Exercise Sheet 8

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1. *Spherical wave.* Consider wave equation in 3D

$$\frac{\partial^2\phi}{\partial t^2} - a_0^2\nabla^2\phi = 0.$$

Show that the solution for the spherical, outgoing wave with a source at point $(0,0)$ and the initial condition

$$\phi(r,0) = \frac{f(r)}{r}$$

reads

$$\phi(r,t) = \frac{f(r-at)}{r}.$$

2. *Bernoulli equation for ideal gas.* Consider stationary flow of compressible, inviscid, ideal gas and show that the Bernoulli equation can be written in the following, alternative forms

$$h + \frac{u^2}{2} = c_pT + \frac{u^2}{2} = \frac{a^2}{\kappa-1} + \frac{u^2}{2} = \frac{\kappa p}{(\kappa-1)\rho} + \frac{u^2}{2} = const$$

3. *Poisson adiabath.* Relation between density ρ and the density at the stagnation point ρ_0 in the isentropic flow (derived during lecture) reads

$$\frac{\rho}{\rho_0} = \left(1 + \frac{\kappa-1}{2}M^2\right)^{\frac{1}{\kappa-1}}.$$

Derive corresponding relations for p/p_0, a/a_0 and T/T_0.

4. *Parameters at the stagnation and at critical points.* Consider isentropic flow of an ideal gas and derive relations between flow parameters at the stagnation point and corresponding parameters at the critical point, where $M = 1$

$$\frac{T_*}{T_0} = \frac{2}{\kappa+1}, \quad \frac{p_*}{p_0} = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}, \quad \frac{\rho_*}{\rho_0} = \left(\frac{2}{\kappa+1}\right)^{\frac{1}{\kappa-1}}, \quad \frac{a_*}{a_0} = \left(\frac{2}{\kappa+1}\right)^{\frac{1}{2}}.$$

5. *Flow in de Laval nozzle*. Consider flow of an ideal gas in a convergent-divergent de Laval nozzle and derive the ratio of the cross-sectional areas S/S_* as a function of the local Mach number. Discuss the results.

6. *Shock wave*. Using Hugoniot adiabath (discussed during lecture) show that $M_1 > 1$ before the shock wave and $M_2 < 1$ behind the shock wave.