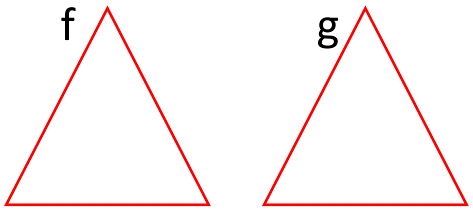
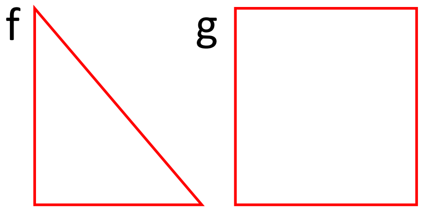
WSTĘP DO OPTYKI FOURIEROWSKIEJ

dr hab. Rafał Kasztelanic

WYKŁAD 2

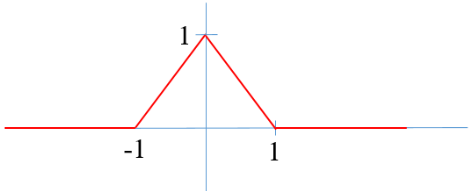
Splot





Splot

$$\Lambda(x) = \begin{cases} 1 - |x| & |x| < 1 \\ 0 & |x| \geq 1 \end{cases}$$



$$h(x) = \int_{-\infty}^{\infty} f(x - x') g(x') \, dx'$$

Splot

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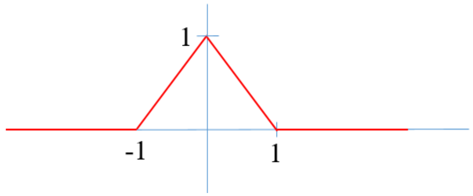
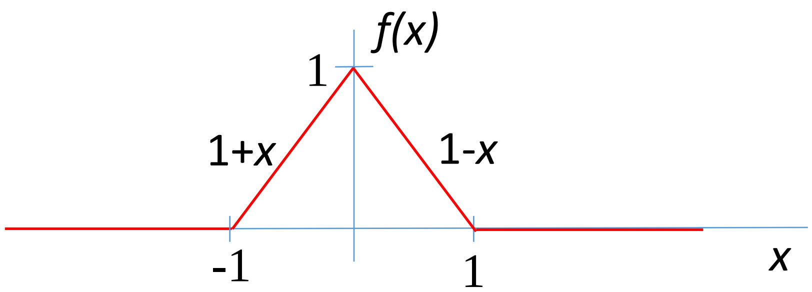


$$h(x) = \int_{-\infty}^{\infty} f(x - x')\, g(x')\, dx'$$



Splot

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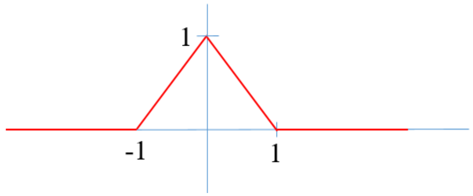

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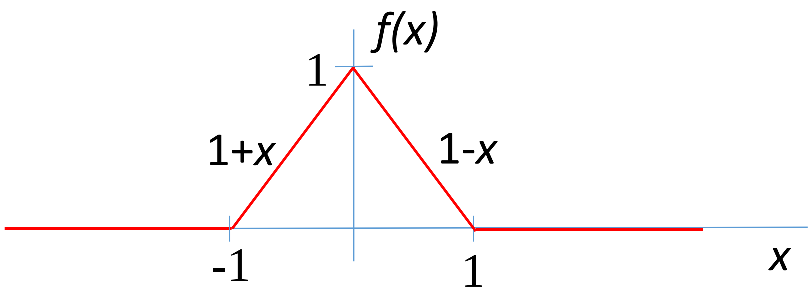

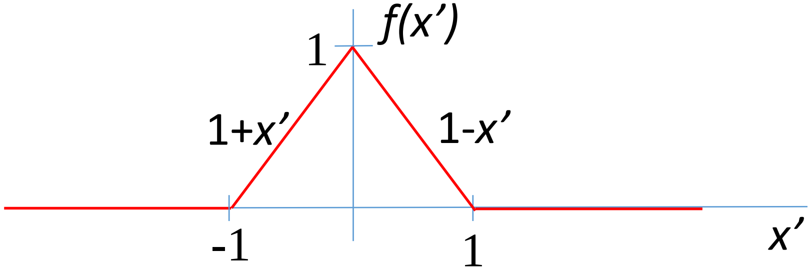

Splot

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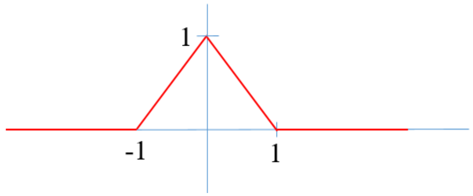

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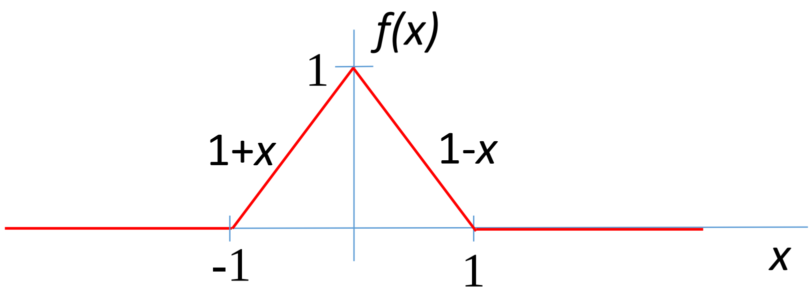

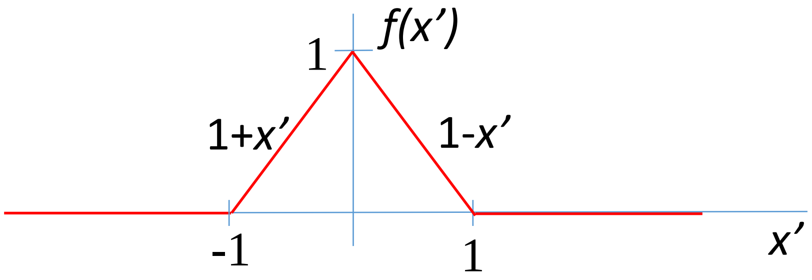

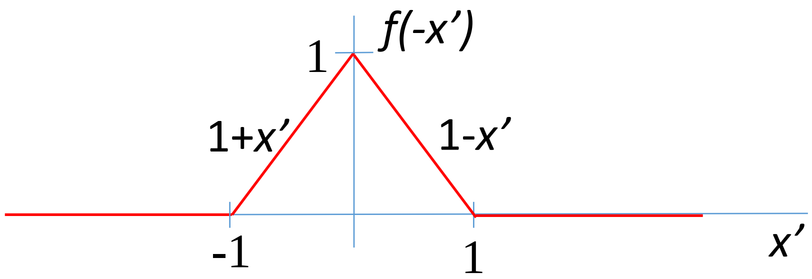

Splot

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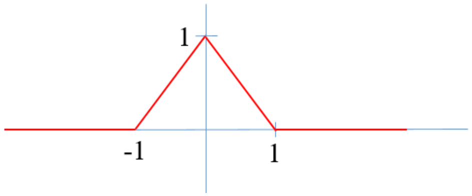

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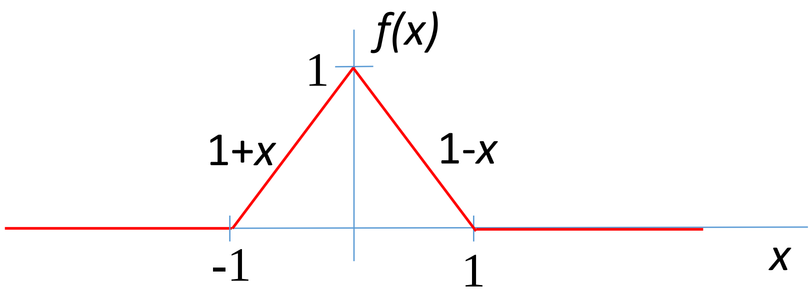
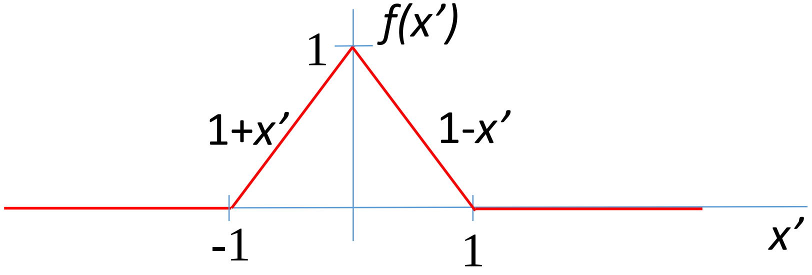
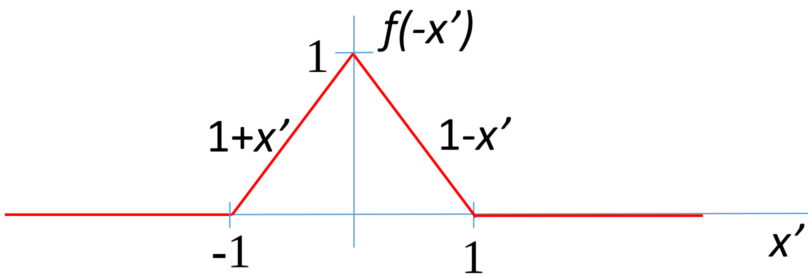
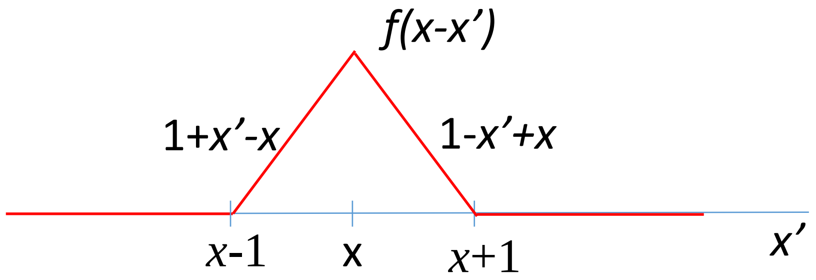


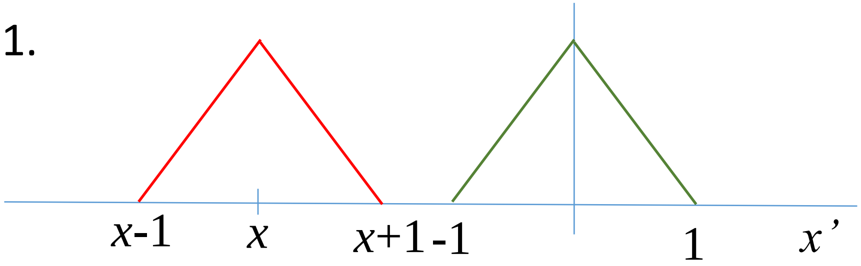






Splot

1.

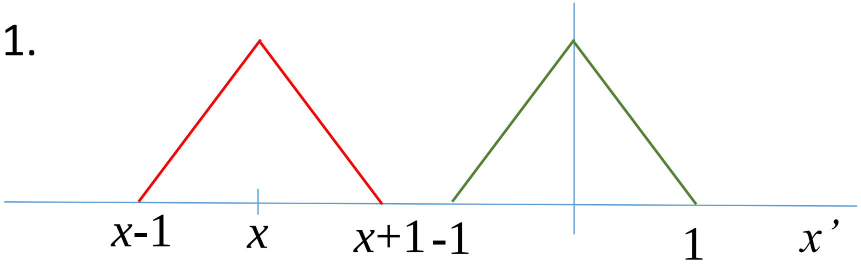


$$x + 1 < -1 \;\; \rightarrow \;\; x < -2$$

$$\boxed{h(x) = 0}$$

Splot

1.



$$x + 1 < -1 \quad \rightarrow \quad x < -2$$

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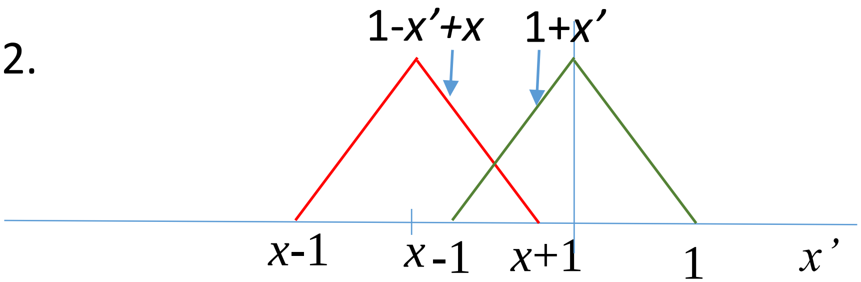
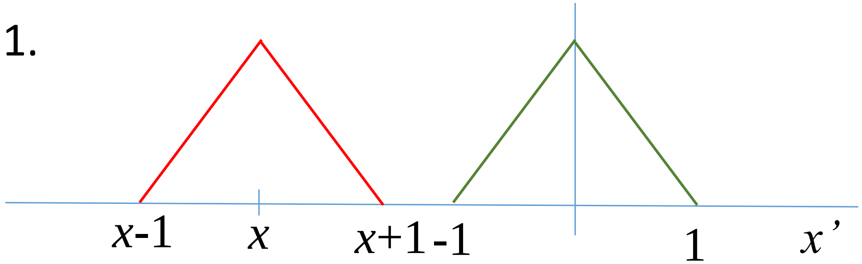
2.



$$-1 < x + 1 < 0 \quad \rightarrow \quad -2 < x < -1$$

Splot

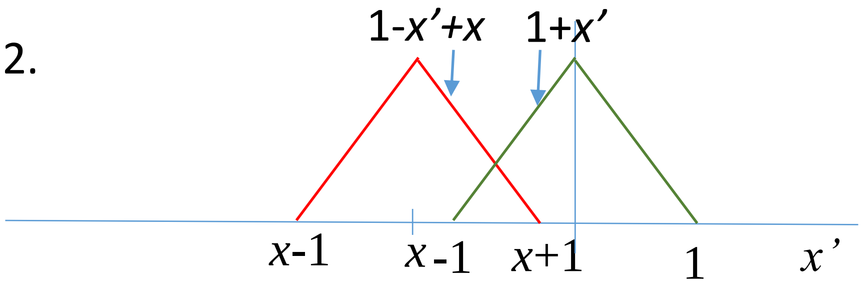
1.



$$x + 1 < -1 \rightarrow x < -2$$

$$\boxed{h(x) = 0}$$

2.



$$-1 < x + 1 < 0 \rightarrow -2 < x < -1$$

$$h(x) = \int_{-1}^{x+1} (1 + x')(1 - x' + x)\, dx' =$$

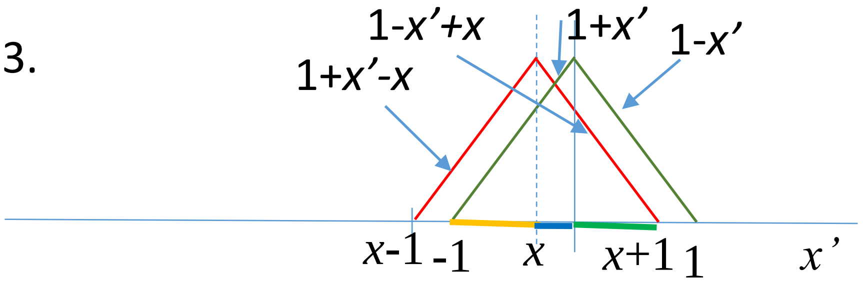
$$= \int_{-1}^{x+1} (1 + x - x'^2 + xx')dx' = (1 + x)\int_{-1}^{x+1} dx' + x\int_{-1}^{x+1} x'dx' - \int_{-1}^{x+1} x'^2 dx' =$$

$$-(x+1)(x+2) \qquad -\frac{x}{2}\left((x+1)^2 - 1\right) \qquad -\frac{1}{3} - \frac{(x+1)^3}{3}$$

$$\boxed{h(x) = \frac{1}{6}(x+2)^3}$$

Splot

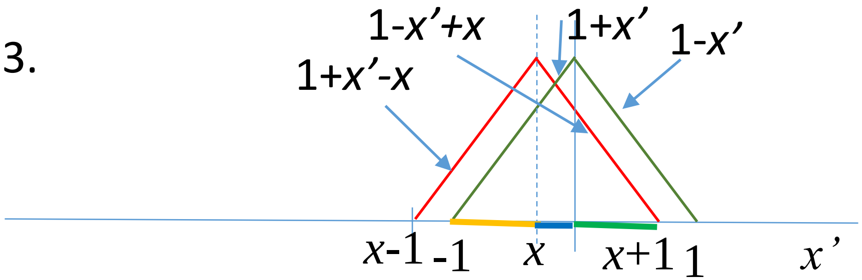
3.



$$0 < x + 1 < 1 \;\; \rightarrow \;\; -1 < x < 0$$

Splot

3.



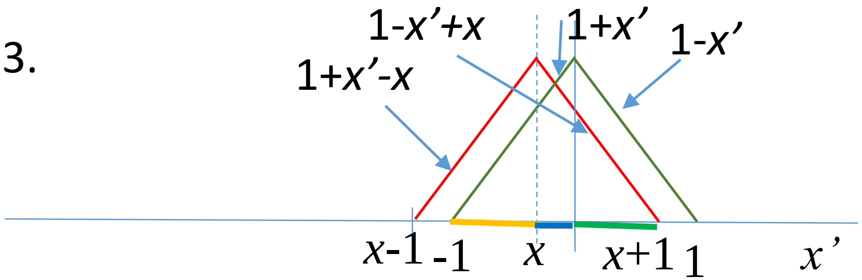
$$0 < x + 1 < 1 \ \rightarrow \ -1 < x < 0$$

$$h(x) = \underbrace{\int_{-1}^{x} (1 + x')(1 + x' - x)\, dx'}_{\frac{1}{6}(-x^3 + 3x + 2)} + \underbrace{\int_{x}^{0} (1 + x')(1 - x' + x)\, dx'}_{-\frac{1}{6}x(x^2 + 6x + 6)} + \underbrace{\int_{0}^{x+1} (1 - x')(1 - x' + x)\, dx'}_{\frac{1}{6}x(-x^3 + 3x + 2)}$$

$$\boxed{h(x) = \frac{1}{3}(-x^3 + 3x + 2) - \frac{1}{6}x(x^2 + 6x + 6)}$$

Splot

3.



$$0 < x + 1 < 1 \quad \rightarrow \quad -1 < x < 0$$

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$$h(x) = \frac{1}{3}(-x^3 + 3x + 2) - \frac{1}{6}x(x^2 + 6x + 6)$$

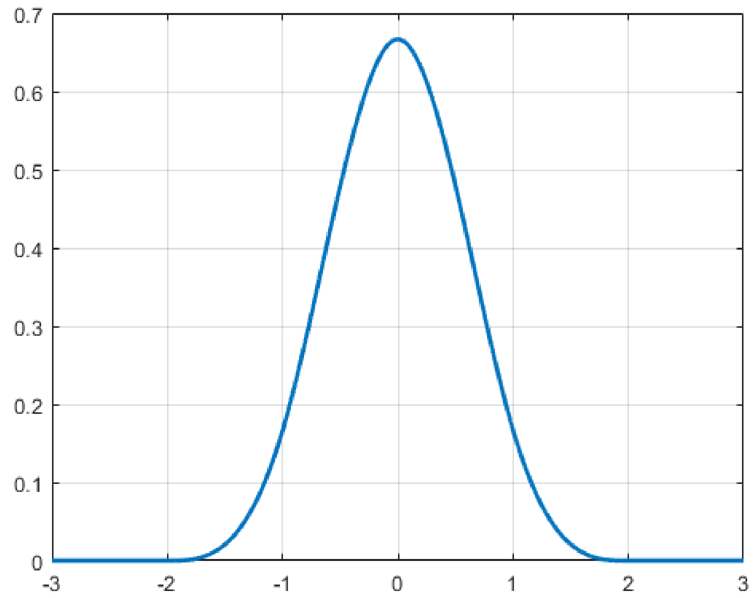
4. $-1 < x - 1 < 0 \quad \rightarrow \quad 0 < x < 1$ — Symetrycznie do 3

5. $0 < x - 1 < 1 \quad \rightarrow \quad 1 < x < 2$ — Symetrycznie do 2

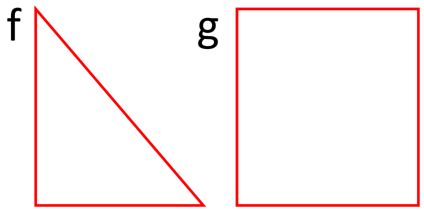
6. $1 < x - 1 \quad \rightarrow \quad 2 < x$ — Symetrycznie do 1

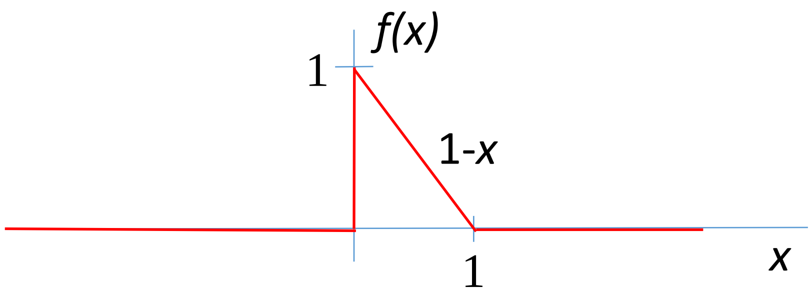
Splot

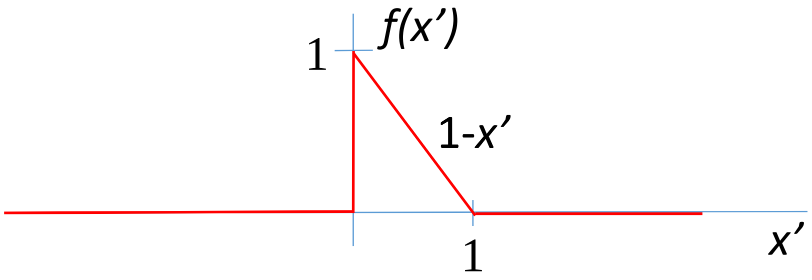
$$h(x) = \begin{cases} 0 & x < -2 \\ \frac{1}{6}(x+2)^3 & -2 < x < -1 \\ \frac{1}{3}(-x^3+3x+2) - \frac{1}{6}x(x^2+6x+6) & -1 < x < 0 \\ \frac{1}{3}(x^3-3x+2) + \frac{1}{6}x(x^2-6x+6) & 0 < x < 1 \\ -\frac{1}{6}(x-2)^3 & 1 < x < 2 \\ 0 & 2 < x \end{cases}$$

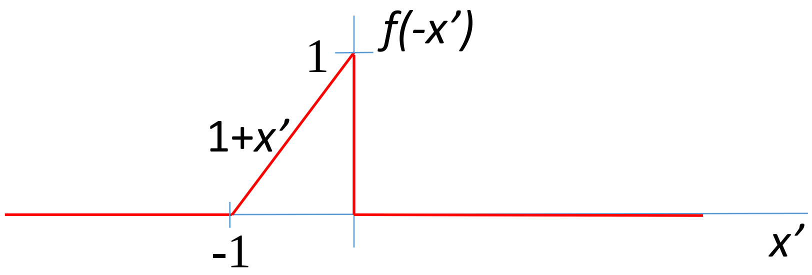


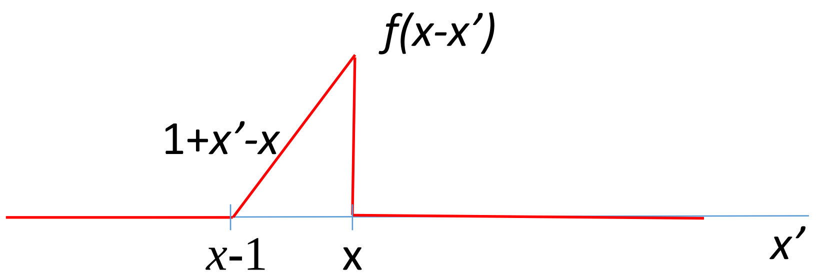
Splot





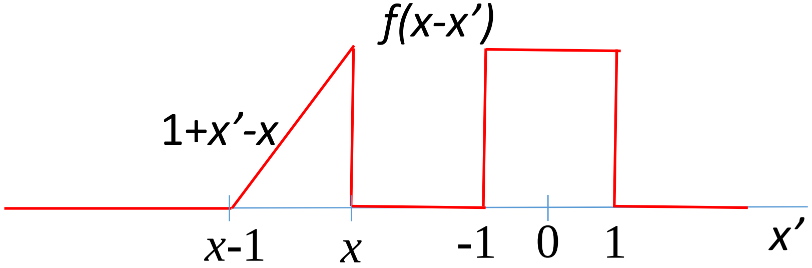






Splot

1.

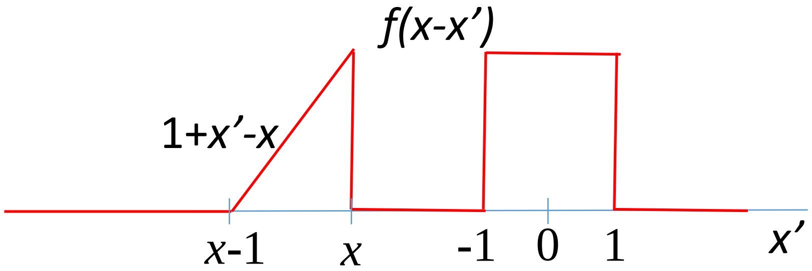


$x < -1$

$$h(x) = 0$$

Splot

1.



$x < -1$

$$h(x) = 0$$

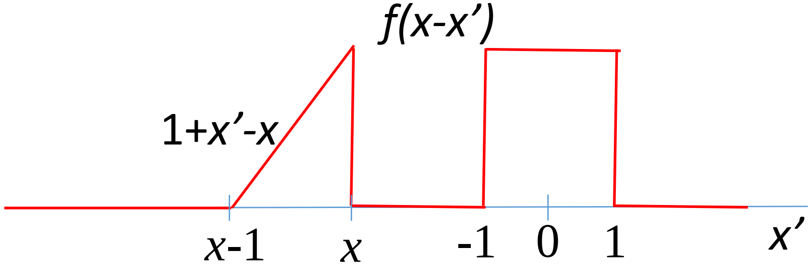
2. $-1 < x < 0$

$$h(x) = \int_{-1}^{x} (1 + x' - x)dx' = (1 - x)\int_{-1}^{x} dx' + \int_{-1}^{x} x'dx' =$$

$$= (1 - x)(x + 1) + \frac{x^2}{2} + \frac{1}{2} = \frac{1}{2}(1 - x^2)$$

Splot

1.



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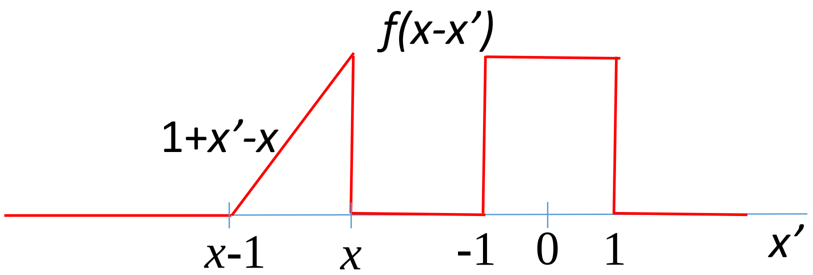
3. $0 < x < 1$

$$h(x) = \frac{1}{2}$$

Splot

1.



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3. $0 < x < 1$

$$h(x) = \frac{1}{2}$$

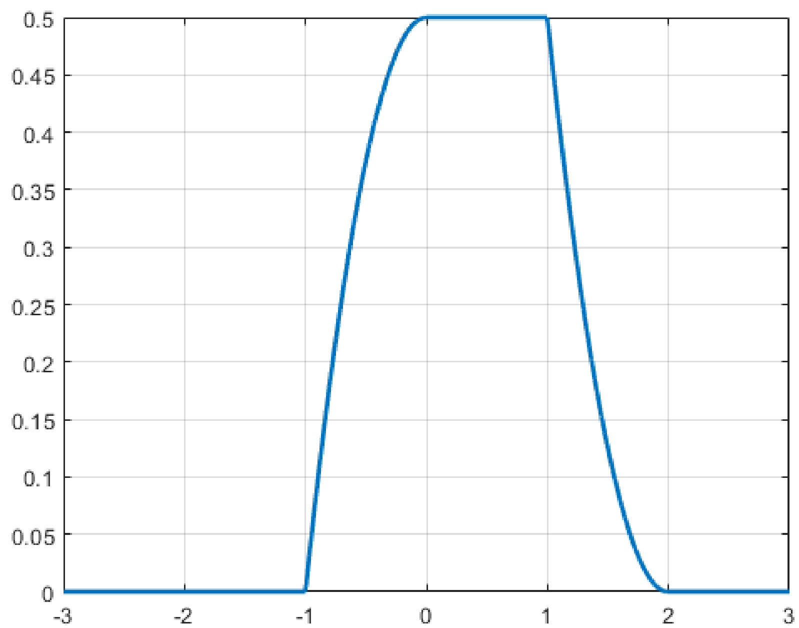
4. $0 < x - 1 < 1 \quad \rightarrow \quad 1 < x < 2$

$$h(x) = \int_{x-1}^{1} (1 + x' - x)dx' = (1 - x) \int_{x-1}^{1} dx' + \int_{x-1}^{1} x'dx' =$$

$$= (1 - x)(2 - x) + \frac{1}{2}[1 - (x - 1)^2] = \frac{x^2}{2} - 2x + 2$$

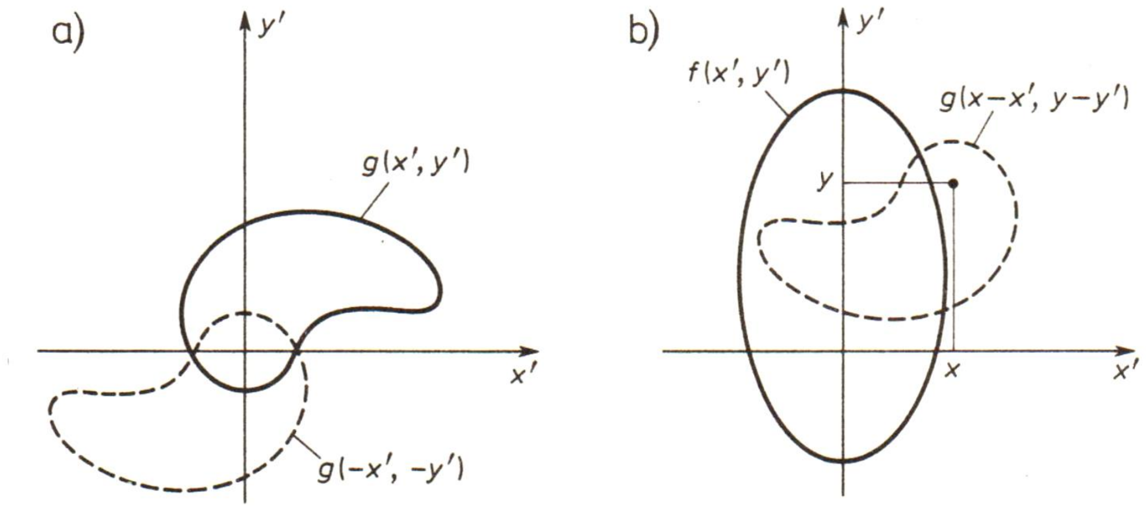
Splot

$$h(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{2}(1 - x^2) & -1 < x < 0 \\ \frac{1}{2} & 0 < x < 1 \\ \frac{x^2}{2} - 2x + 2 & 1 < x < 2 \\ 0 & 2 < x \end{cases}$$



Splot.m

Splot 2D



Korelacja

$$\varphi(x) = \int_{-\infty}^{\infty} f(x')g(x' - x)dx'$$

$$\varphi(x) = f(x) \star g(x)$$

Splot:

$$h(x) = \int_{-\infty}^{\infty} f(x - x')g(x')\,\mathrm{d}x'$$

Korelacja

$$\varphi(x) = \int_{-\infty}^{\infty} f(x')g(x'-x)dx'$$

$$\varphi(x) = f(x) \star g(x)$$

Splot:

$$h(x) = \int_{-\infty}^{\infty} f(x-x')g(x')\,dx'$$

Wasności:

Korelacja nie jest przemienna:

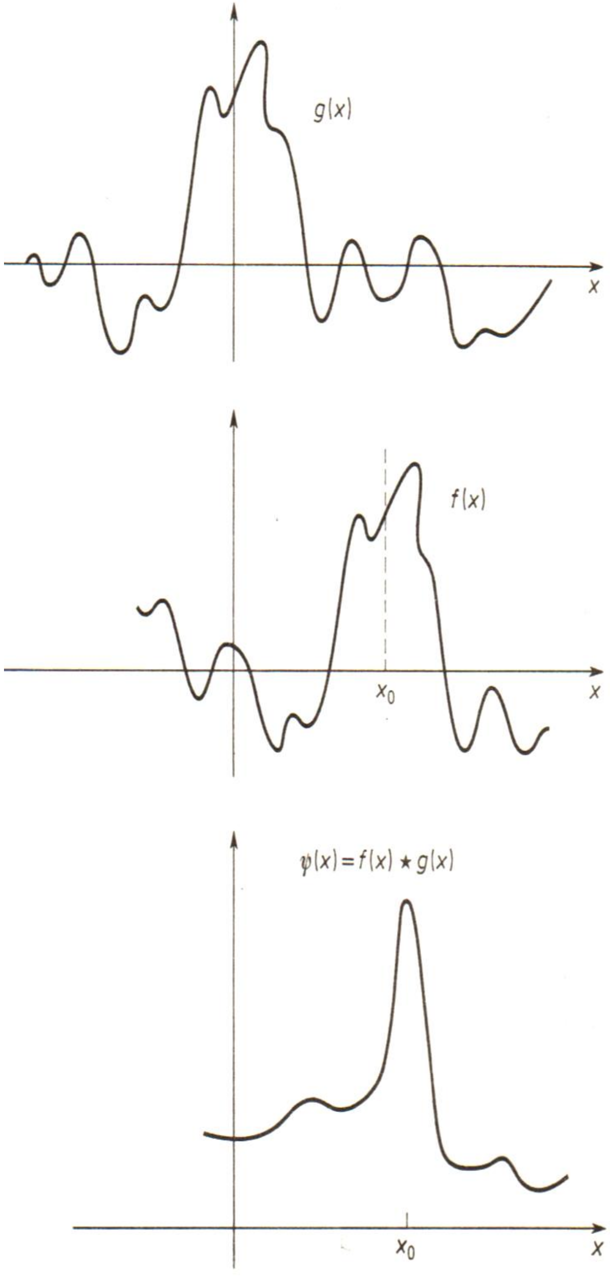
$$f(x) \star g(x) \neq g(x) \star f(x)$$

Korelacja jest równa splotowi z odwróconą funkcją *g*:

$$f(x) \star g(x) = f(x) \otimes g(-x)$$

Gdy g(x) jest funkcją parzystą to korelacja jest równoważna splotowi

Korelacja



Autokorelacja

Autokorelacja gdy $g(x) = f(x)$

Współczynnik autokorelacji: $\gamma(x) = \dfrac{\varphi(x)}{\varphi(0)}$

Moduł autokorelacji osiąga największą wartość w ‚0’ : $|\varphi(x)| \leq \varphi(0)$

Transformacja Fouriera

Szereg Taylora:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ \dots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$$

SzeregTaylora.mlx

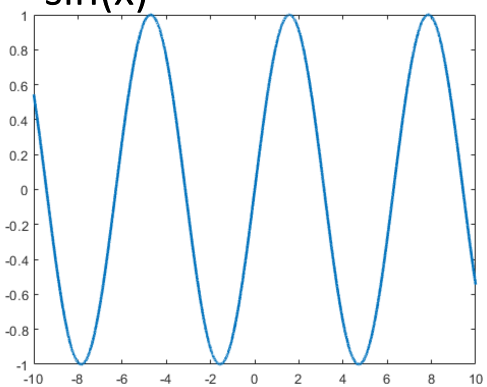
Transformacja Fouriera

Szereg Taylora:

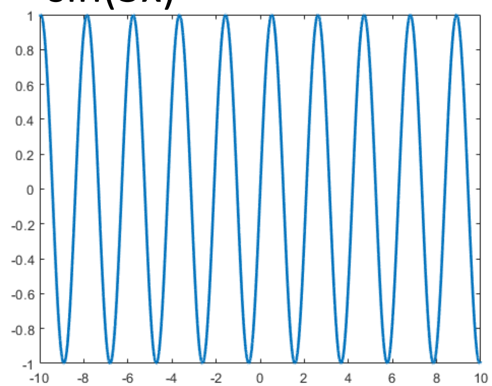
$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ \dots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$$

Suma sinusów:

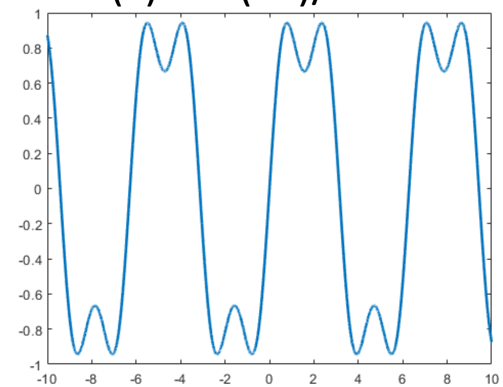
sin(x)



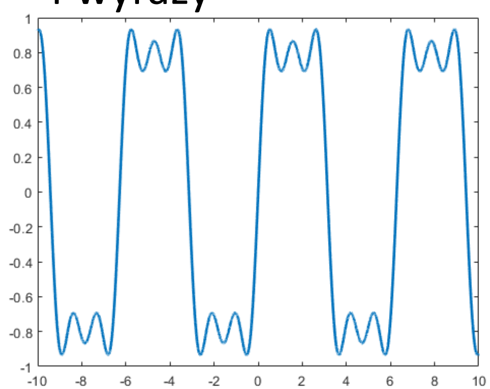
sin(3x)



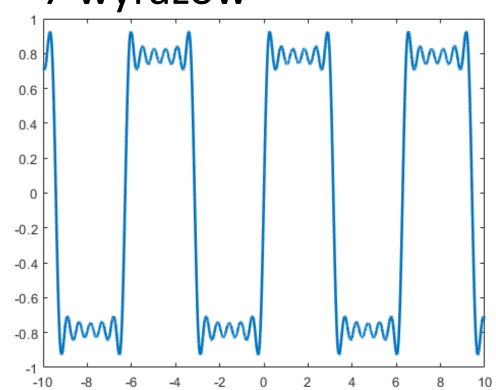
sin(x)+sin(3x)/3



4 wyrazy



7 wyrazów



$$\sum_{n=1}^{N} \frac{\sin[(2n-1)x]}{2n-1}$$

ProstokatSzereg.mlx