Session 7

1. Starting from the inviscid shallow water equations, show that if the flow is initially independent of z, it must stay so.

2. Show that the vertical velocity within a shallow water system is given by

$$w = \frac{z - \eta_b}{h}\frac{Dh}{Dt} + \frac{D\eta_b}{Dt}$$

   Interpret this result, showing that it gives sensible answers at the top and bottom of the fluid layer.

3. Show that in a two-layer shallow water system the pressures p_1 and p_2 in, respectively, the top and the bottom layer are

$$p_1 = \rho_1 g(h_1 + h_2 + \eta_b) - \rho_1 g z,$$

$$p_2 = \rho_1 g h_1 + \rho_2 g(h_2 + \eta_b) - \rho_2 g z.$$

4. Linearize the non-rotating, flat-bottomed shallow water equations about a state with uniform velocity $\boldsymbol{u} = (u_0, 0)$. Assume two-dimensional motion in the x, z plane. Show that wave solutions propagate with the speed

$$c = U \pm \sqrt{gH}$$