

THERMODYNAMICS



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ATMOSPHERE

A MULTI-COMPONENT MULTI-PHASE SYSTEM

- Gaseous constituents: major gases with fixed proportions by volume (dry air)
 - Nitrogen N_2 78.08%
 - Oxygen O_2 20.95%
 - Argon Ar 0.934%

99,96%
- Variable vapors and minor gases:
 - Carbon dioxide (CO_2)
 - Neon, Helium, Nitrous Oxide, Ozone, Methane, Sulfur compounds, organics
 - **Water vapor (0 – 4 % by volume)**
- **Water** marks influences atmospheric motion in the lower atmosphere on all temporal and spatial scales for two main reasons:
 - It readily changes phase, thereby significantly affecting the local temperature through latent heat release or absorption.
 - It fosters diverse and complex interactions with radiant energy.

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- In cloud physics, we consider:
 - **Dry air:** a gas mixture consisting primarily of nitrogen and oxygen, with all other constituents present in negligible quantities. Its molar mass is **28.96 g/mol**.
 - **Water:** a substance that can exist in three physical phases: vapor, liquid, and ice.
 - **Moist air:** a mixture of dry air and water vapor.
 - **Moist atmosphere:** a system comprising dry air alongside water vapor, liquid water, and/or ice.

SUBSCRIPTS NOTATION

d	dry air
v	water vapor
l	liquid water
i	solid water (ice)
c	condensate
t	total water (irrespective of phase)
s	saturated state, or process
e	equivalent (all condensate) reference state
ℓ	liquid-free (all vapor) reference state

DESCRIPTION OF MAJOR CONSTITUENTS OF THE ATMOSPHERE

- Total mass of the system: $m = m_d + m_v + m_l + m_i$
 - d – dry air
 - v – water vapor
 - l – liquid water
 - i – ice
- To describe the amount of matter, we use:
 - Specific mass: $q_x = m_x/m$
 - It is useful in a thermodynamic description of an equilibrium system (the normalizing mass is invariant as long as the basic flow describes the motion of the dry air).
 - Mixing ratio: $r_x = m_x/m_d$
 - It is helpful when considering the possibility that different components of a mass element have their own velocities.

- In cloud physics we distinguish between:
 - equilibrium condensed phases associated with clouds, which evolve with the thermodynamic state in a nearly **reversible** way, and
 - larger hydrometeors which evolve in an **irreversible** way
 - Larger hydrometeors, like raindrops and most forms of ice, develop through irreversible microphysical processes such as the collision and coalescence of water droplets.
 - These are more difficult to approximate as equilibrium phases. Because they are larger, non-equilibrium phases of water in the atmosphere are also more dilute and short-lived.
- The present lecture (Thermodynamics) focuses mostly on reversible thermodynamic processes.

THE CONTINUUM & LOCAL EQUILIBRIUM ASSUMPTION

- Thermodynamic systems in local equilibrium are conceptualized as mass elements, commonly referred to as **air** or **fluid parcels**.
- Formally, this concept is valid for mass elements that are sufficiently small to occupy a spatial scale much smaller than that of macroscopic thermodynamic variations (L_{macro}), yet large enough to remain significantly greater than the mean free path of the molecules (λ_{mol}).

$$\lambda_{mol} < L_{parcel} < L_{macro}$$

- Diffusion rapidly homogenizes the atmosphere on scales smaller than the **Kolmogorov length scale**

- $\eta = (\nu^3 / \varepsilon)^{1/4}$

- ν - viscosity of the atmosphere, ε - turbulent dissipation rate
 - In vigorous cumulus clouds: $\varepsilon = 0.05 \text{ m}^2/\text{s}^3$, $\nu = 1.5 \cdot 10^{-5} \text{ m}^2/\text{s}$
 - $\eta = 0.5 \text{ mm}$; this is several thousand times larger than the mean free path of an air molecule, making the concept of an air parcel a useful one.

DILUTE NATURE OF THE CONDENSATE

- Typically, the specific cloud water content (or the mixing ratio) within a cloud is less than 1 g kg^{-1} , which is significantly lower than the specific humidity of water vapor.
- What is the volume fraction of the liquid water condensate?

$$\frac{V_l}{V_d} = \frac{m_l}{\rho_l} \cdot \frac{\rho_d}{m_d}$$

$$\rho_d = 1 \frac{\text{kg}}{\text{m}^3}; \quad \rho_l = 10^3 \frac{\text{kg}}{\text{m}^3}; \quad \frac{m_l}{m_d} = 10^{-3} \frac{\text{kg}}{\text{kg}}$$

$$\frac{V_l}{V_d} = 10^{-6}$$

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- The dilute/discrete nature of the condensate can conflict with the concept of an equilibrium thermodynamic system, as the condensate is not continuously distributed on the Kolmogorov scale.
 - In the presence of the condensate (droplets, ice) that is not continuously distributed, the concept of an air parcel must be expanded.
 - One often imagines an air parcel as being on the scale of around 1 m^3 .
 - Strictly speaking, volumes of air this large cannot be assumed to have a uniform temperature, but the error introduced by this approximation is typically much smaller than those associated with other approximations invoked in the description of such systems.

SUBSCRIPTS NOTATION

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GAS PHASE DESCRIPTION

- Vapor pressure, e – partial pressure of water vapor [mb , hPa]
- Absolute humidity, $\rho_v = 1/v_v$ – concentration of water vapor [kg/m^3]
- Specific humidity, q_v – the ratio of the masses of vapor and mixture, [kg/kg , g/kg]

$$q_v = \frac{m_v}{m} = \frac{\rho_v}{\rho} = \frac{\rho_v}{\rho_d + \rho_v}$$

$$q_v = \varepsilon \frac{e}{p - (1 - \varepsilon)e} \approx \varepsilon \frac{e}{p}$$

$$\rho_v = \frac{e}{R_v T} = \varepsilon \frac{e}{R_d T}$$

$$\rho_d = \frac{p_d}{R_d T} = \frac{p - e}{R_d T}$$

One can analogically define:

specific dry air mass: $q_d = \frac{m_d}{m}$

$$\varepsilon = \frac{R_d}{R_v} \approx 0.622$$

GAS PHASE DESCRIPTION...

- **Mixing ratio, r_v** – the ratio of the masses of vapor and dry air [kg/kg, g/kg]

$$r_v = \frac{m_v}{m_d} = \frac{\rho_v}{\rho_d}$$

$$r_v = \varepsilon \frac{e}{p - e} \approx \varepsilon \frac{e}{p}$$

$$\rho_v = \frac{e}{R_v T} = \varepsilon \frac{e}{R_d T}$$
$$\rho_d = \frac{p_d}{R_d T} = \frac{p - e}{R_d T}$$

One can analogically define:

dry air mixing ratio: $r_d = \frac{m_d}{m_d} \equiv 1$

The mixing ratio is approximately equal to the specific humidity

$$r_v = \frac{m_v}{m_d} = \frac{m_v}{m - m_v} = \frac{q_v}{1 - q_v} \cong q_v$$

The specific humidity (q_v) and the mixing ratio (r_v) can often be used interchangeably. Both are conserved for an individual parcel outside of condensation regions.

By contrast, measures of absolute concentration such as e and ρ_v change for an individual air parcel due to changes in its pressure, even if the mass of vapor remains fixed.

- Relative humidity (f) is defined as the ratio of the actual partial water vapor pressure (e) to the saturation vapor pressure (e_s).

$$f = \frac{e}{e_s(T)}$$

It can be approximated as the ratio of the actual water vapor mixing ratio (r_v) to the saturation mixing ratio (r_s).

$$f \cong \frac{r_v}{r_s(p, T)} \cong \frac{q_v}{q_s(p, T)}$$

SPECIFIC HUMIDITY IN THE PRESENCE OF A CONDENSED PHASE

$$q_v = \frac{m_v}{m} = \frac{m_v}{m_d + m_v + m_c} \quad / \frac{V}{V}$$

$$q_v = \frac{\rho_v}{\rho_d + \rho_v + \frac{m_c}{V}}$$

$$q_v = \frac{\varepsilon e}{p - e(1 - \varepsilon) + \varepsilon e \frac{q_c}{q_v}}$$

$$q_v = \frac{\varepsilon e}{p - e(1 - \varepsilon)} (1 - q_c)$$

$$\left\{ \begin{array}{l} \rho_v = \frac{e}{R_v T} = \varepsilon \frac{e}{R_d T} \\ \rho_d = \frac{p_d}{R_d T} = \frac{p - e}{R_d T} \\ \\ V = V_g + V_c \approx V_g \\ V_g = \frac{m_v}{\rho_v} = \frac{m_v R_d T}{\varepsilon e} \\ \\ \frac{m_c}{V} = \frac{\varepsilon e}{R_d T} \cdot \frac{m_c}{m_v} = \frac{\varepsilon e}{R_d T} \cdot \frac{q_c}{q_v} \end{array} \right.$$

Specific condensate mass $q_c = \frac{m_c}{m}$ is of order of $10^{-4} - 10^{-3}$

EQUATION OF STATE

Assuming an air parcel to be composed of an ideal mixture of ideal gases, potentially in the presence of condensate, the equation of state is that of an ideal gas of variable composition:

$$p = p_d + e = \left(\frac{m_d R_d}{V} + \frac{m_v R_v}{V} \right) T \quad R_d = 287 \text{ J K}^{-1} \text{ kg}^{-1} \quad R_v = 461 \text{ J K}^{-1} \text{ kg}^{-1}$$

Defining the density of the air parcel (including the condensate) as $\rho = m/V$,
where $m = m_d + m_v + m_c$

$$p = \frac{m_d R_d + m_v R_v}{m} \rho T = (q_d R_d + q_v R_v) \rho T$$

allows to formulate the equation of state as: $p = \rho R T$ *R is a state-dependent gas constant*

$$R = q_d R_d + q_v R_v = (1 - q_v - q_c) R_d + q_v R_v = R_d [1 + \varepsilon' q_v - q_c]$$

$$\begin{aligned} m &= m_d + m_v + m_c & /m \\ 1 &= q_d + q_v + q_c \end{aligned}$$

$$\varepsilon' = \left(\frac{R_v}{R_d} - 1 \right) \approx 0.608$$

DENSITY TEMPERATURE, VIRTUAL TEMPERATURE

To avoid dealing with a state-dependent gas constant, we define a **density temperature**, T_ρ

$$T_\rho = T(1 + 0.608q_v - q_c)$$

$$p = \rho R_d T_\rho$$

Here, the $+0.608q_v$ term represents the buoyancy-enhancing effect of the lighter water vapor, while the $-q_c$ term represents the weight-loading (buoyancy-reducing) effect of the suspended condensate droplets.

In the absence of water, the **density temperature** reduces to the air temperature.

It can be interpreted as the temperature that a dry air parcel would have if it possessed the same density and pressure as the given moist air parcel.

In meteorological literature, the density temperature in the absence of condensate ($q_c = 0$) is commonly called the **virtual temperature**.

$$T_v = T(1 + \varepsilon' q_v) = T(1 + 0.608q_v)$$

BUOYANCY

For a given pressure, the density temperature uniquely determines the air density, making it a crucial parameter for calculating buoyancy.

$$\rho = p/R_d T_\rho$$

The buoyancy (b) of a fluid parcel is governed by the density difference between the parcel (ρ) and its ambient environment (reference state; ρ_0).

$$b \equiv -g \frac{\rho - \rho_0}{\rho_0}$$

Assume that, locally, the density is expressed as a perturbation from a reference state density (ρ_0): $\rho = \rho_0 + \rho'$

Linearizing the equation of state ($p = \rho R_d T_\rho$) yields:

$$p_0 + p' = (\rho_0 + \rho') R_d (T_{\rho_0} + T'_\rho) \quad /p_0$$

$$\frac{p'}{p_0} = \frac{\rho'}{\rho_0} + \frac{T'_\rho}{T_{\rho_0}} + \cancel{\frac{\rho' T'_\rho}{\rho_0 T_{\rho_0}}}$$

BUOYANCY

$$\frac{p'}{p_0} = \frac{\rho'}{\rho_0} + \frac{T'_\rho}{T_{\rho_0}}$$

Neglecting the pressure perturbation term ($p' \approx 0$) allows us to directly equate the relative density deficit to the relative density temperature excess:

$$b \equiv -g \frac{\rho - \rho_0}{\rho_0} = -g \frac{\rho'}{\rho_0} \approx g \frac{T'_\rho}{T_{\rho_0}}$$

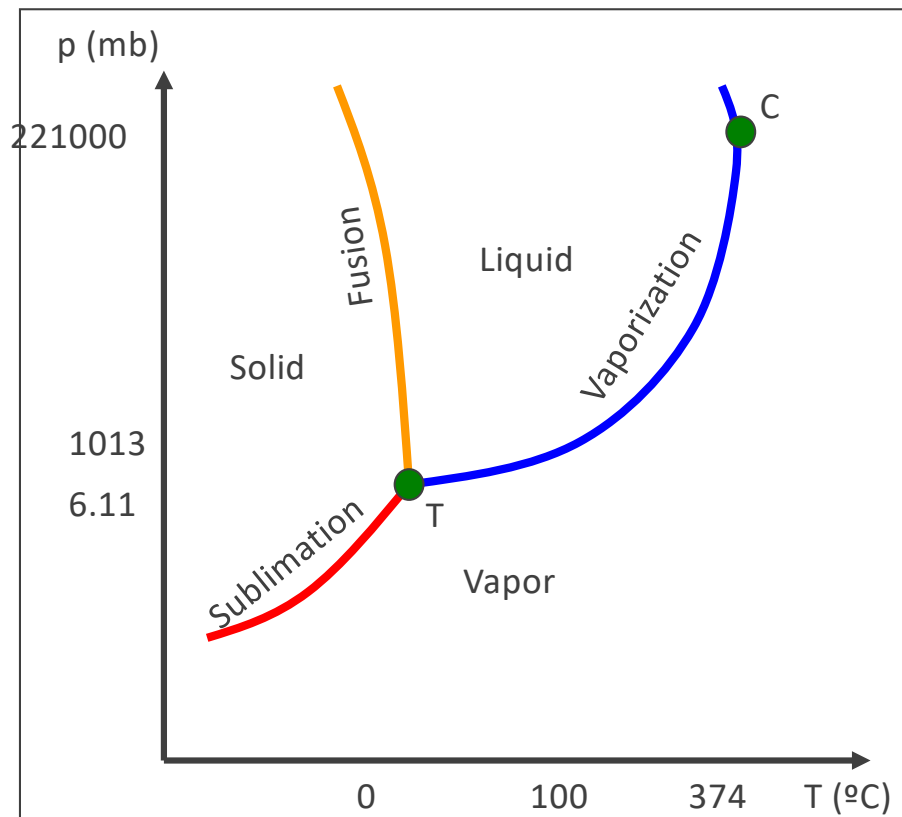
The approximation of buoyancy using density temperature rests on the assumption that relative pressure perturbations are much smaller than relative density perturbations ($|p'/p_0| \ll |\rho'/\rho_0|$). It is the basis for the anelastic approximation.

It is equivalent to the assumption that the parcel's pressure adjusts instantaneously to the environmental pressure ($p \approx p_0$).

CLAUSIUS-CLAPEYRON EQUATION



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Condensed water (liquid or ice) in the atmosphere is typically in equilibrium with water vapor (i.e., the air parcel is saturated).

The Clausius-Clapeyron equation relates this equilibrium vapor pressure to the temperature of the heterogeneous system.

CLAUSIUS-CLAPEYRON EQUATION

The Clausius-Clapeyron equation relates the equilibrium saturation vapor pressure (e_s) directly to the temperature (T) of this heterogeneous system:

$$\frac{de_s}{dT} = \frac{Le_s}{R_v T^2}$$

where:

- e_s is the saturation vapor pressure,
- T is the absolute temperature,
- L is the latent heat (of vaporization L_v or sublimation L_s),
- R_v is the individual gas constant for water vapor.

The Clausius-Clapeyron equation

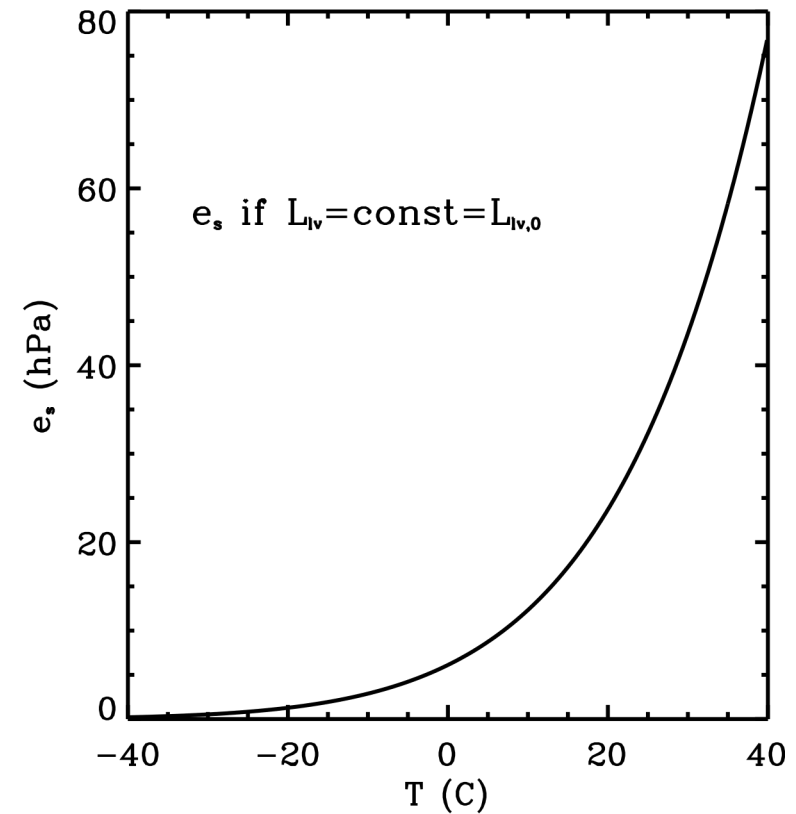
$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2}$$

can be easily integrated:

- $L_v = \text{const} = L_{v0}$ – latent heat at $T_0 = 273,16\text{K}$

$$\ln\left(\frac{e_s}{e_{s0}}\right) = -\frac{L_{v0}}{R_v} \left(\frac{1}{T} - \frac{1}{T_0}\right)$$

$$e_s = e_{s0} \exp\left[-\frac{L_{v0}}{R_v} \left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$



$$T_0 = 0^\circ\text{C}$$

$$e_{s0} = 611 \text{ Pa}$$

$$L_{v0} = 2.5 \cdot 10^6 \text{ J/kg}$$

The Clausius-Clapeyron equation

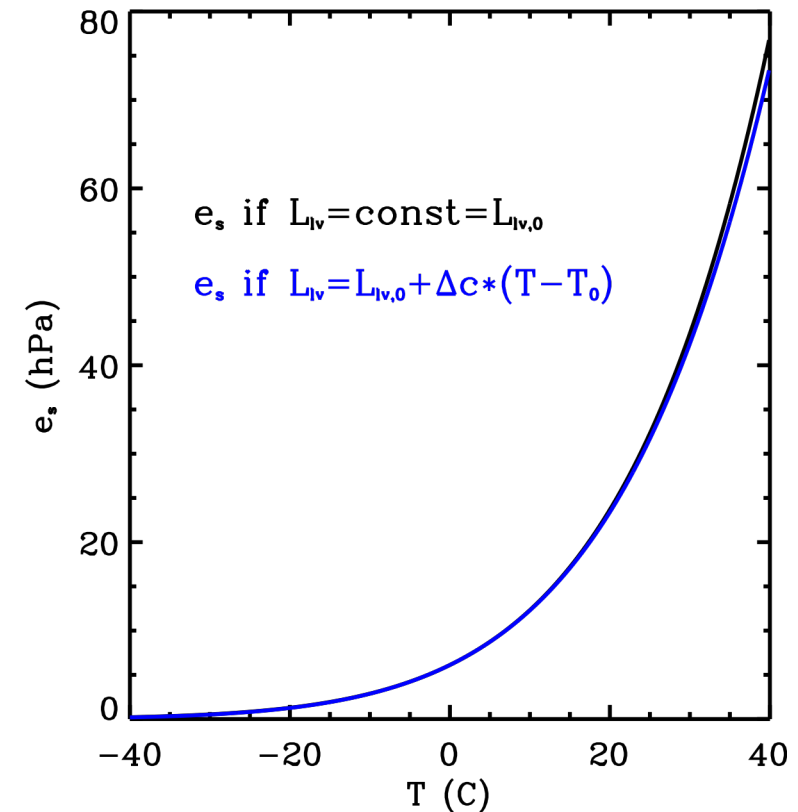
$$\frac{de_{sl}}{dT} = \frac{L_v e_s}{R_v T^2}$$

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- $L_v = L_{v0} + \Delta c(T - T_0)$, where $\Delta c = c_{pv} - c_l = \text{const} = 2317 \frac{\text{J}}{\text{kgK}}$

$$\ln\left(\frac{e_s}{e_{s0}}\right) = -\frac{L_{v0} - \Delta c T_0}{R_v} \left(\frac{1}{T} - \frac{1}{T_0}\right) + \frac{\Delta c}{R_v} \ln \frac{T}{T_0}$$

$$e_s = e_{s0} \left(\frac{T}{T_0}\right)^{\Delta c/R_v} \exp\left[-\frac{L_{v0} - \Delta c T_0}{R_v} \left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

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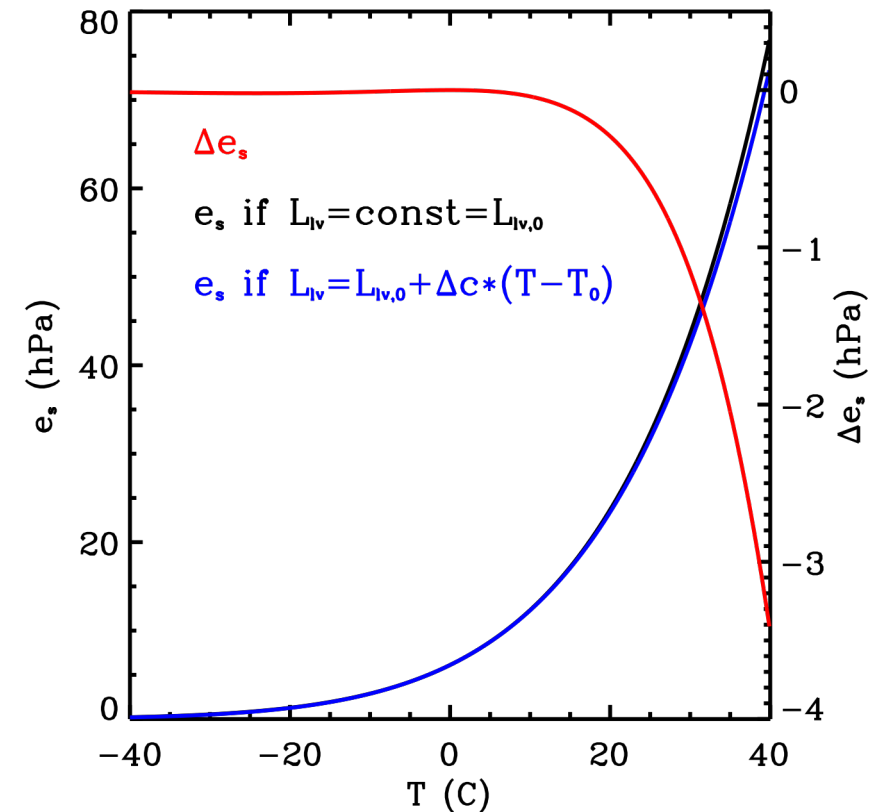
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$$e_s = e_{s0} \left(\frac{T}{T_0}\right)^{\Delta c/R_v} \exp\left[-\frac{L_{v0} - \Delta c T_0}{R_v} \left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$



$$T_0 = 0^\circ\text{C}$$

$$e_{s0} = 611 \text{ Pa}$$

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APPROXIMATED SOLUTIONS OF CLAUSIUS-CLAPEYRON EQUATION

Magnus-Tetens formula

Murray, F. W., On the computation of saturation vapor pressure, J. Appl. Meteorol., 6, 203-204, 1967

$$e_s(T) = e_{s0} \exp \left[\frac{17.27(T - 273.15)}{(T - 273.15) + 237.7} \right]$$

Bolton formula

Bolton, D., The computation of equivalent potential temperature, Monthly Weather Review, 108, 1046-1053, 1980

$$e_s(T) = e_{s0} \exp \left[\frac{17.67(T - 273.15)}{(T - 273.15) + 243.5} \right]$$

POTENTIAL TEMPERATURES



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POTENTIAL TEMPERATURES

The first and second laws dictate how temperature changes between a given state and a **reference state** defined by its pressure p_0 , and phase distribution, given by the triplet $\{p_0, q_v, q_l\}$.

The temperature in this reference state is called **the potential temperature** (denoted by θ) as it measures the temperature the system would have to have in the reference state for the entropy of this state to be identical to that of the given state.

Potential temperature is invariant under an isentropic process, but its properties and absolute values depend on the choice of the reference.

The potential temperature, rather than **the sensible temperature**, is often preferred as a state variable because it is invariant for reversible transformations of the air parcel.

It provides a convenient way to compare air parcels in different parts of the atmosphere, where, for instance, the pressure or humidity may vary.

POTENTIAL TEMPERATURE θ

Reference state: $\{p_0, q_v, q_l\} = \{10^5 Pa, 0, 0\}$

$$\theta = T \left(\frac{p_0}{p_d} \right)^\kappa \quad \kappa = \frac{R_d}{c_{pd}}, \quad p_0 = 1000 \text{ hPa}$$

EQUIVALENT POTENTIAL TEMPERATURE θ_e

Reference state $\{p_0, q_v, q_l\} = \{10^5 \text{ Pa}, 0, q_t\}$

All the vapor is condensed into liquid at $p_0 = 1000 \text{ hPa}$.

For historical reasons this particular reference state is called **an equivalent state** and denoted by subscript 'e'.

$$\theta_e = T \left(\frac{p_0}{p} \right)^{\frac{R_e}{c_e}} \Omega_e \exp \left(\frac{q_v L_v}{c_e T} \right)$$

$$\Omega_e = \left(\frac{R}{R_e} \right)^{\frac{R_e}{c_e}} \left(\frac{e}{e_s} \right)^{-\frac{q_v R_v}{c_e}}$$

$$R_e = q_d R_d$$

$$c_e = c_{pd} + q_t (c_l - c_{pd})$$

$$R = R_d + q_t (R_v - R_d)$$

Ω_e is a factor that is very near unity, and (because $q_t \ll 1$) depends only very weakly on the thermodynamic state.

e/e_s is relative humidity

SATURATED EQUIVALENT POTENTIAL TEMPERATURE

$$\theta_s = \theta \exp\left(\frac{q_s L_v}{c_{pd} T}\right)$$

θ_s is only a function of temperature and pressure, so it measures the thermal structure of the atmosphere.

θ_s is constant following a saturated pseudo-adiabat.

The word *pseudo* arises because formally the process corresponding to constant θ_s is similar to a reversible adiabat, but the removal of condensate upon condensation, as implied by the use of q_s instead of q_t implies a loss of condensate enthalpy by the system, hence it is not truly adiabatic.

The difference between θ_s and θ_e measures the subsaturation, as $\theta_e < \theta_s$.

EQUIVALENT POTENTIAL TEMPERATURE

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θ_e is the temperature that an air parcel would attain if it was brought to 1000 hPa with all water vapor contained in the parcel condensed.

Owing to the latent heat release θ_e is higher than θ .

Unlike the dry adiabatic process, in which θ can be found by bringing the air parcel to 1000 hPa, there is no atmospheric process in which the air parcel reaches 1000 hPa while condensing out all the water vapor.

θ_e is a hypothetical moisture quantity.

The most often an approximate form of θ_e is used:

$$\theta_e = \theta \exp \left(\frac{q_v L_v}{c_{pd} T} \right)$$

LIQUID WATER POTENTIAL TEMPERATURE

The counterpart to the equivalent potential temperature is the **liquid-water potential temperature**, which is the temperature an air parcel would have if were reversibly brought to the reference state $\{p_0, q_v, q_l\} = \{10^5 \text{ Pa}, q_v, 0\}$.

$$\theta_\ell = T \left(\frac{p_0}{p} \right)^{\frac{R_\ell}{c_{p\ell}}} \Omega_\ell \exp \left(- \frac{q_l L_v}{c_{p\ell} T} \right) \quad \Omega_\ell = \left(\frac{q_t}{q_v} \right)^{\frac{q_t R_v}{c_{p\ell}}} \left(\frac{R}{R_\ell} \right)^{\frac{R_\ell}{c_{p\ell}}} \left(\frac{e}{e_s} \right)^{- \frac{q_l R_v}{c_{p\ell}}}$$

$$c_{p\ell} = c_{pd} + q_t (c_{pv} - c_{pd})$$

$$R_\ell = q_d R_d + q_t R_v$$

$$R = R_d + q_t (R_v - R_d)$$

Approximate form: $\theta_\ell = \theta \exp \left(- \frac{q_l L_v}{c_{pd} T} \right)$

ADIABATIC LAPSE RATE

The temperature of an air parcel moving vertically changes due to expansion work.

It is useful to write the first law in form of enthalpy:

$$dh = c_{pd}dT = \delta q + vdp \quad \text{in adiabatic process } \delta q = 0$$

$$c_{pd}dT - vdp = 0$$

Hydrostatic equation:

$$dp = -\rho g dz$$

$$\rho = \frac{1}{v}$$

$$vdp = -g dz$$

$$c_{pd}dT + g dz = 0$$

$$\Gamma_d = -\frac{dT}{dz} = \frac{g}{c_{pd}}$$

$$\Gamma_d = \frac{g}{c_{pd}} = \frac{9.81 \text{ m/s}^2}{1004 \text{ J/kg} \cdot \text{K}} \approx 9.8 \frac{\text{K}}{\text{km}}$$

SATURATED MOIST ADIABATIC LAPSE RATE

$$\Gamma_s \equiv - \left. \frac{dT}{dz} \right|_{\theta_e} = \gamma \Gamma_d$$

γ - normalized saturated moist adiabatic lapse rate

$$\gamma \equiv \frac{c_{pd}}{c_p} \left[\frac{1 + q_s \beta_T \left(\frac{R_v}{R} \right)}{1 + q_s \beta_T \left(\frac{L_v}{c_p T} \right)} \right] \equiv \frac{c_{pd}}{c_p} \gamma'$$

$$\beta_T \equiv \frac{\partial \ln q_s}{\partial \ln T} = \frac{L_v}{R_v T} \frac{\partial \ln q_s}{\partial \ln p} \approx \frac{L_{lv}}{R_v T} \approx \frac{5400 \text{ K}}{T}$$

$$c_p = q_d c_{pd} + q_s c_{pv} + q_l c_l$$

$$R = q_d R_d + q_s R_v$$

$$q_s + q_l = \text{const}$$

SATURATED MOIST PSEUDO-ADIABATIC GRADIENT

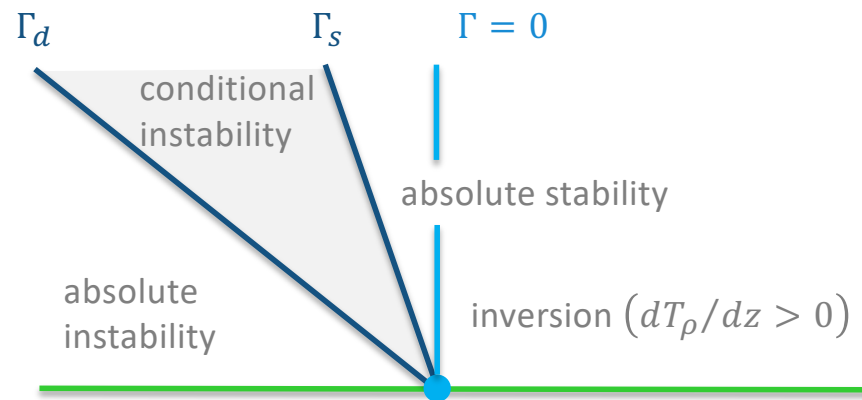
The presence of water vapor and liquid phase is neglected in gas constant and specific heat.

$$\Gamma_{s,pa} = \gamma_{pa} \Gamma_d$$

γ_{pa} - normalized saturated moist adiabatic lapse rate

$$\gamma_{pa} = \frac{1 + q_s \frac{L_{lv}}{R_d T}}{1 + q_s \frac{L_{lv}^2}{c_{pd} R_v T^2}}$$

STABILITY

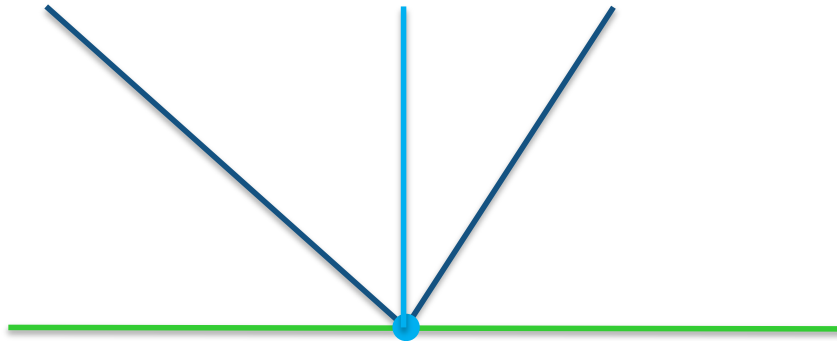


STABILITY

Dry parcel movements

$d\theta/dz < 0$
unstable

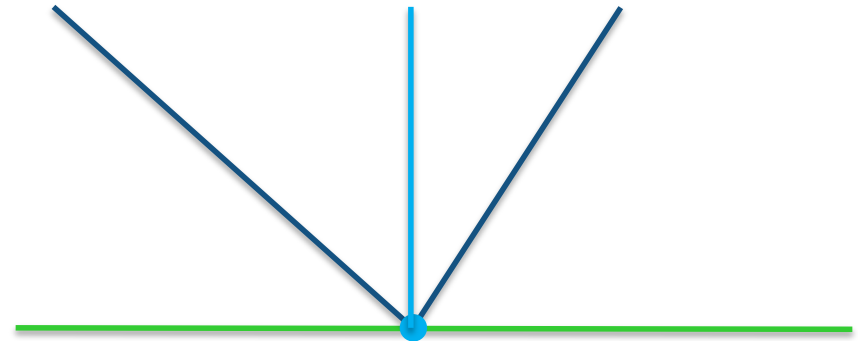
$d\theta/dz > 0$
stable



Moist parcel movements

$d\theta_s/dz < 0$
unstable

$d\theta_s/dz > 0$
stable



WATER CONDENSED IN SATURATED ADIABATIC ASCENT

Adiabatic enthalpy equation of a closed system: dry air, water vapor, condensed water:

$$dh = c_p dT + L_{lv} dq_s ; \quad dh = \delta q + v dp \quad \rightarrow \quad 0 = c_p dT + L_{lv} dq_s - v dp$$

$$dq_l = -dq_s$$

$$dq_l = \frac{c_p}{L_{lv}} dT - \frac{v}{L_{lv}} dp$$

$$dp = -\frac{g}{v} dz$$

$$dq_l = \frac{c_p}{L_{lv}} \left(\frac{dT}{dz} + \frac{g}{c_p} \right) dz$$

$$\frac{dT}{dz} = -\Gamma_s$$

$$\frac{g}{c_p} = \frac{c_{pd}}{c_p} \Gamma_d$$

$$dq_l = \frac{c_{pd}}{L_{lv}} \left(\Gamma_d - \frac{c_p}{c_{pd}} \Gamma_s \right) dz = \frac{c_{pd}}{L_{lv}} \Gamma_d (1 - \gamma') dz$$

WATER CONDENSED IN SATURATED ADIABATIC ASCENT

Because $c_p \cong c_{pd}$ then $\Gamma_s = \gamma' \Gamma_d$ one can express q_l as:

$$dq_l = \frac{c_{pd}}{L_{lv}} (\Gamma_d - \Gamma_s) dz = \frac{c_{pd}}{L_{lv}} \Gamma_d (1 - \gamma') dz$$

Let's assume: $L_{lv} = \text{const}$

$$q_l(z) = \frac{c_{pd}}{L_{lv}} \int_{z_{cl.base}}^z (\Gamma_d - \Gamma_s) dz' = \frac{c_{pd}}{L_{lv}} \Gamma_d \int_{z_{cl.base}}^z (1 - \gamma') dz'$$

The amount of water condensed in a rising adiabatic parcel increases with the height above the cloud base and increases with increasing temperature at the cloud base.

For shallow clouds (cloud depth not bigger than ca. 300-500 m, for instance stratocumulus clouds) it can be assumed that the amount of condensed water increases linearly with height above the cloud base (h).

The rate of this increase (c_q) is approximately **constant** and depends on temperature and pressure at the cloud base (T_0, p_0).

$$q_l(h) = c_q(T_0, p_0) \cdot h \quad \text{where} \quad h = z - z_{cl.base}$$

$$c_q \approx \frac{c_{pd}}{L_{lv}} (\Gamma_d - \Gamma_s) = \frac{c_{pd}}{L_{lv}} \Gamma_d (1 - \gamma') \quad \left[\frac{g}{kg \cdot m} \right]$$

For $T_0 = 20^\circ\text{C}$ and $p_0 = 1000 \text{ hPa}$

$$c_q = \frac{c_{pd}}{L_{lv}} (\Gamma_d - \Gamma_s) \approx 2 \cdot 10^{-3} \frac{g}{kg \cdot m}$$

Liquid Water Content (LWC) is the amount of liquid water per unit volume:

$$LWC = q_l \cdot \rho ; \quad \rho - \text{density of the air}$$

$$d \frac{LWC}{\rho} = \frac{c_{pd}}{L_{lv}} (\Gamma_d - \Gamma_s) dz$$

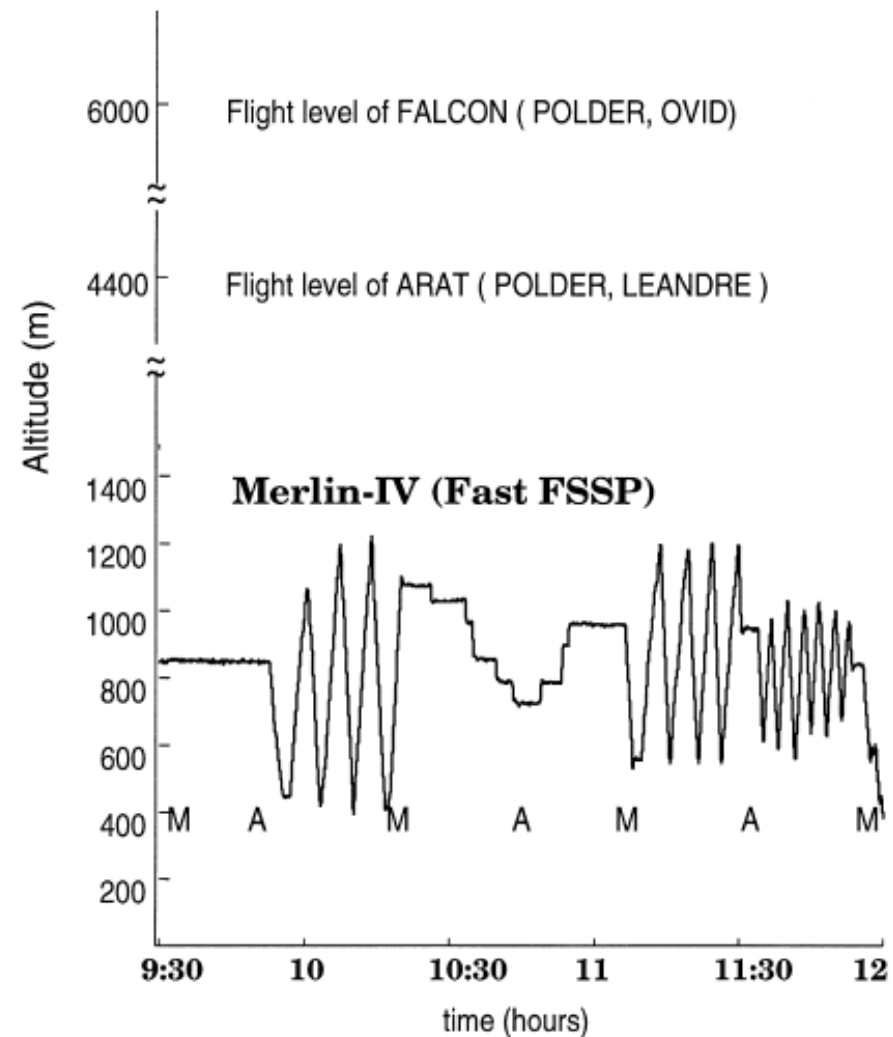
$$LWC(z) = \rho(z) \frac{c_{pd}}{L_{lv}} \int_{z_{cl.base}}^z (\Gamma_d - \Gamma_s) dz' = \rho(z) \frac{c_{pd}}{L_{lv}} \Gamma_d \int_{z_{cl.base}}^z (1 - \gamma') dz'$$

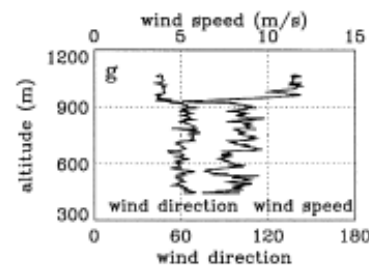
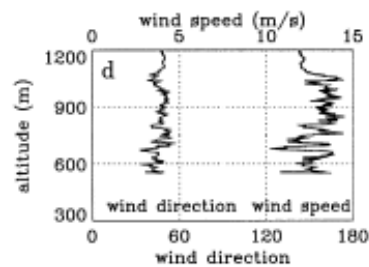
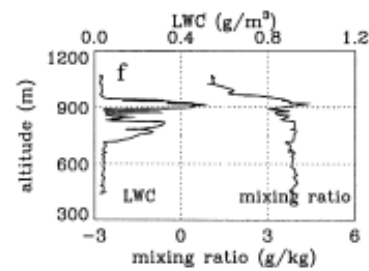
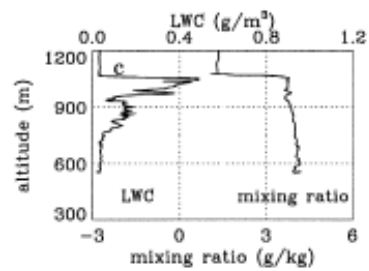
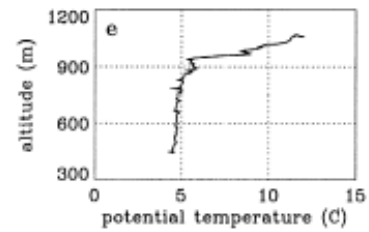
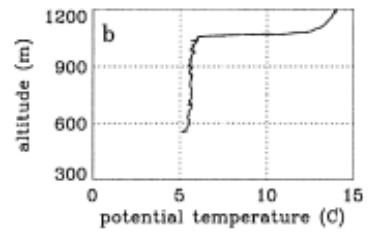
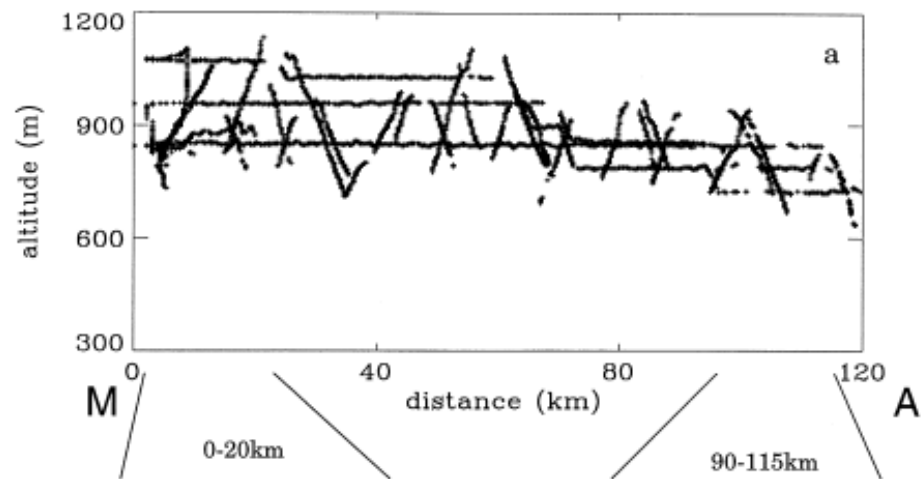
For shallow clouds one can assume that the air density is constant, therefore:

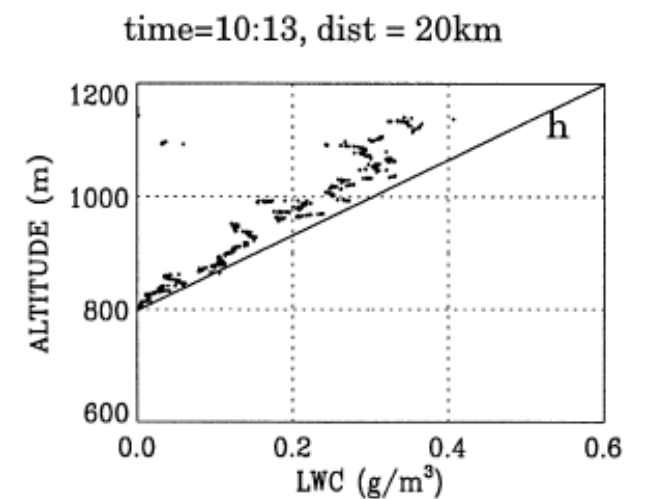
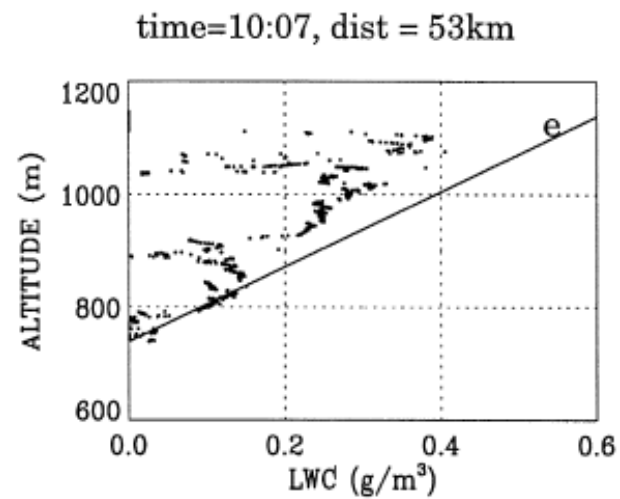
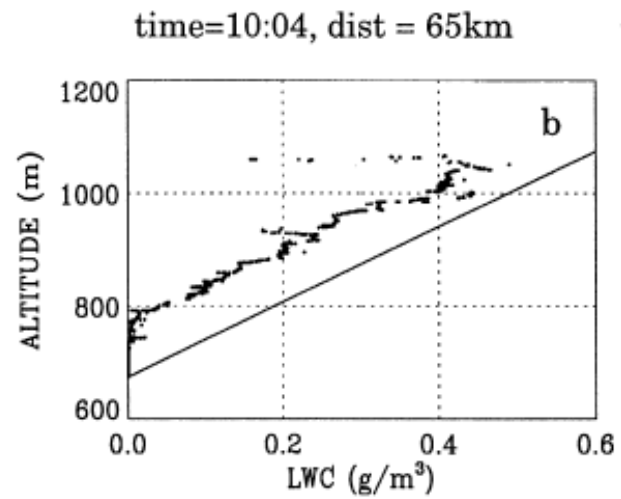
$$LWC(z) = c_w(T_0, p_0) \cdot (z - z_{cl.base}) = c_w(T_0, p_0) \cdot h$$

$$c_w \approx \rho \frac{c_{pd}}{L_{lv}} (\Gamma_d - \Gamma_s) = \rho(T_0, p_0) c_q(T_0, p_0) \quad \left[\frac{g}{m^4} \right]$$

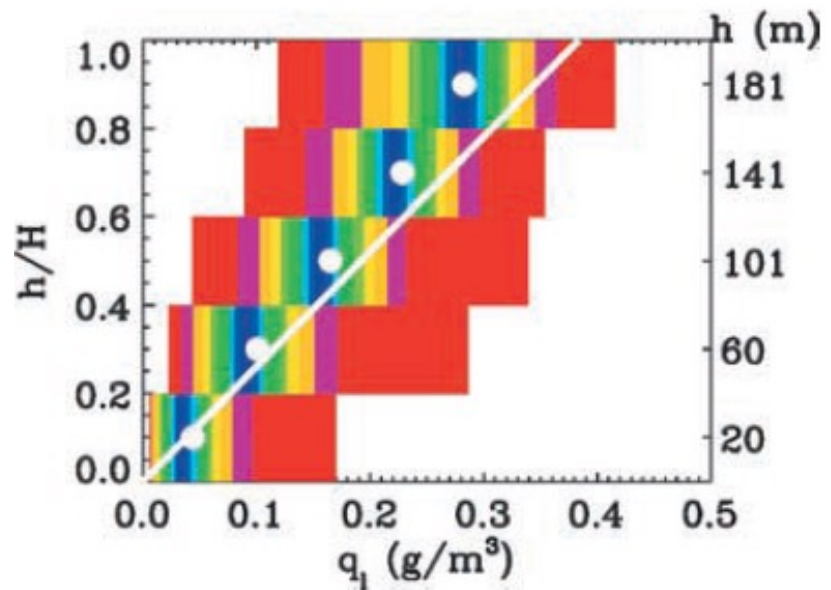
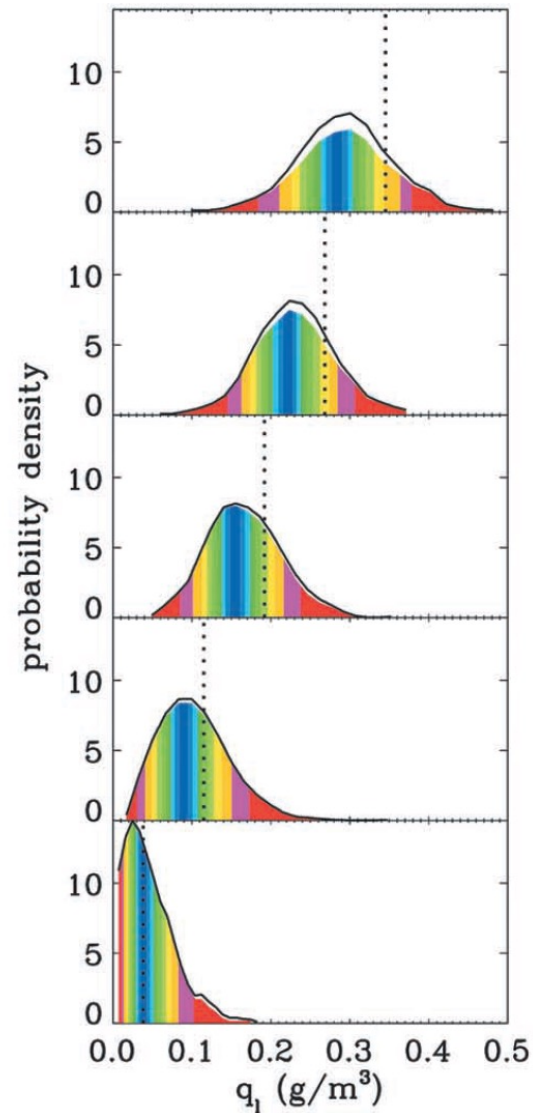
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