 (1 + x' - x) dx' = (1 - x) \int_{x-1}^1 dx' + \int_{x-1}^1 x' dx' =$$
$$= (1 - x)(2 - x) + \frac{1}{2} [1 - (x - 1)^2] = \frac{x^2}{2} - 2x + 2$$

$$h(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{2}(1 - x^2) & -1 < x < 0 \\ \frac{1}{2} & 0 < x < 1 \\ \frac{x^2}{2} - 2x + 2 & 1 < x < 2 \\ 0 & 2 < x \end{cases}$$



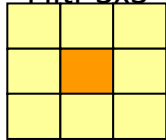
Splot 2D



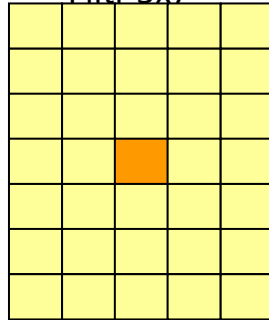
Spot realizacja cyfrowa

Wielkość filtru

Filtr 3x3

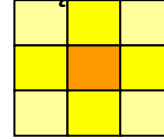


Filtr 5x7

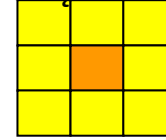


Sąsiedztwo

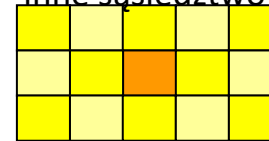
4 sąsiadów



8 sąsiadów



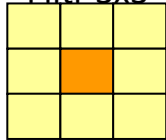
inne sąsiedztwo



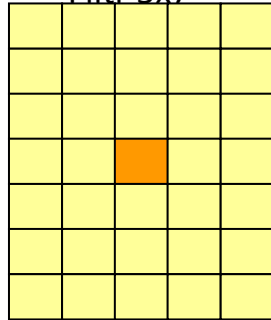
Splot realizacja cyfrowa

Wielkość filtru

Filtr 3x3

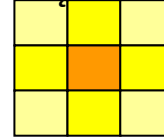


Filtr 5x7

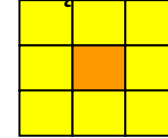


Sąsiedztwo

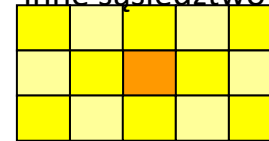
4 sąsiadów



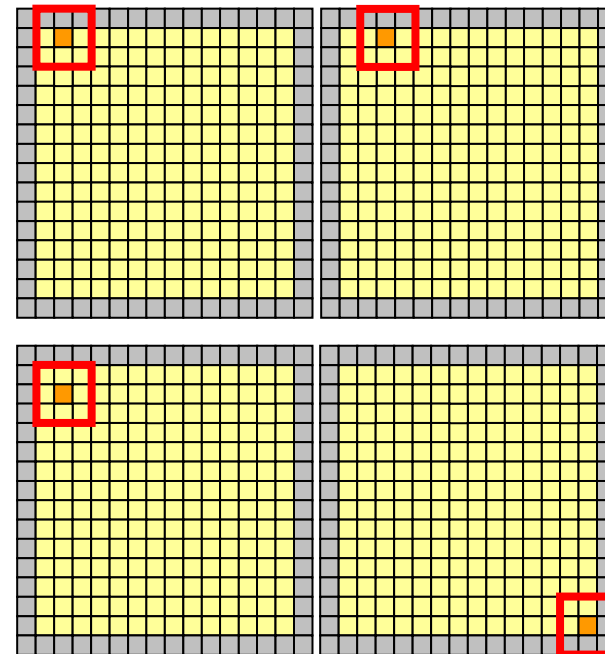
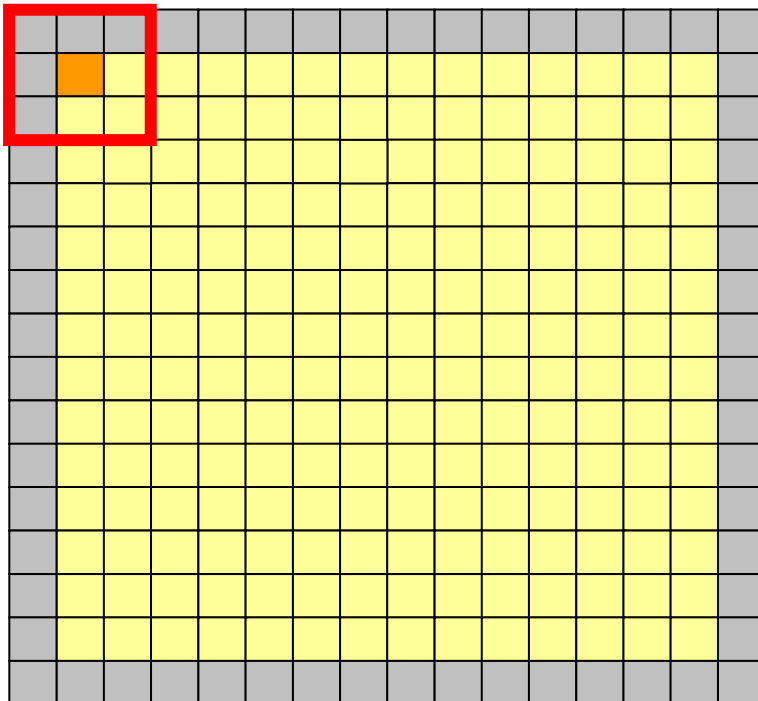
8 sąsiadów



inne sąsiedztwo



Działanie filtru 3x3



Spot realizacja cyfrowa

okno filtru



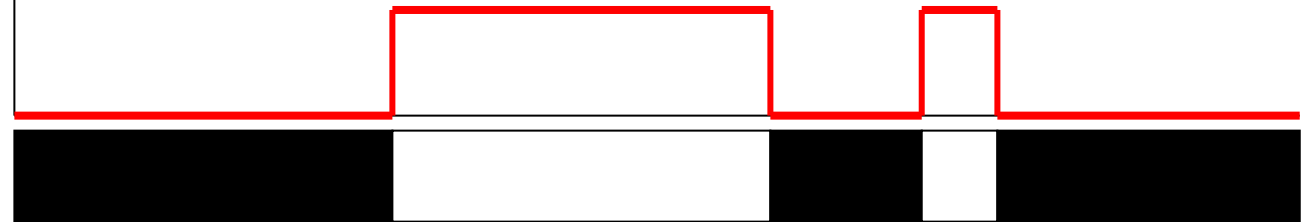
wiersz obrazu rastrowego



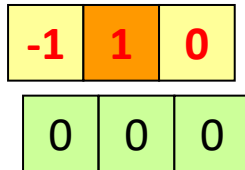
a)

b)

c)

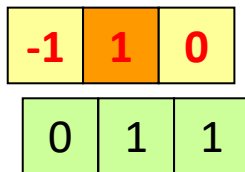


a)



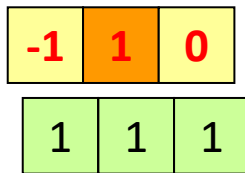
$$S = -1 * 0 + 1 * 0 + 0 * 0 = 0$$

b)



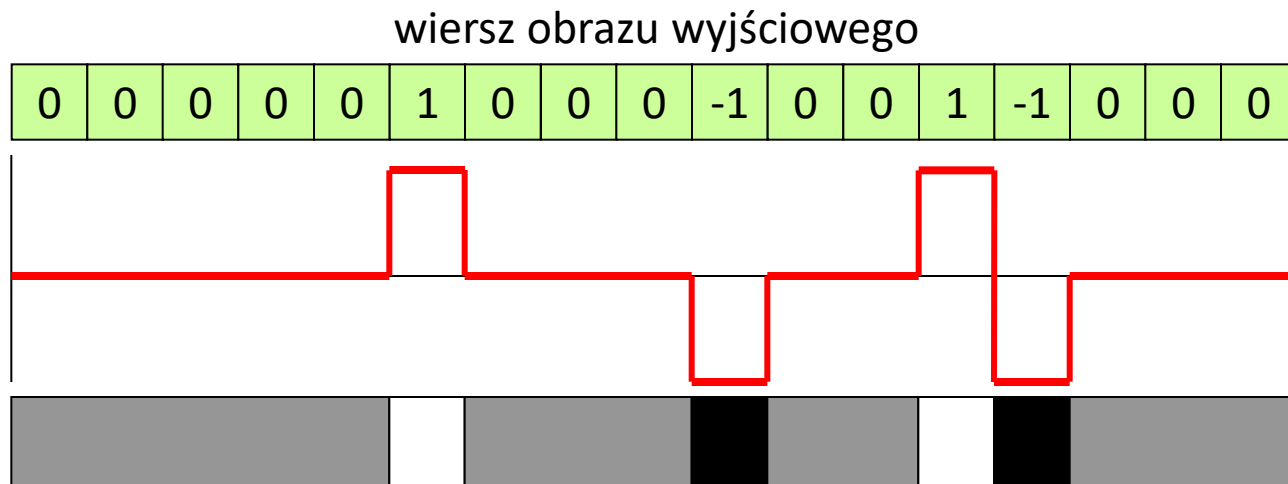
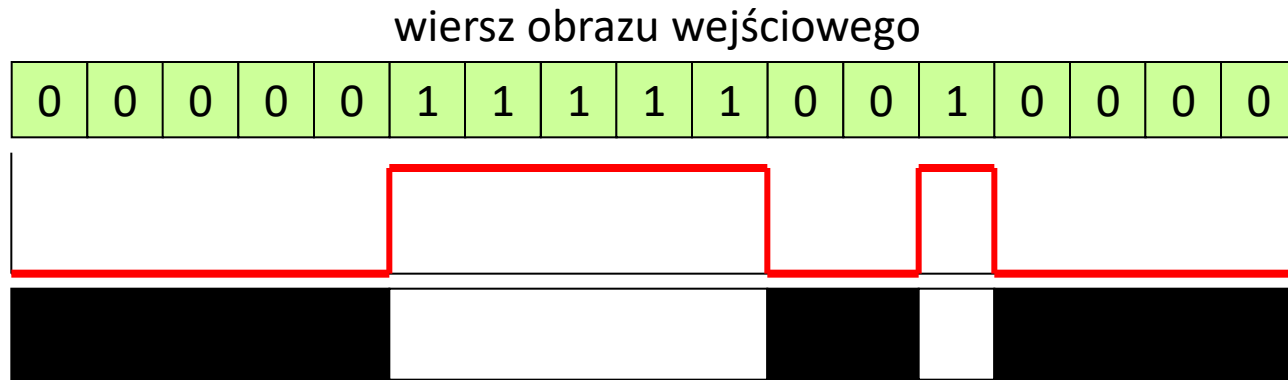
$$S = -1 * 0 + 1 * 1 + 0 * 1 = 1$$

c)



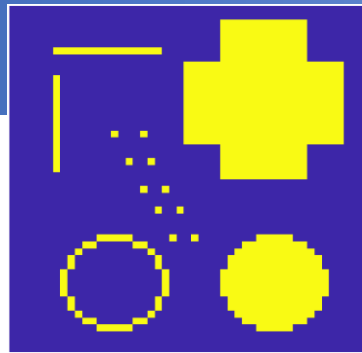
$$S = -1 * 1 + 1 * 1 + 0 * 1 = 0$$

Spot realizacja cyfrowa



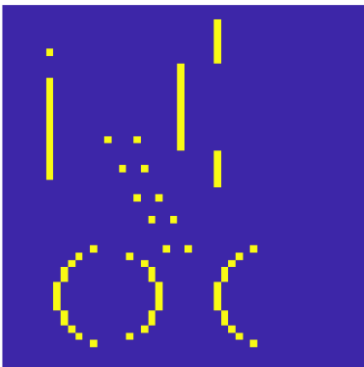
Spot realizacja cyfrowa

Przykładowe filtrów w przetwarzaniu obrazów



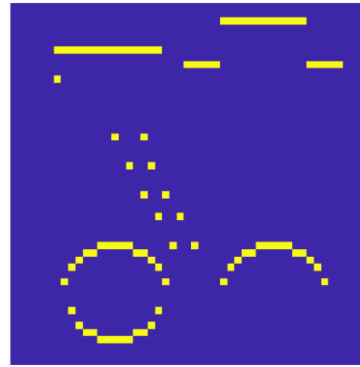
Gradient X:

-1	1	0
----	---	---



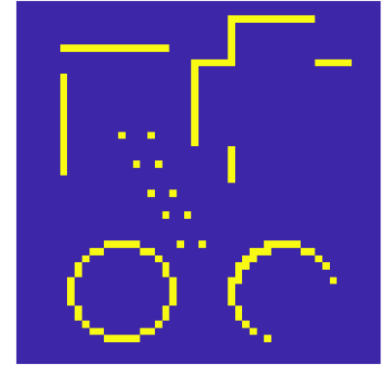
Gradient Y:

-1
1
0

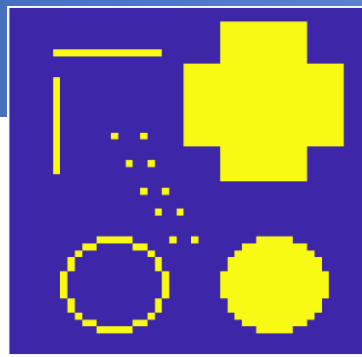


Gradient XY:

-1	-1	0
-1	3	0
0	0	0



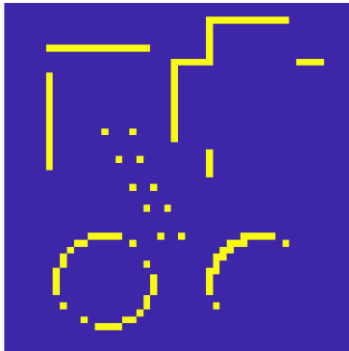
Spot realizacja cyfrowa



Przykładowe filtrów w przetwarzaniu obrazów

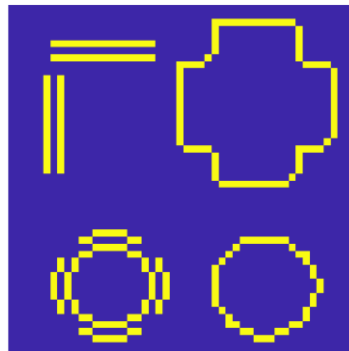
Roberts:

-1	0	0
0	1	0
0	0	0



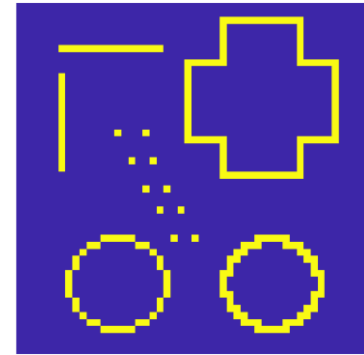
Sobel:

-1	0	1
-2	0	2
-1	0	1



Wyodrębiający:

-1	-1	-1
-1	8	-1
-1	-1	-1



Uśredniający:

1	1	1
1	1	1
1	1	1



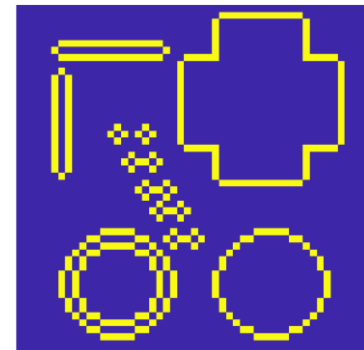
Gaussowski:

1	2	1
2	4	2
1	2	1



Laplacea:

0	1	0
1	-4	1
0	1	0



Korelacja

$$\varphi(x) = \int_{-\infty}^{\infty} f(x')g(x' - x)dx'$$

$$\varphi(x) = f(x) \star g(x)$$

Splot:

$$h(x) = \int_{-\infty}^{\infty} f(x - x')g(x')dx'$$

Korelacja

$$\varphi(x) = \int_{-\infty}^{\infty} f(x')g(x' - x)dx'$$

$$\varphi(x) = f(x) \star g(x)$$

Splot:

$$h(x) = \int_{-\infty}^{\infty} f(x - x')g(x')dx'$$

Własności:

Korelacja nie jest przemienna:

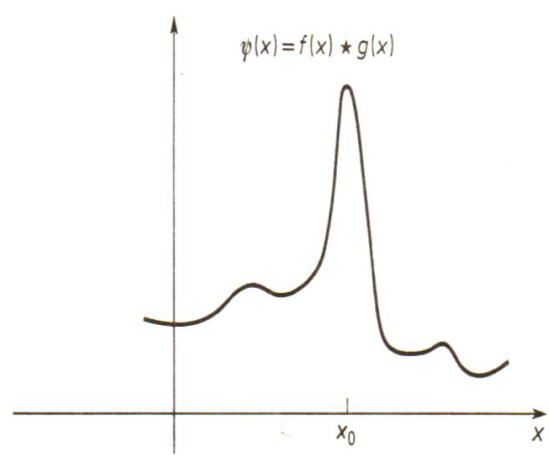
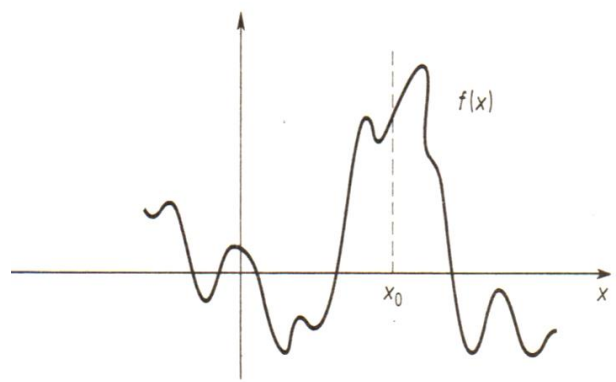
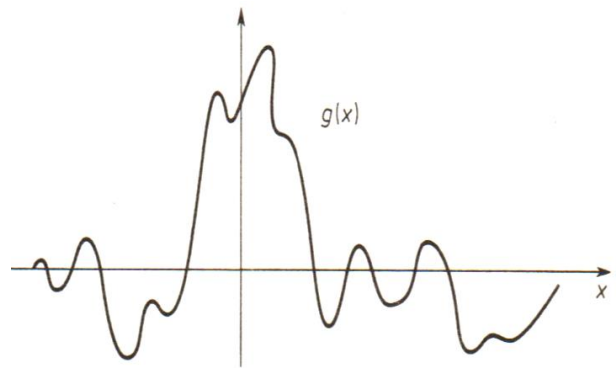
$$f(x) \star g(x) \neq g(x) \star f(x)$$

Korelacja jest równa splotowi z odwróconą funkcją g :

$$f(x) \star g(x) = f(x) \otimes g(-x)$$

Gdy $g(x)$ jest funkcją parzystą to korelacja jest równoważna splotowi

Korelacja



Autokorelacja

Autokorelacja gdy $g(x) = f(x)$

Współczynnik autokorelacji: $\gamma(x) = \frac{\varphi(x)}{\varphi(0)}$

Moduł autokorelacji osiąga największą wartość w ,0' : $|\varphi(x)| \leq \varphi(0)$

Transformacja Fouriera

Szereg Taylora:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$$

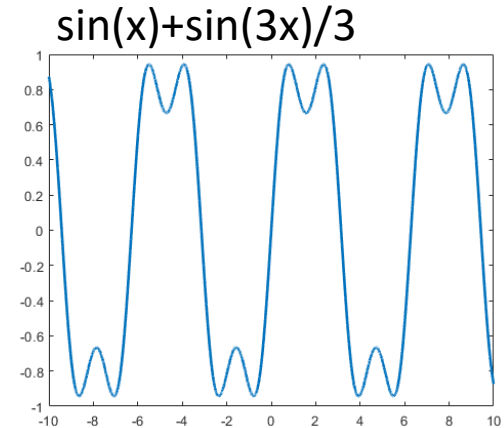
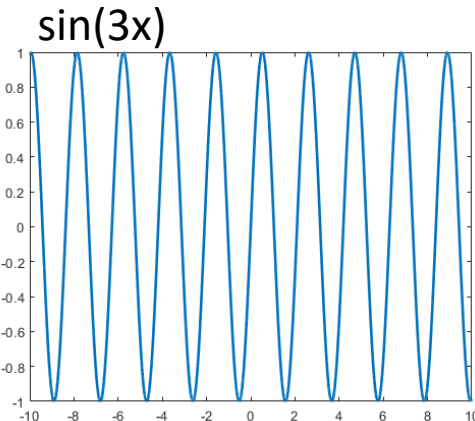
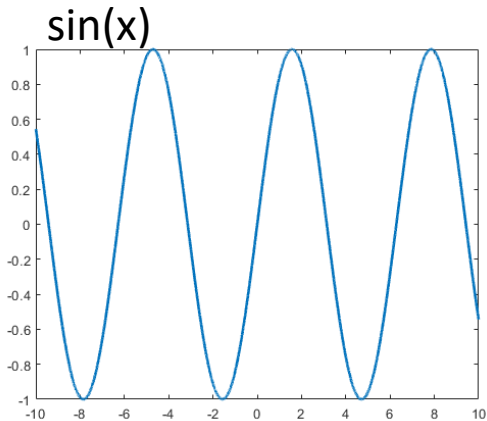
SzeregTaylora.mlx

Transformacja Fouriera

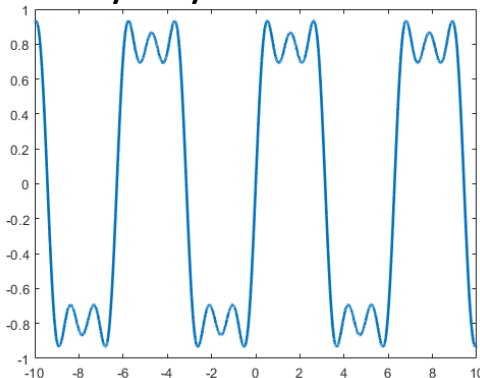
Szereg Taylora:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$$

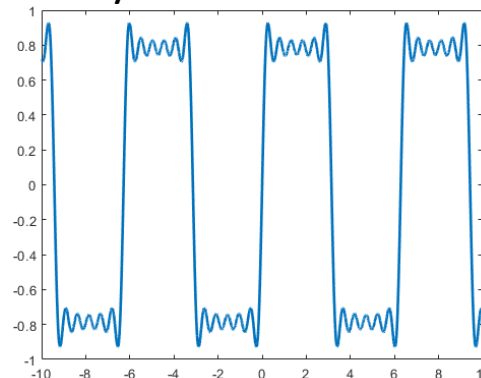
Suma sinusów:



4 wyrazy



7 wyrazów



$$\sum_{n=1}^N \frac{\sin[(2n-1)x]}{2n-1}$$

Transformacja Fouriera

Funkcję periodyczną można przedstawić jako sumę sinusów:

$$f(t) = \sum_{k=1}^n [A_k \sin(2\pi\omega_k t + \pi/2) + B_k \sin(2\pi\omega_k t)]$$

Wygodniej to przepisać jako sumę sinusa i cosinusa:

$$f(t) = \sum_{k=1}^n [A_k \cos(2\pi\omega_k t) + B_k \sin(2\pi\omega_k t)]$$

Np. dla funkcji:

$$f_1 = \frac{1}{2} \sin(\pi t) + 2 \sin(4\pi t) + 4 \cos(2\pi t)$$

amplitudy

częstości