

Midterm Exam Training Problems

1. Assuming that the geostrophic wind is purely zonal ($v_g = 0$) and independent of height, the wind profile in the atmospheric Ekman layer reads:

$$u = u_g[1 - e^{-\gamma z} \cos(\gamma z)].$$

$$v = u_g e^{-\gamma z} \sin(\gamma z).$$

where $\gamma = \sqrt{f/2A}$ and A is the eddy diffusivity. Evaluate the frictionally induced ageostrophic transport (U, V), defined as:

$$U = \int_0^\infty (u - u_g) dz.$$

$$V = \int_0^\infty (v - v_g) dz.$$

How is the ageostrophic transport oriented with respect to the friction force (τ_x, τ_y) at the gradient wind level?

2. Given the following expression of geopotential field

$$\Phi = \Phi_0(p) + cf \left\{ -y \left[\cos\left(\frac{\pi p}{p_0}\right) + 1 \right] + \frac{\sin(k(x-ct))}{k} \right\}.$$

Where Φ_0 is a function of p alone, c is a constant speed, k is a zonal wavenumber and $p_0 = 1000 \text{ hPa}$, obtain the expression for the corresponding geostrophic wind. What is the gradient of temperature in horizontal directions?

3. Determine the ratio of maximum possible regular anticyclonic gradient wind speed to the geostrophic wind speed for the same pressure gradient (regular anticyclonic implicates that $R < 0$, and we take square root with negative sign before it, in formula for gradient wind).

4. The actual wind is directed 30° to the right of the geostrophic wind. If the geostrophic wind is 20 m/s , what is the rate of change of wind speed? Let $f = 10^{-4} \text{ s}^{-1}$.

5. The planet Venus rotates about its axis so slowly, that to a reasonable approximation the Coriolis parameter may be set equal to zero. For steady, frictionless motion parallel to latitude circles the momentum equation then reduces to a type of cyclostrophic balance

$$\frac{u^2 \tan(\varphi)}{a} = -\frac{1}{\rho} \frac{\partial p}{\partial y}.$$

By transforming this expression to isobaric coordinates, show that the thermal wind equation in this case can be expressed in the form

$$\omega_r^2(p_2) - \omega_r^2(p_1) = -\frac{R_d \ln(p_2/p_1)}{a \sin(\varphi) \cos(\varphi)} \frac{\partial \bar{T}}{\partial y}.$$

where R_d is the gas constant, a the radius of the planet, and $\omega_r = u/a \cos(\varphi)$ is the relative angular velocity. How must $\bar{T}$ (vertically averaged temperature) vary with respect to latitude in order that ω_r be a function of pressure only? If the zonal velocity at about 60km height above the equator ($p = 2.9 \times 10^5 \text{Pa}$) is 100m/s and the zonal velocity vanishes at the surface of the planet ($p_0 = 9.5 \times 10^6 \text{Pa}$), what is the vertically averaged temperature difference between the equator and pole assuming that ω_r depends only on pressure? The planetary radius is $a = 6100 \text{km}$, and the gas constant is $R_d = 187 \frac{\text{J}}{\text{kgK}}$.

6. Find the geopotential profile $\Phi(p)$ in a hydrostatic atmosphere of uniform temperature. What is the geopotential height at the $p = 300 \text{hPa}$ surface for the temperature $T = 250 \text{K}$ and surface pressure $p_s = 1000 \text{hPa}$? Find the geopotential profile $\Phi(p)$ in a hydrostatic atmosphere of constant potential temperature θ .