Supplementary material 1

We say that a result is physical if U is **real** and **non-negative**.

We have balanced flow equation:

$$\frac{U^2}{R} = -fU + fU_g,$$

which we can simplify to a quadratic formula:

$$U^2 + fRU + fU_g = 0.$$

We can solve it for U:

$$U = -\frac{fR}{2} \pm \sqrt{\frac{f^2R^2}{4} + fRU_g},$$

and here we have 3 parameters which can change the sign: R, U_g, and the sign before the square root. This means that we have 2^3 possibilities.

We can change this expression to help us analyze the signs better:

$$U = -\frac{fR}{2} \pm \left|\frac{fR}{2}\right| \sqrt{1 + \frac{4U_g}{fR}}$$

First division will be with regards to the sign of R. If it is positive, then the parcels will turn left following the motion, if negative, they will turn right following the motion.

The second division will be with regards to the sign of U_g.

Last division will be with regards to the sign before the square root.

1. Positive R - the first term on the right is negative, therefore the second term on the right needs to be larger than the first term for U to be positive.

   (a) Positive geostrophic wind - the second term under the square root sign is positive, so we add or subtract something which is positive and greater than the first term on the right. If we add, then we have a physical solution: **a cyclonic flow (regular low)**. If we subtract, then our velocity is negative, and our solution is nonphysical.

   (b) Negative geostrophic wind - the second term under the square root sign is negative. If it is greater than one, then the result is non-real. If it is smaller than one, then U is negative. Therefore both signs yield nonphysical results.

2. Negative R - the first term on the right is positive, therefore the second term on the right needs to be smaller than the first term for U to be positive.

   (a) Positive geostrophic wind - the second term under the square root is negative. If it is bigger than one, then the result is not real. If it is smaller than one, then we have two solutions, in which one solution is $U > -\frac{fR}{2}$ and is called **an anticyclonic flow (anomalous high)**, and the other one is $U < -\frac{fR}{2}$ and is called **an anticyclonic flow (regular high)**.

   (b) Negative geostrophic wind - the second term under the square root is positive, and bigger than the first term on the right hand side, therefore taking the plus sign yields physical result: **an antibaric flow (anomalous low)**. Taking the minus sign yields negative U.

One last remark: the **anomalous** and **regular** refer to the wind velocity. **Anomalous** is rather big and uncommon, **regular** is smaller and common. **High** and **low** refer to the spatial structure of the pressure.