Exercise Sheet 9

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1. *Small-amplitude waves in deep water* Show that in deep water with small-amplitude waves on the surface, the particle paths are circular, with the radius of the circles decreasing exponentially with depth. How do the trajectories change when the depth is finite and equal to h?

2. Consider a small-amplitude wave at the interface between a warmer fluid of density ρ_1 and a colder fluid of density ρ_2 . Show that the dispersion relation $\omega(k)$ is

$$\omega^2 = gk\frac{(\rho_2 - \rho_1)}{(\rho_2 + \rho_1)}. \tag{1}$$

3. Oscilations in a container. Liquids have a (sometimes unfortunate) tendency to slosh back and forth in a container. The tea in a teacup on a train which is rocking may slosh out of the cup; liquid in road tankers must be prevented from sloshing, or else the tanker becomes uncontrollable. Here, we study the motion of a liquid with a free surface in a container. The container has rigid walls at

$$\begin{cases} x = 0, \; x = l, \\ y = 0, \; y = b, \\ z = -h. \end{cases} \tag{2}$$

At equilibrium, the free surface coincides with the plane $z = 0$. Assume the velocity potential has the form

$$\phi(x, y, z, t) = X(x)Y(y)Z(z)e^{-i\omega t}. \tag{3}$$

Calculate the velocity potential $\phi(x, y, x, t)$ and the dispersion relation $\omega = \omega(k)$ for the motion of a liquid with a free surface in a rectangular container.