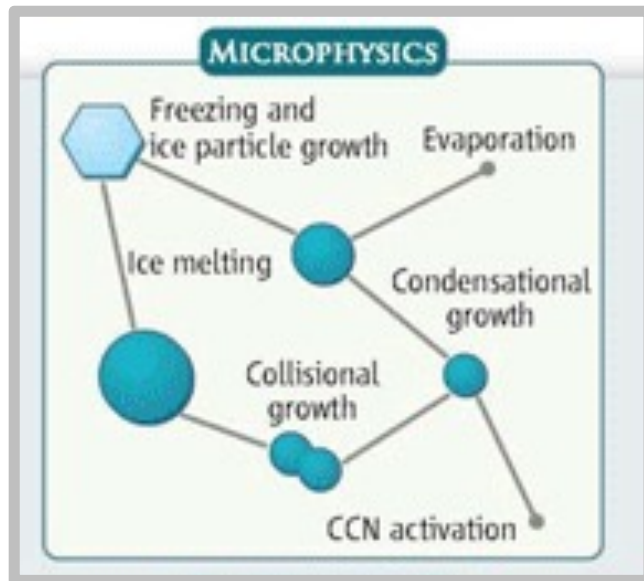


MICROPHYSICS



UNIVERSITY
OF WARSAW

A MATTER OF SCALE



100 μm

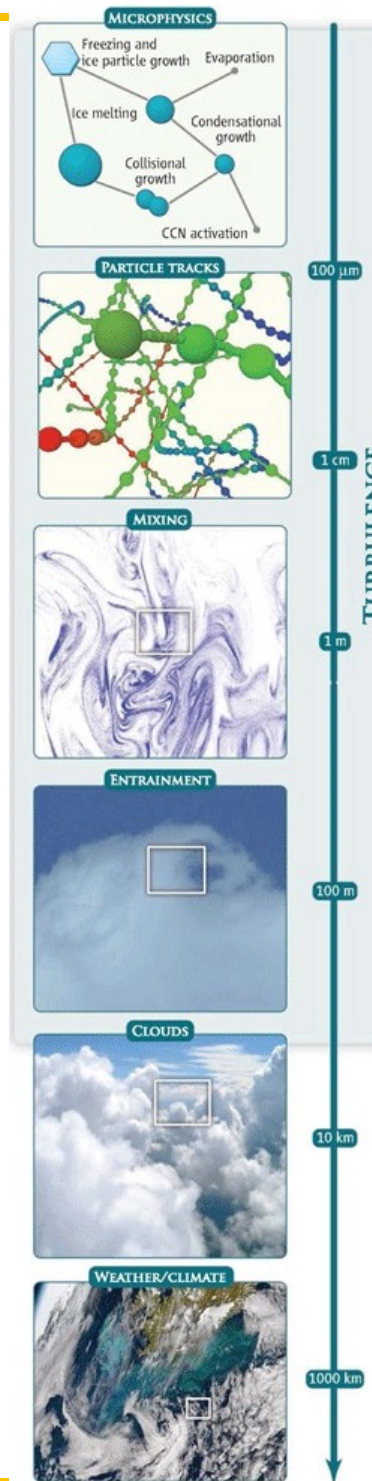
1 cm

1 m

100 m

1 km

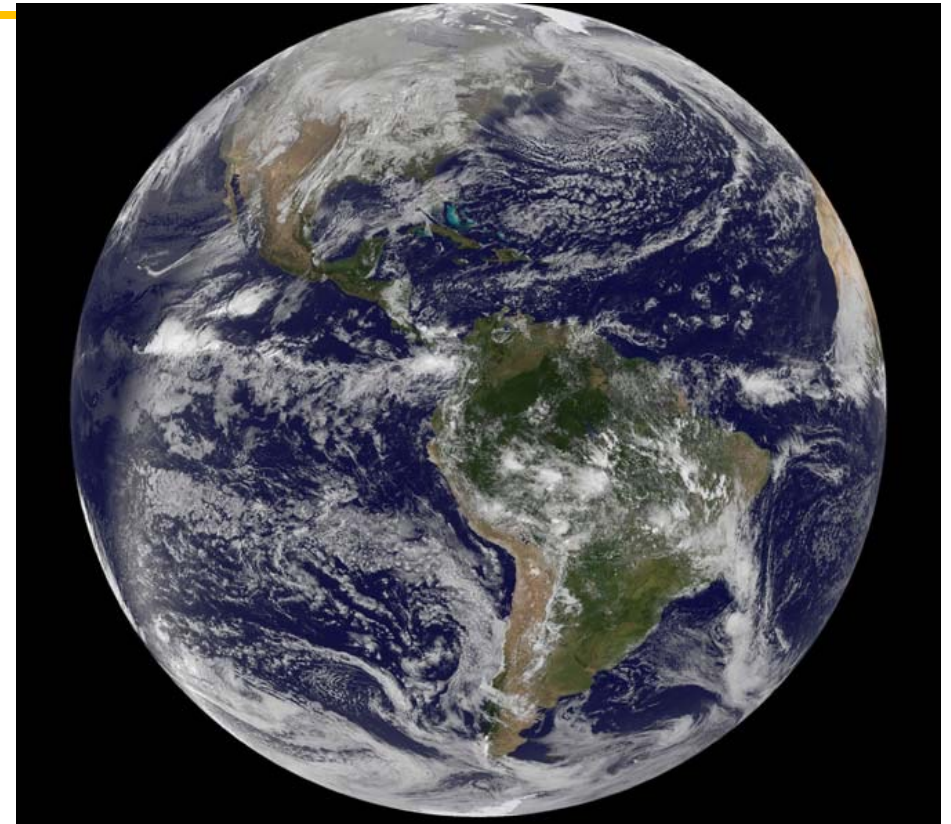
1000 km



E Bodenschatz et al. Science 2010;327:970-971

- A cloud is a medium composed of liquid water and/or ice [particles](#) immersed in a field of water vapor.

For a cloud to be visible, its particles must exist in a sufficiently high concentration.



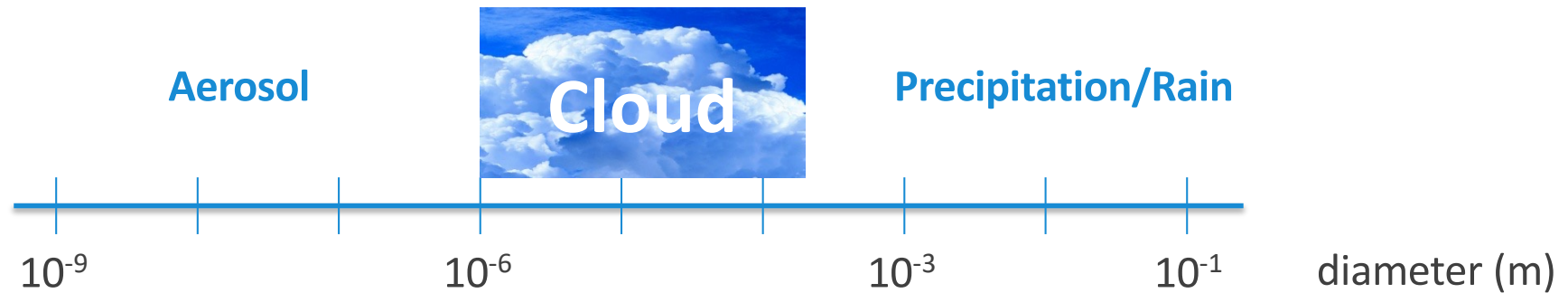
- The description of the formation and evolution of cloud [particles](#) is a primary goal of **cloud microphysics**.
- The spatial coordinates, sizes, and/or shapes of every cloud [particle](#) at any instant in time would provide the most exhaustive information about a cloud.

TYPES AND SIZES OF PARTICLES

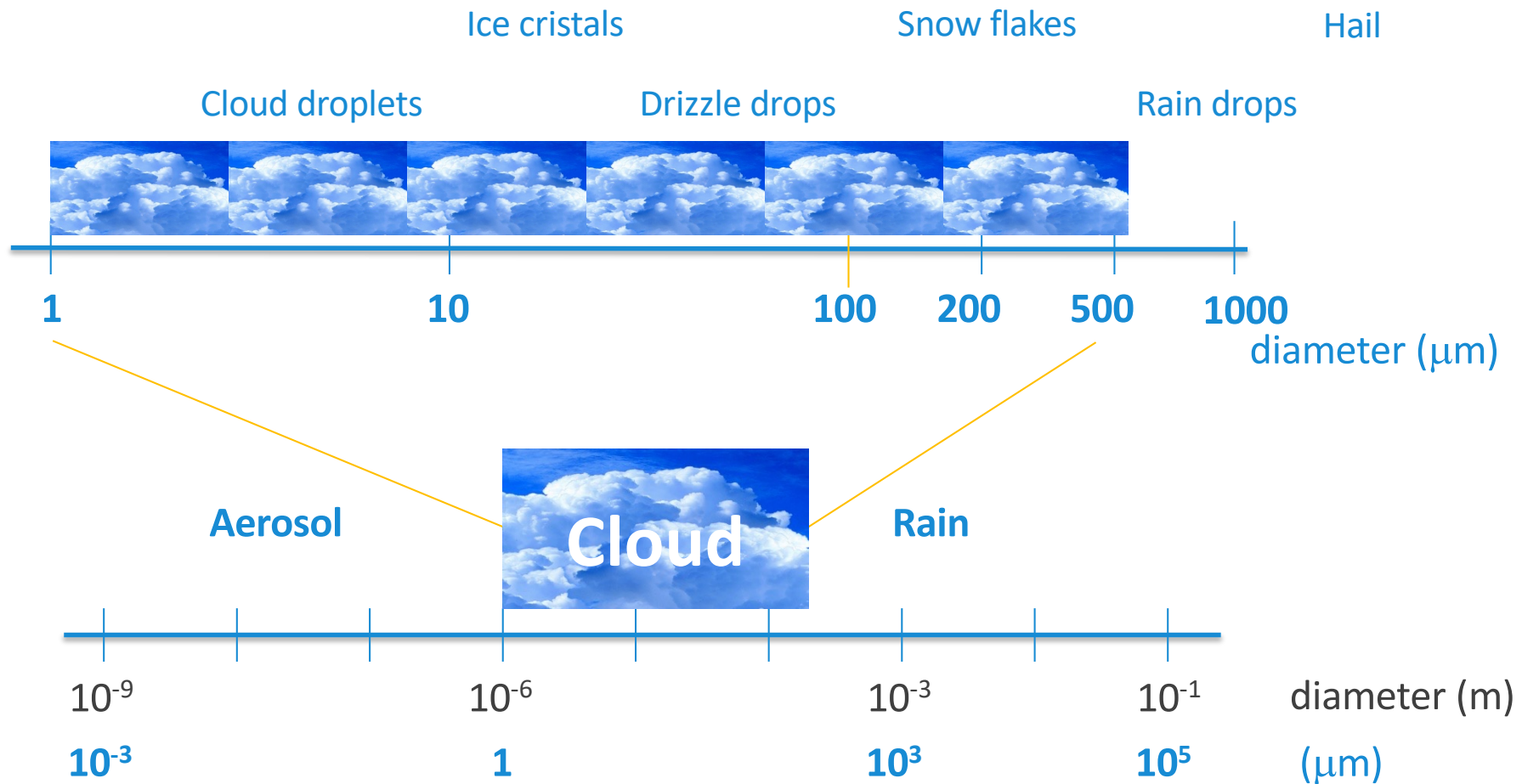


UNIVERSITY
OF WARSAW

SIZES OF CLOUD PARTICLES



SIZES OF CLOUD PARTICLES



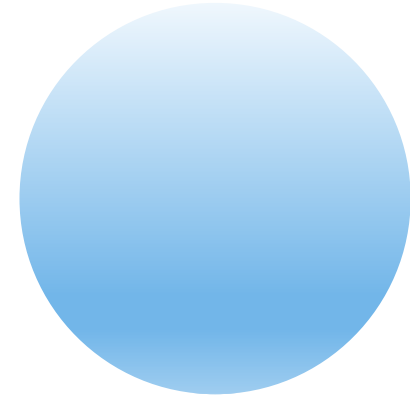
Typical condensation
nucleus
 $1\ \mu\text{m}$



Typical cloud droplet
 $10\ \mu\text{m}$



Large cloud droplet
 $100\ \mu\text{m}$



Tip-top of a typical raindrop
 $1\ \text{mm}$



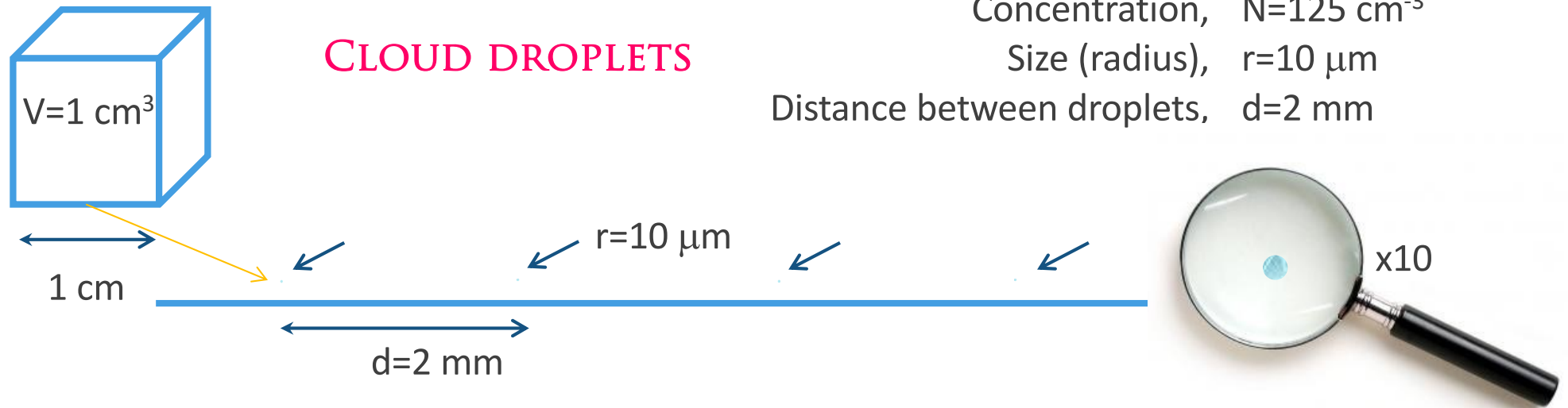
Cloud particles are categorized according to their size (diameter):

- Condensation nucleus: $<1\ \mu\text{m}$
- Cloud droplets: $1\text{--}30\ \mu\text{m}$
- Drizzle drops: $30\text{--}600\ \mu\text{m}$
- Raindrops: $>600\ \mu\text{m}$

This division reflects the processes involved in the formation of these particles.

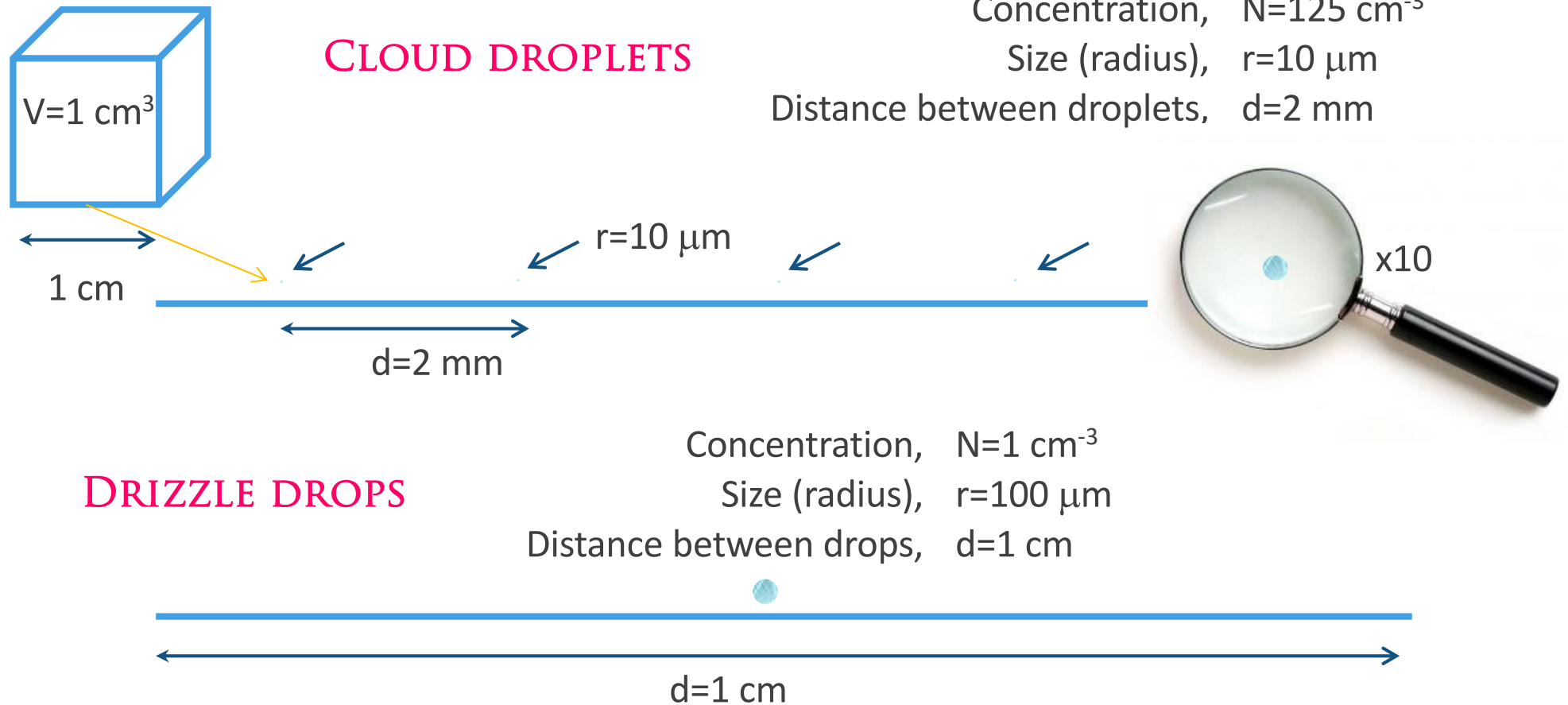
CLOUD DROPLETS, DRIZZLE, RAIN DROPS

CONCENTRATION, SIZE, DISTANCES BETWEEN PARTICLES



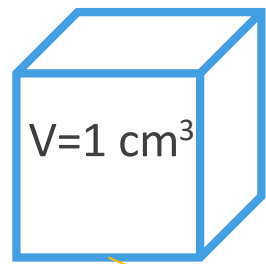
CLOUD DROPLETS, DRIZZLE, RAIN DROPS

CONCENTRATION, SIZE, DISTANCES BETWEEN PARTICLES



CLOUD DROPLETS, DRIZZLE, RAIN DROPS

CONCENTRATION, SIZE, DISTANCES BETWEEN PARTICLES



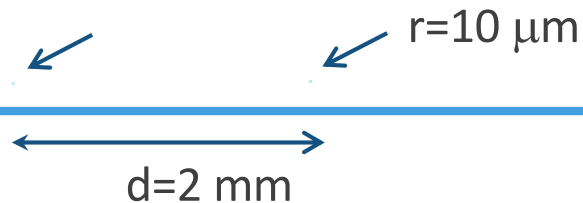
1 cm

CLOUD DROPLETS

Concentration, $N=125 \text{ cm}^{-3}$

Size (radius), $r=10 \text{ } \mu\text{m}$

Distance between droplets, $d=2 \text{ mm}$



DRIZZLE DROPS

Concentration, $N=1 \text{ cm}^{-3}$

Size (radius), $r=100 \text{ } \mu\text{m}$

Distance between drops, $d=1 \text{ cm}$



PRECIPITATION/RAIN DROPS



Concentration, $N=1 \text{ dm}^{-3}$

Size (radius), $r=1000 \text{ } \mu\text{m}$

Distance between drops, $d=10 \text{ cm}$

HAILSTONE... THE RECORD

The largest recorded hailstone in the United States by diameter 8 inches (20 cm) and weight 1.93 pounds (0.88 kg). The hailstone fell in Vivian, South Dakota on July 23, 2010.



Rain drop, $r = 1 \text{ mm}$

Image: NOAA

CONCENTRATION AND SIZE

1. CLOUD DROPLETS

Concentration: $10\text{--}500\text{ cm}^{-3}$

Size (radius): $1\text{--}20\text{ }\mu\text{m}$

2. DRIZZLE DROPS

Concentration: $0.1\text{--}1\text{ cm}^{-3}$

Size (radius): $30\text{--}300\text{ }\mu\text{m}$

3. PRECIPITATION/RAINDROPS

Concentration: 1 dcm^{-3}

Size (radius): 1 mm



UNIVERSITY
OF WARSAW

HOW TO DESCRIBE CLOUD MICROPHYSICAL PROPERTIES?

- Particle Size Distribution (PSD)
- Moments of the PSD (e.g., numer concentration, mean radius, mean volume radius)
- Integrated cloud properties (e.g., liquid water path, cloud optical thickness)



UNIVERSITY
OF WARSAW

CLOUD PROCESSES SPAN A WIDE RANGE OF SPATIAL AND TEMPORAL SCALES

- **Lower limit: Microscale**
 - Droplet size: micrometers
 - Droplet spacing: millimeters to centimeters
 - Key challenge:
 - Direct in-situ observation of droplet-scale processes in natural clouds is extremely difficult - if not impossible.
- **Upper limit: Macroscale**
 - Spatial extend: hundreds of meters to hundreds of kilometers
 - Key challenge: Macroscale characterization must simultaneously:
 - reflect mean cloud properties
 - accurately reproduce global radiative and/or dynamical impacts.

Cloud microphysical characteristics are always defined relative to a given volume or mass of air.



Spatial coordinates, sizes, and shapes of every cloud particle at any given instant provide the most exhaustive information about a cloud.

Is the exact position of an individual cloud particle important for describing cloud microphysics?

NO!

Because an identical microscopic configuration will never occur again.

Therefore, to describe cloud particle populations, we must rely on distribution functions.

WARM CLOUDS



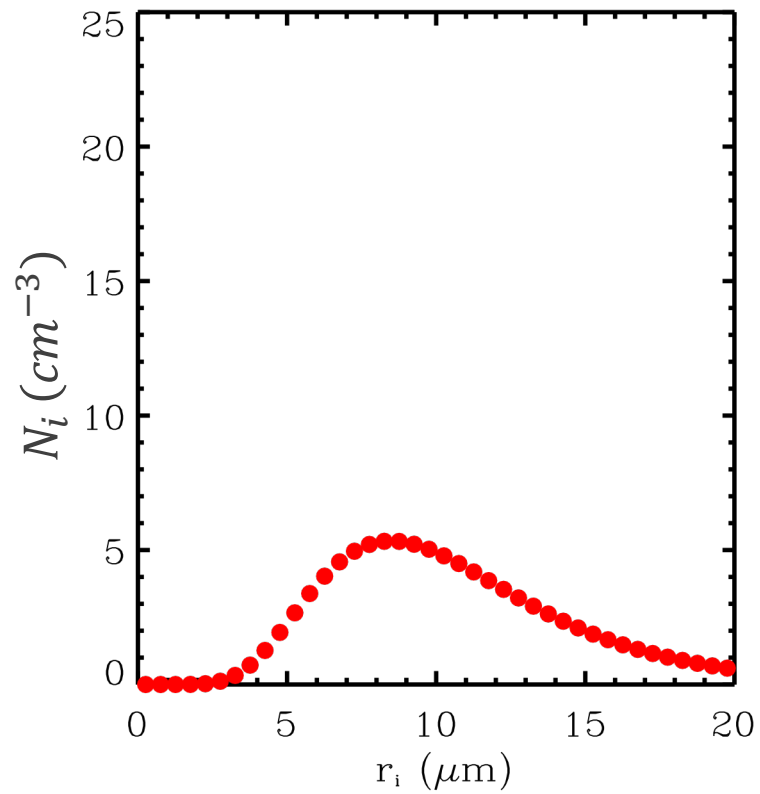
UNIVERSITY
OF WARSAW

PARTICLE SIZE DISTRIBUTION (PSD)

The particle size distribution (or particle spectrum) provides information on the number of particles of a given size per unit volume of a cloud.

In a discrete bin representation, (N_i, r_i) denotes the particle concentration $N_i(\text{cm}^{-3})$ per unit volume associated with a representative radius $r_i(\mu\text{m})$. Most commonly, N_i represents the total number concentration of particles with radii falling within the size bin $(r_i, r_i + \Delta r_i)$.

$$N_i, r_i + \Delta r_i \quad N = \sum_i N_i = 100 \text{ cm}^{-3}$$

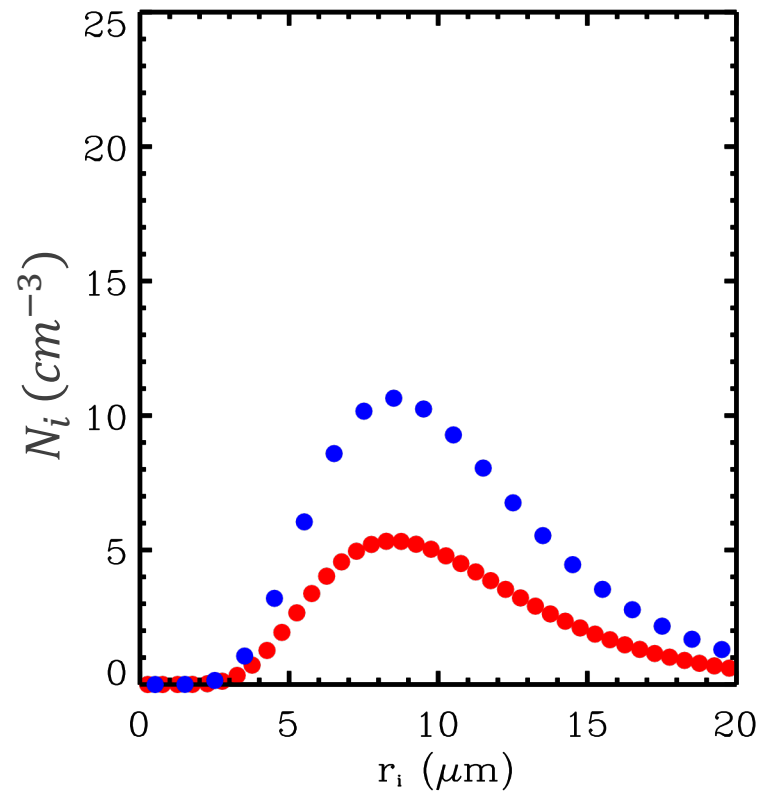


$$\Delta r_i = 0.5 \mu\text{m}$$

$$N_i, r_i + \Delta r_i \quad N = \sum_i N_i = 100 \text{ cm}^{-3}$$

$$\Delta r_i = 1 \mu\text{m}$$

$$\Delta r_i = 0.5 \mu\text{m}$$

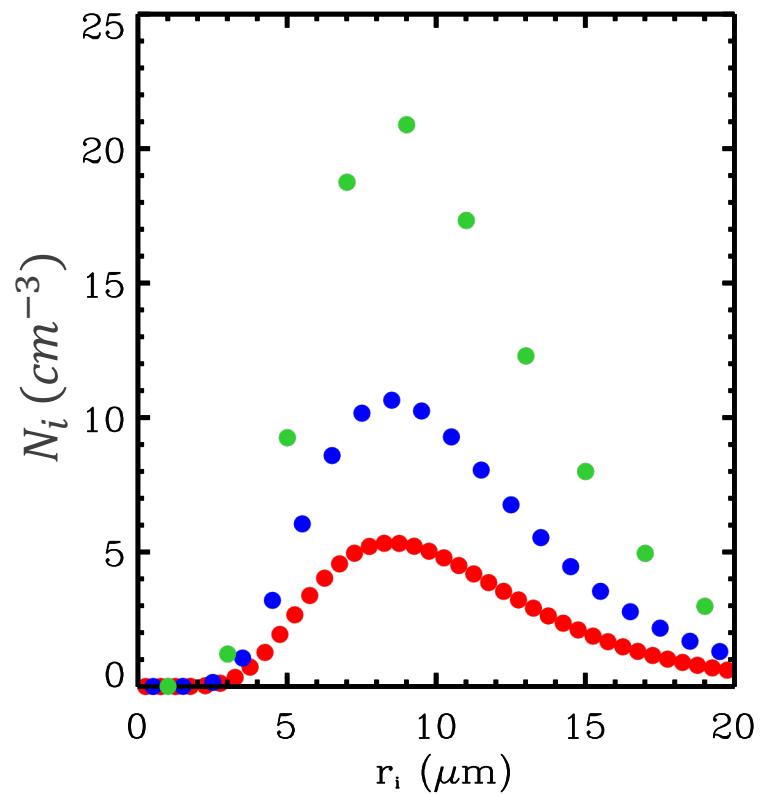


$$N_i, r_i + \Delta r_i \quad N = \sum_i N_i = 100 \text{ cm}^{-3}$$

$\Delta r_i = 2 \mu\text{m}$

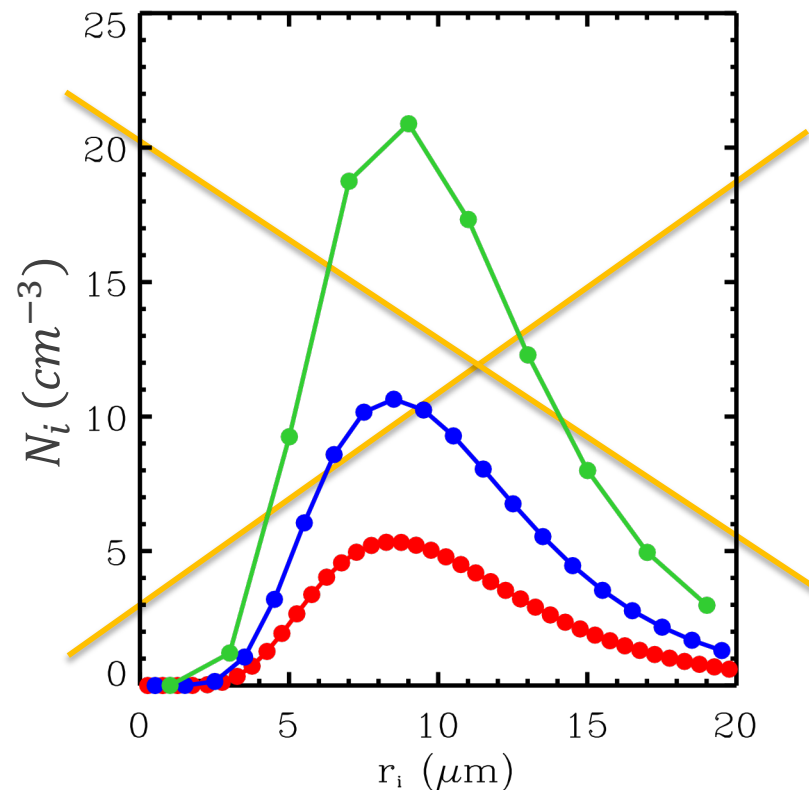
$\Delta r_i = 1 \mu\text{m}$

$\Delta r_i = 0.5 \mu\text{m}$



Connecting raw discrete points (N_i, r_i) with straight lines is mathematically and physically misleading:

- The quantity N_i is not a point measurement; it is the integrated count of particles within the interval $(r_i, r_i + \Delta r_i)$



- A straight line connecting two bin midpoints implies that particle concentration varies linearly between those two precise sizes. In reality, the data inside a bin is aggregated—you only know the average concentration over the interval Δr_i , not the exact slope between adjacent bins.

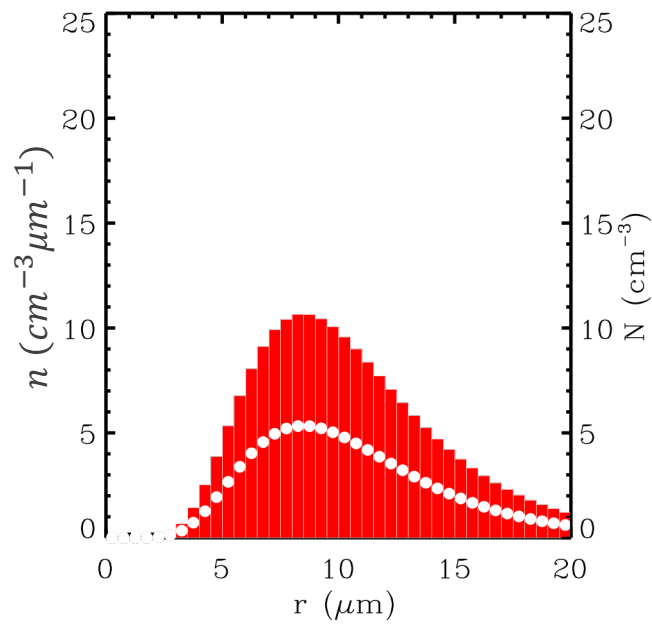
PARTICLE SIZE DISTRIBUTION (PSD)

The particle size distribution (or particle spectrum) provides information on the number of particles of a given size per unit volume of a cloud.

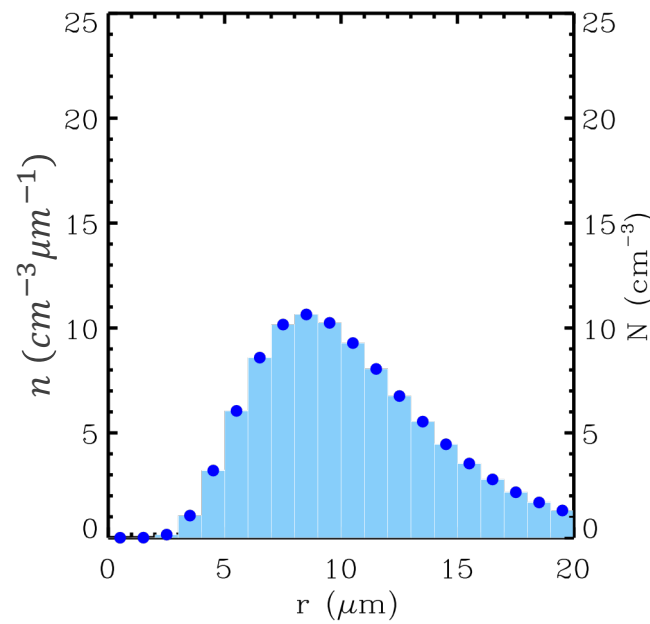
In a discrete bin representation, (N_i, r_i) denotes the particle concentration $N_i(\text{cm}^{-3})$ per unit volume associated with a representative radius $r_i(\mu\text{m})$. Most commonly, N_i represents the total number concentration of particles with radii falling within the size bin $(r_i, r_i + \Delta r_i)$.

To eliminate the effect of varying bin widths Δr_i , raw concentrations N_i are normalized to define the **spectral number density** $n_i = N_i / \Delta r_i$.

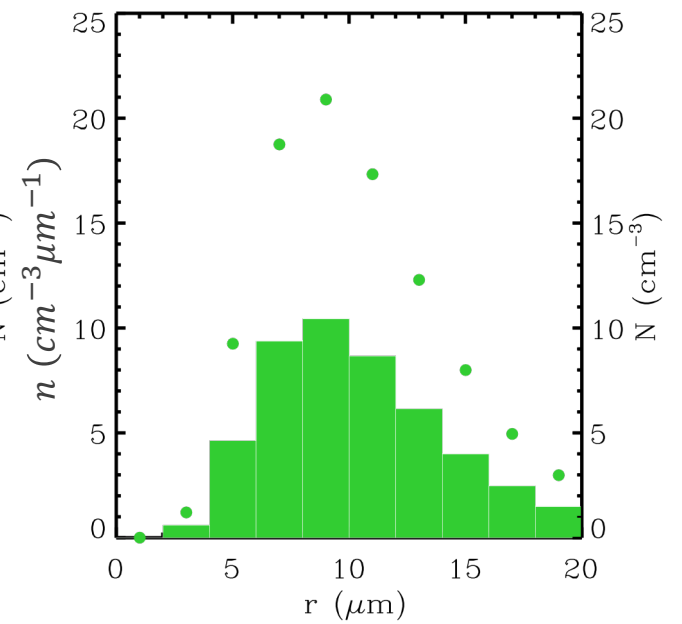
$$n_i = N_i / \Delta r_i \quad N = \sum_i n_i \Delta r_i = 100 \text{ cm}^{-3}$$



$$\Delta r_i = 0.5 \mu\text{m}$$



$$\Delta r_i = 1 \mu\text{m}$$



$$\Delta r_i = 2 \mu\text{m}$$

PARTICLE SIZE DISTRIBUTION (PSD)

The particle size distribution (or particle spectrum) provides information on the number of particles of a given size per unit volume of a cloud.

In a discrete bin representation, (N_i, r_i) denotes the particle concentration $N_i(\text{cm}^{-3})$ per unit volume associated with a representative radius $r_i(\mu\text{m})$. Most commonly, N_i represents the total number concentration of particles with radii falling within the size bin $(r_i, r_i + \Delta r_i)$.

To eliminate the effect of varying bin widths Δr_i , raw concentrations N_i are normalized to define the **spectral number density** $n_i = N_i / \Delta r_i$.

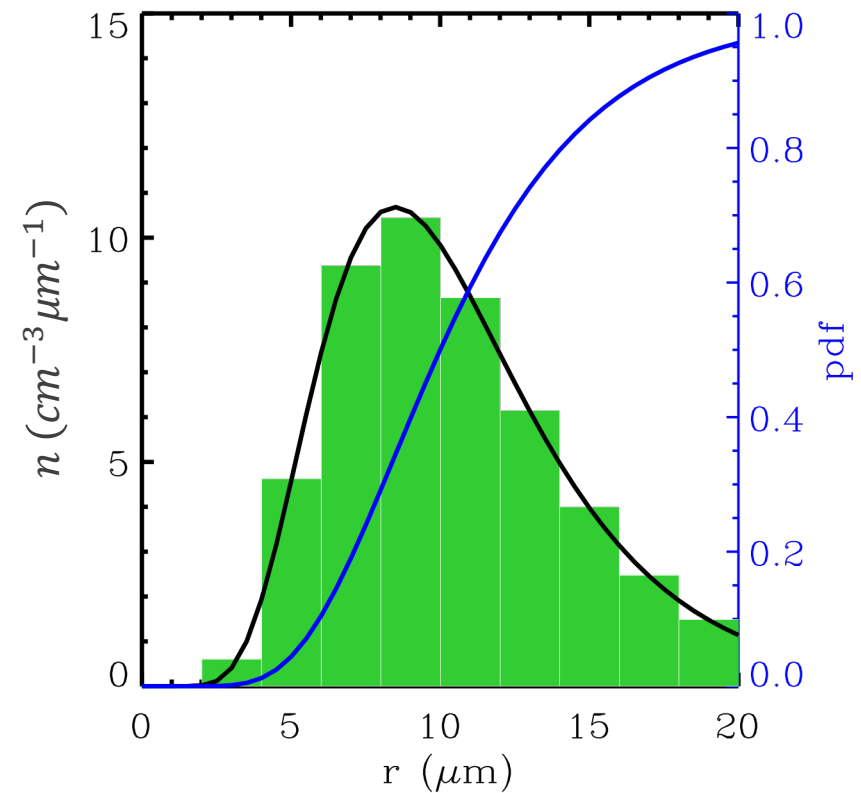
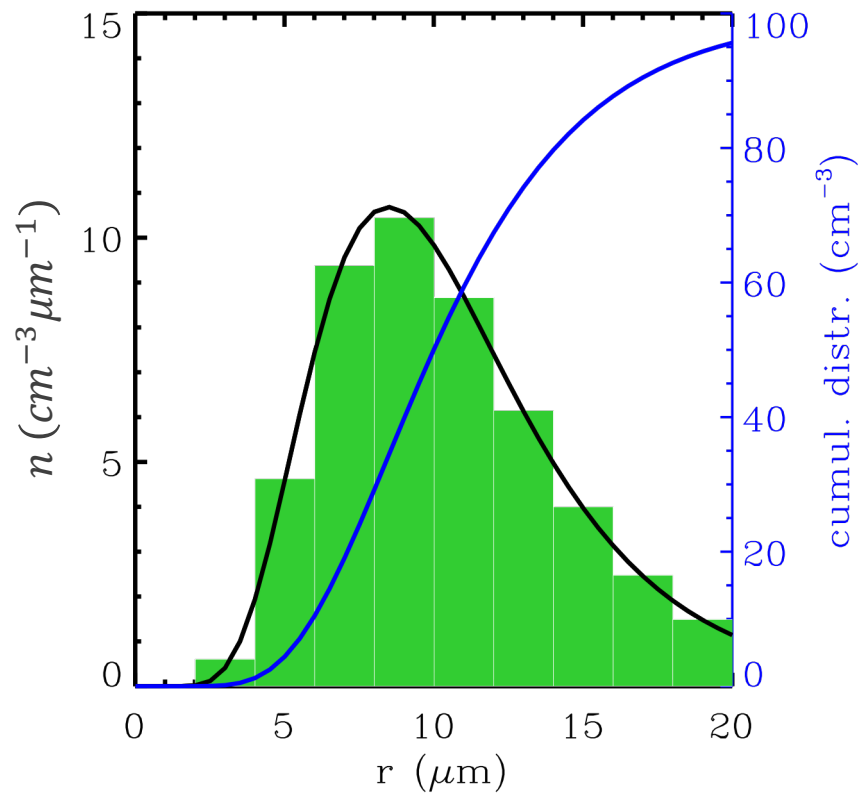
For many purposes the particle density function is expressed by a continuous analytical function $n(r)$, where $n(r)dr$ is the number of particles in the infinitesimal size interval $(r, r + dr)$.

$$n_i = N_i / \Delta r_i [cm^{-3} \mu m^{-1}], N = \sum_i n_i \Delta r_i$$

Continuous analytical function: $n(r) [cm^{-3} \mu m^{-1}]$, $N = \int_0^\infty n(r) dr$

Cumulative distribution $P(r) = \int_0^r n(x) dx$

OR probability density distribution $P(r) = \frac{1}{N} \int_0^r n(x) dx$



CLOUD MICROPHYSICAL PARAMETERS

- Particle Size Distribution (PSD) encapsulates a complete description of particle number concentration across the size spectrum
- Full spectral information is often unnecessary for characterizing bulk microphysical behavior.
- In practice, key thermodynamic, optical, and radiative properties of a cloud volume can be effectively captured by the statistical moments of the normalized size density function, $n(r)/N$, where N is the total droplet number concentration

CLOUD MICROPHYSICAL PARAMETERS

$$M_j = N\langle r^j \rangle = \sum_i r_i^j N_i = \int_0^\infty r^j n(r) dr, \quad j = 0, 1, 2, \dots \quad M_j - j^{th} \text{ moment of particle size distribution}$$

j	PARAMETER		APPLICATION	
0	Concentration	N		
1	Mean radius	$\bar{r} = \langle r \rangle$		
2	Mean surface radius	$r_s = \langle r^2 \rangle^{1/2}$	Extinction [m^{-1}]	$\sigma_{ext} = Q_{ext} \pi N r_s^2 [m^{-1}]$
3	Mean volume radius	$r_v = \langle r^3 \rangle^{1/3}$	Liquid Water Content Specific mass, Mixing ratio	$LWC = \frac{4}{3} \pi \rho_l N r_v^3 [g m^{-3}]$ $q_l = LWC / \rho_a [g kg^{-1}]$
6	no name	$\langle r^6 \rangle$	Radar reflectivity	$Z \propto \langle r^6 \rangle$

$$\bar{r} < r_s < r_v$$

INTEGRATED CLOUD CHARACTERISTICS

LIQUID WATER PATH (LWP)

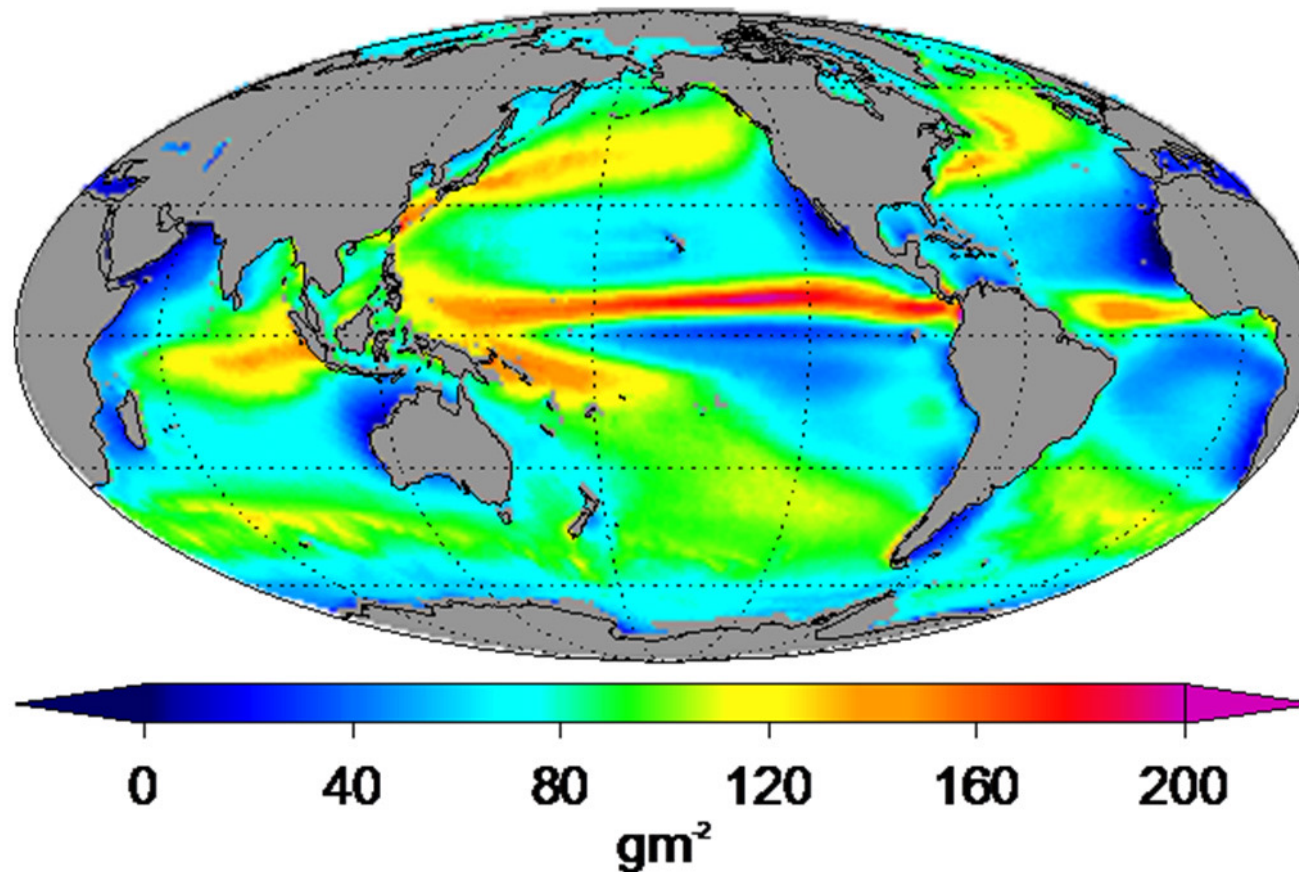
Liquid Water Path in units of $[g/m^2]$ is a measure of the total amount of liquid water present in a column of atmosphere (cloud).

LWP is an important quantity in understanding radiative transfer in the atmosphere. It is defined as the integral of liquid water content between two points in the atmosphere.

$$LWP = \int_{h_{base}}^{h_{top}} LWC \cdot dh \quad [gm^{-2}]$$

$$LWP = \int_{h_{base}}^{h_{top}} \frac{4}{3} \pi \rho_l N(h) r(h)_v^3 \cdot dh$$

LIQUID WATER PATH



EFFECTIVE RADIUS $r_e = \frac{r_v^3}{r_s^2}$

The effective radius (r_e) describes the optical properties of cloud particles because it relates the mass of liquid water in a cloud to its total projected cross-sectional area (which determines how much light the cloud scatters and absorbs).

Liquid Water Content $LWC = \frac{4}{3}\pi\rho_l N r_v^3 \Rightarrow r_v^3 = \frac{3 \cdot LWC}{4\pi\rho_l N}$

In optics, atmospheric physics, and radiative transfer, **extinction** refers to the total attenuation (loss of intensity) of electromagnetic radiation (light) as it passes through a medium containing gases, aerosols, or cloud droplets.

$$\sigma_{ext} = Q_{ext}\pi N r_s^2 \Rightarrow r_s^2 = \frac{\sigma_{ext}}{Q_{ext}\pi N}$$

$$r_e = \frac{r_v^3}{r_s^2} = \frac{3}{4} \frac{Q_{ext}}{\rho_l} \frac{LWC}{\sigma_{ext}}$$

The effective radius (r_e) is always larger than the mean volume radius (r_v) in any real droplet distribution ($r_e > r_v$).

While r_v measures the average physical size of a droplet based on cloud mass, r_e is shifted toward larger droplet sizes because it weights the distribution by surface area.

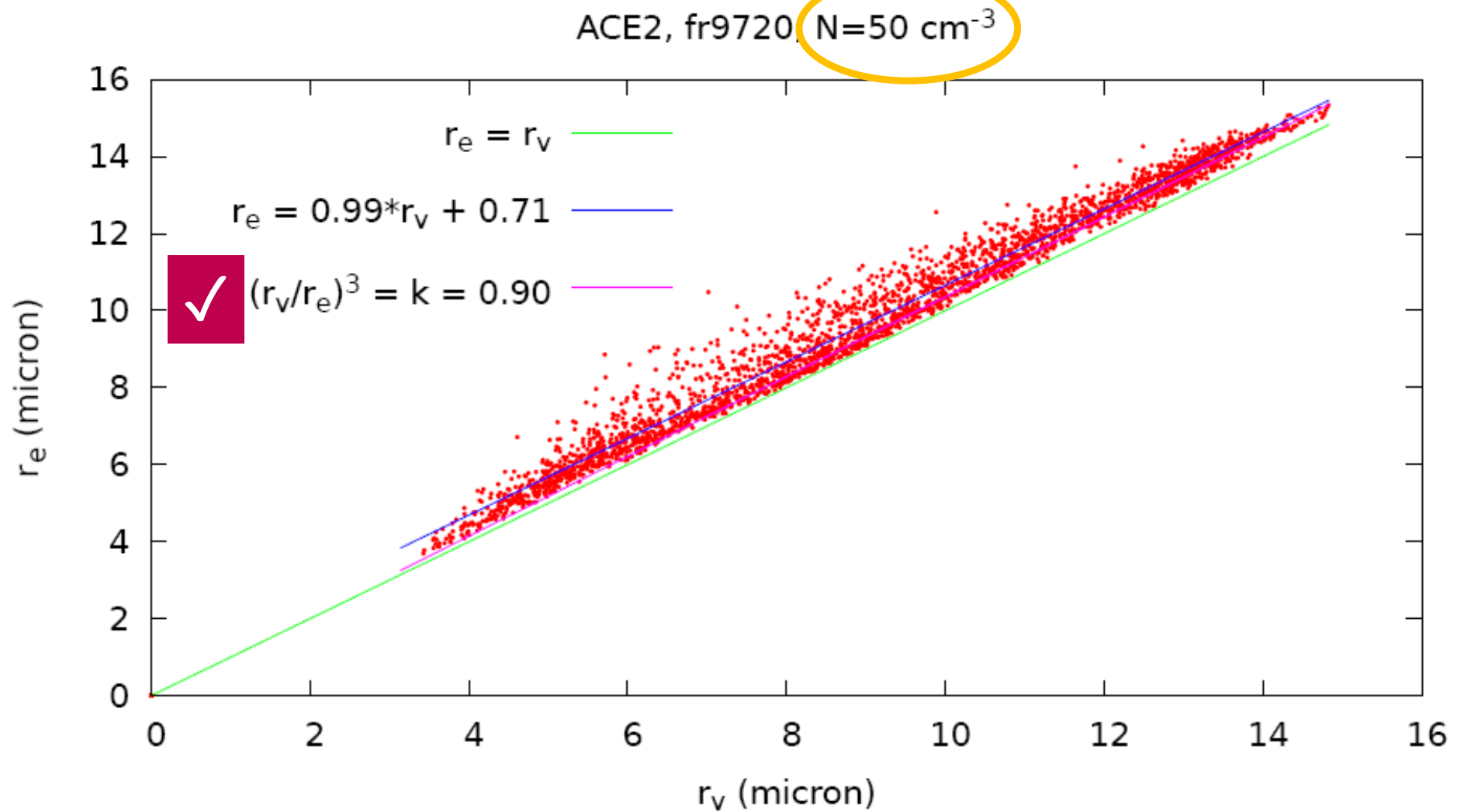
In cloud microphysics (e.g., Martin et al., 1994), it is useful to express the effective radius in terms of the mean volume radius. This relationship is commonly parameterized using the dimensionless factor k :

$$r_v^3 = k \cdot r_e^3 \quad \Rightarrow \quad r_e = k^{-1/3} r_v$$

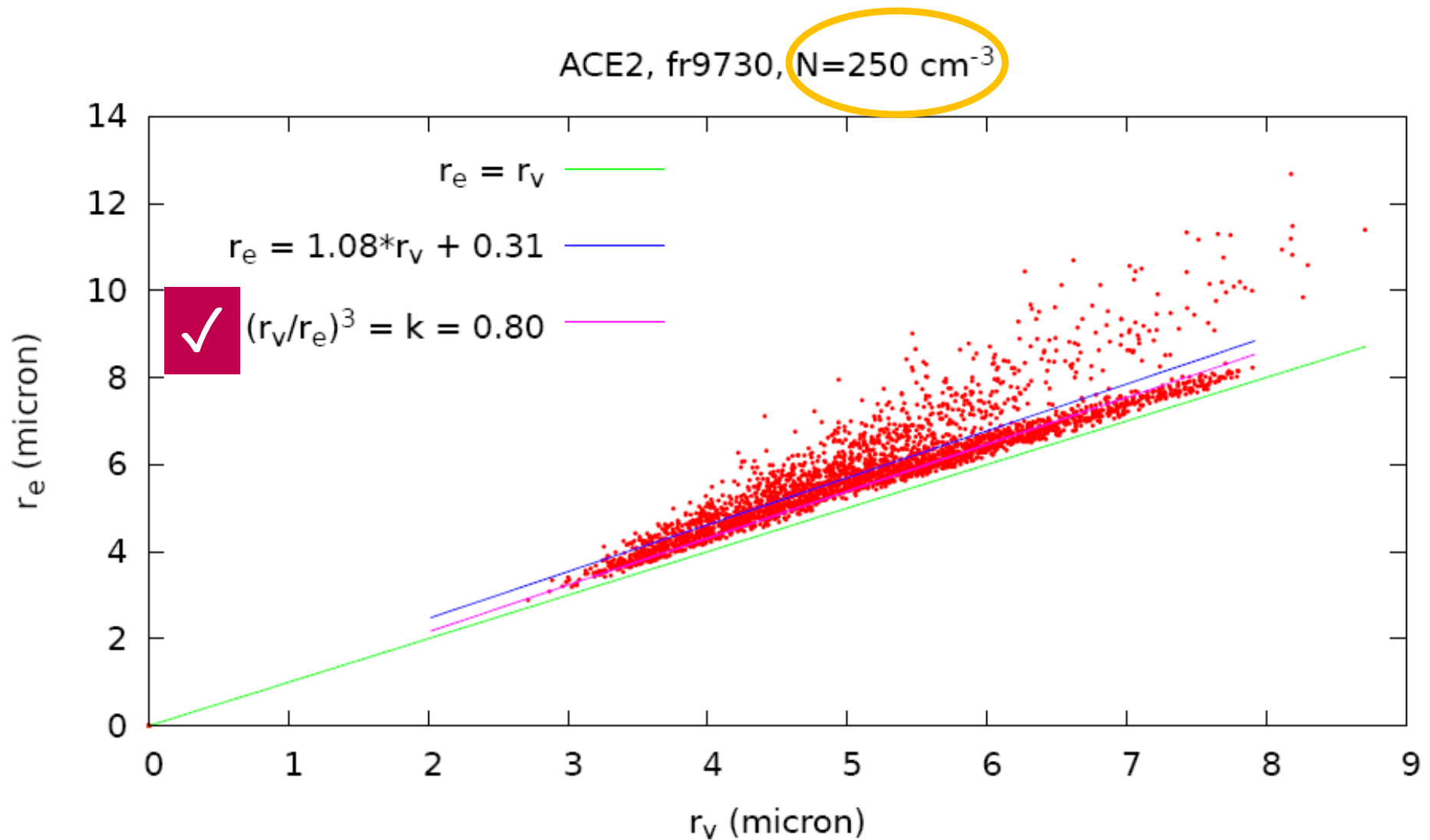
The value of k depends on the width and shape of the size distribution (often linked to aerosol pollution):

- **Maritime Clouds (Clean Air):**
 - $k \approx 0.80 \pm 0.07$
 - $r_e \approx 1.08 r_v$ (narrower size spectrum)
- **Continental Clouds (Polluted Air):**
 - $k \approx 0.67 \pm 0.07$
 - $r_e \approx 1.14 r_v$ (broader size spectrum due to varied CCN activation)

EFFECTIVE RADIUS (r_e) - VERSUS - MEAN VOLUME RADIUS (r_v)



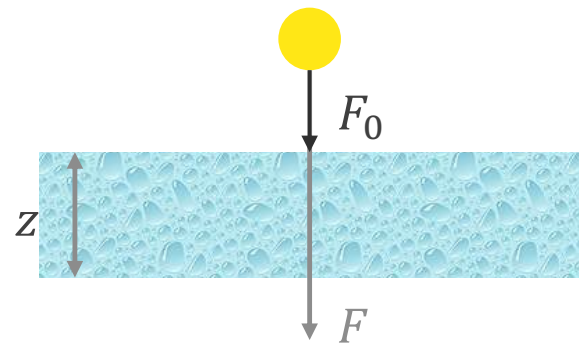
EFFECTIVE RADIUS (r_e) - VERSUS - MEAN VOLUME RADIUS (r_v)



INTEGRATED CLOUD CHARACTERISTICS

OPTICAL THICKNESS (DEPTH)

Optical depth (τ) serves as a dimensionless measure of the total extinction—the combined effect of absorption and scattering—experienced by radiation as it traverses an optically active medium such as a cloud.



$$\tau = \int_{h_{base}}^{h_{top}} \sigma_{ext} \cdot dz$$

$$F = F_0 e^{-\tau}$$

Beer-Lambert law

Optical depth can be viewed as a coordinate transformation. Instead of using physical distance (z), we rescale space to a dimensionless coordinate (τ), where $\tau = 1$ means that only $e^{-1} \approx 0.368$ of the energy is transmitted without being scattered.

INTEGRATED CLOUD CHARACTERISTICS

OPTICAL THICKNESS (DEPTH)

Optical depth is a measure of extinction integrated over a layer:

$$\tau = \int_{h_{base}}^{h_{top}} \sigma_{ext} \cdot dz = \pi Q_{ext} \int_{h_{base}}^{h_{top}} N r_s^2 \cdot dz$$

$$\tau = \frac{3Q_{ext}}{4\rho_l} \int_{h_{base}}^{h_{top}} \frac{LWC}{r_e} \cdot dz$$

$$\text{if } r_e = \text{const} \quad \tau = \frac{3Q_{ext}}{4\rho_l} \frac{LWP}{r_e}$$

$$r_e = \frac{r_v^3}{r_s^2} \Rightarrow r_s^2 = \frac{r_v^3}{r_e}$$

$$r_v^3 = \frac{3}{4\pi} \cdot \frac{LWC}{N\rho_l}$$

- If $\tau \ll 1$, the cloud is optically thin (transparent).
- If $\tau \gg 1$, the cloud is optically thick (opaque).

IS ASSUMING A CONSTANT r_{eff} VALID WHEN ESTIMATING LIQUID WATER PATH?

Assuming a constant r_e provides a first-order estimate of optical depth, but it neglects spatial variations in droplet size and aerosol-cloud interactions, leading to notable systematic errors.

Assuming a vertically uniform r_e equal to $r_e(h_{\text{cltop}})$ underestimates the cloud optical depth by approximately 15% to 20% for a given liquid water path.

CLOUD OPTICAL THICKNESS

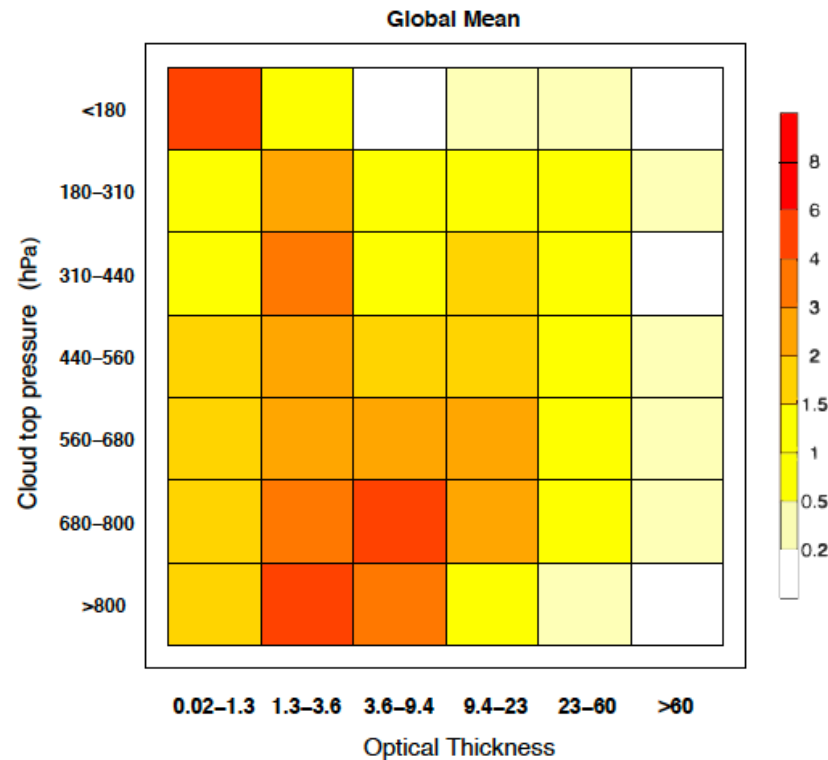


Figure 1.7 The global CTP- τ joint histogram from ISCCP in frequency of occurrence (%). Data is from Tselioudis et al, 2013

Tselioudis, George, Rossow, William, Zhang, Yuanchong, and Konsta, Dimitra. 2013. Global Weather States and Their Properties from Passive and Active Satellite Cloud Retrievals. *Journal of Climate*, 26(19), 7734–7746.

CLOUD ALBEDO

Albedo is a measure of the reflectivity of a surface or object. It is defined as the fraction of incoming solar radiation (light) that is diffusely reflected back into space rather than absorbed.

For a non-absorbing cloud (where droplets only scatter visible light without absorbing it, $\omega_0 = 1$), the relationship is derived using the classic two-stream approximation:

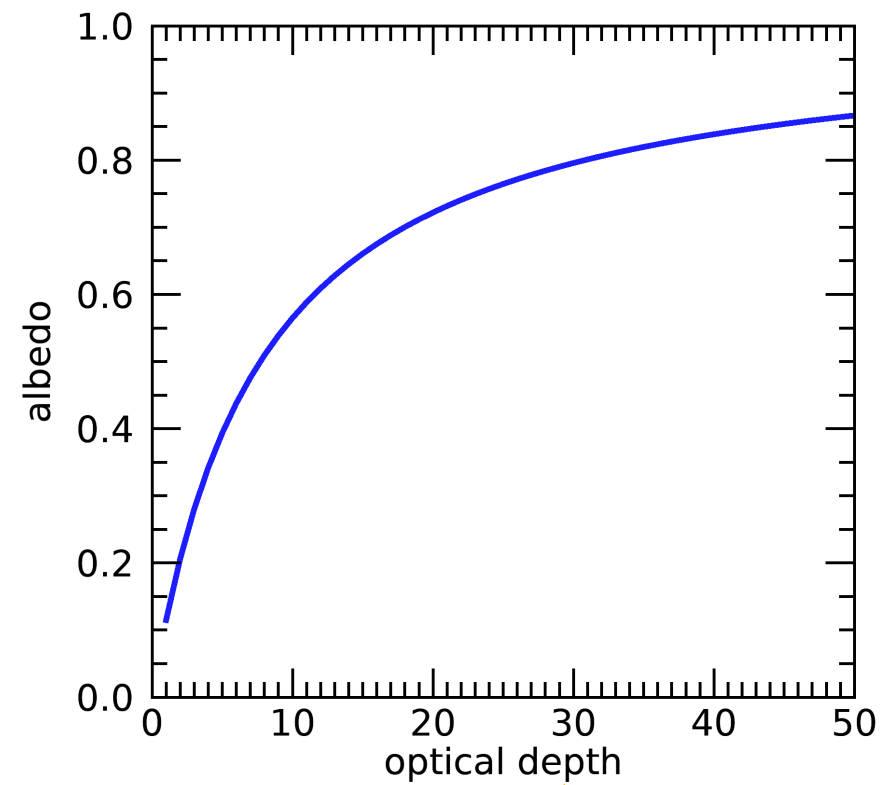
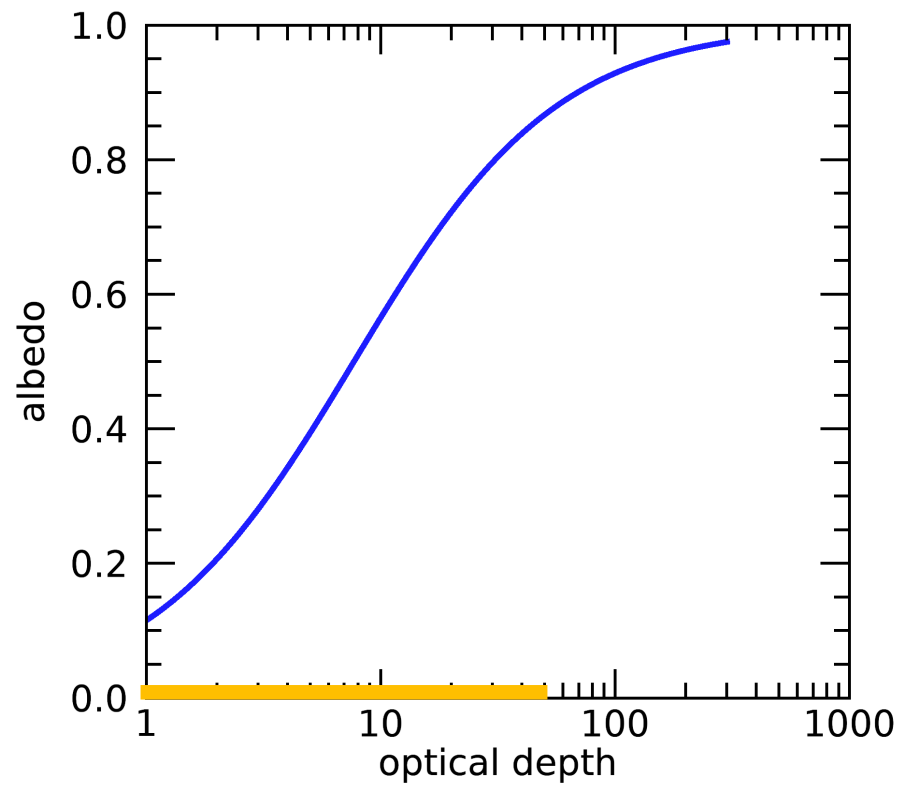
$$A = \frac{(1 - g)\tau}{1 + (1 - g)\tau}$$

Where:

- A = Cloud albedo (fraction of reflected light, between 0 and 1)
- τ = Cloud optical thickness
- g = Asymmetry parameter (typically between 0.83 and 0.87 for liquid water droplets, representing how much light scatters forward vs. backward)

CLOUD ALBEDO

$$A = \frac{\tau}{\tau + 7.7}$$



Linear scale

FORMATION AND GROWTH OF CLOUD PARTICLES



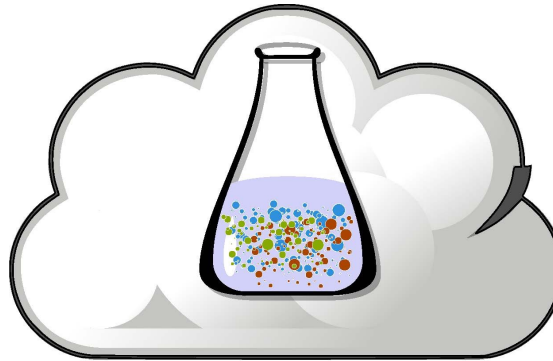
UNIVERSITY
OF WARSAW

WARM CLOUDS

AEROSOL - CLOUD - PRECIPITATION



Heterogeneous
nucleation
Cloud Condensation
Nuclei (CCN)
activation



Diffusional growth
Condensational growth
Collision/coalescence
Drizzle formation



Rain
CCN washout