CLOUD PHYSICS - tutorial 4

Condensational growth of a cloud droplet

The condensational growth of a cloud droplet by water vapor diffusion is described by:

$$r\frac{dr}{dt} = \frac{S - S_{eq}}{F_d + F_k}$$

where

$$F_d = \frac{\rho_l R_v T}{D e_s(T)}, \quad F_k = \left(\frac{L}{R_v T} - 1\right)\frac{\rho_l L}{KT}$$

D is the diffusion coefficient of water vapor in the air. It depends on temperature and pressure: $D = 2.11 \cdot 10^{-5}\left(\frac{T}{T_0}\right)^{1.94}\left(\frac{p_0}{p}\right)\ \mathrm{m^2s^{-1}}$, where $p_0 = 101\,325\,\mathrm{Pa}$.
K is the thermal conductivity coefficient: $K = 4.1868 \cdot 10^{-3}[5.69 + 0.017(T - T_0)]\, Wm^{-1}K^{-1}$.

$$S_{eq} = 1 \quad OR \quad S_{eq} = \frac{r^3 - r_d^3}{r^3 - (1-\kappa)r_d^3} exp(\frac{A}{r}) \approx 1 + \frac{A}{r} - \frac{\kappa r_d^3}{r^3} \quad \text{where} \quad A(T) = \frac{2\sigma}{\rho_l R_v T}$$

Assume that $\sigma = 75.64 \cdot 10^{-3} Nm^{-1}$.

1. Show how $\xi = 1/(F_d + F_k)$ depends on temperature, T, and pressure, p.

2. Calculate the droplet radius growth rate dr/dt as a function of r for different values of saturation S (0.1, 0.3, 0.5%) and S_{eq} expressed either by the κ-Köhler relation (assume $\kappa = 0.67$, $r_d = 0.01 \mu m$), or assume that $S_{eq} = 1$.

   How does it depend on temperature and pressure?

3. Calculate $r(t)$. How does it depend on saturation, temperature, pressure?

4. Corrections (see *A Short Course in Cloud Physics* by R.R. Rogers and M.K. Yau). In condensational growth equation the diffusion and thermal conductivity coefficient have to be replaced by D' and K' as shown below.

   $$D' = D\frac{r}{r + l_\beta}, \quad l_\beta = \frac{D}{\beta}\left(\frac{2\pi}{R_v T}\right)^{1/2}, \quad \beta = 0.04$$

   $$K' = K\frac{r}{r + l_\alpha}, \quad l_\alpha = \left(\frac{K}{\alpha p}\right)\frac{(2\pi R_d T)^{1/2}}{c_v + R_d/2}, \quad \alpha = 1$$

   where l_α and l_β are the length scales.

   Show how these length scales depend on temperature and pressure.

   Show how these corrections influence the condensational growth of cloud droplets. For that purpose show how ξ/ξ_{corr} ($\xi = 1/(F_d + F_k)$) depends on radius, temperature and pressure.