Exercise Sheet 2

Kinematics

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1. Consider a 2D flow which has the Cartesian coordinates:

$$\mathbf{u} = \left(\frac{x}{1+t}, y \right),$$

where $t > -1$.

Determine the equation describing:

   (a) the streamline passing through point $(1, 1)$,

   (b) the trajectory of a particle which started at $t = 0$ from the point $(1, 1)$,

   (c) the position at $t = 1$ of a dye streak released from the point $(1, 1)$ in the interval $t \in (0, 1)$.

Sketch the obtained curves when possible.

2. Consider the following steady 2D flows:

   (a) $\mathbf{u} = (kx, -ky)$ (extensional flow),

   (b) $\mathbf{u} = (ky, 0)$ (shear flow),

   (c) $\mathbf{u} = (ky, -kx)$ (What flow is this?),

where k is a real constant.

   (i) Check, whether these flows are solenoidal (e.g. divergence-free). For each flow, find the equations for the streamlines and sketch them. Find the vorticity.

   (ii) Imagine that at $t = 0$ particles positioned on a circle of radius a around the origin, given by $x^2 + y^2 = a^2$, were marked with a colored dye. Find the equation for this material curve at any time $t > 0$. How does the surface area enclosed by this curve change in time?

3. Lagrangean approach. Consider the following two-dimensional flow defined in Cartesian system:

$$\mathbf{u} = (kx, -ky) \tag{1}$$

where k is a constant. Let the local concentration $c(\mathbf{r}, t)$ of a certain pollutant in this flow be described by

$$c(x, y, t) = \beta x^2 y \exp(-kt),$$

where $y > 0$ and β is a constant. Find the rate of change in time of the substance concentration in the element of fluid moving in the velocity field (1).