Exercise Sheet 11

Questions, comments and corrections: e-mail to Marta.Waclawczyk@fuw.edu.pl

1. Rewrite the stationary 2-D incompressible Navier-Stokes equations in terms of the dimensionless variables

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{R^{-1/2}L}, \quad u^* = \frac{u}{U}, \quad v^* = \frac{v}{R^{-1/2}U}, \quad p^* = \frac{p}{\rho U^2}, \tag{1}$$

where $R = UL/\nu$. By taking the limit $R \to \infty$, derive the boundary layer equations in their dimensionless form

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{\partial^2 u^*}{\partial y^{*2}},$$

$$0 = -\frac{\partial p^*}{\partial y^*}, \qquad \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0.$$

2. *Boundary layer in a flow past a wedge.* Consider the general similarity solution,

$$\psi = F(x) f(\xi), \qquad \xi = y/\delta(x), \tag{2}$$

to the two-dimensional and stationary boundary layer equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2}, \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

The pressure gradient is given in terms of the mainstream irrotational flow $U(x)$,

$$\frac{1}{\rho}\frac{dp}{dx} = -U\frac{dU}{dx}.$$

In Eq. (2), $\psi(x, y)$ denotes the stream function, which is related to the two-dimensional velocity (u, v) by

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}.$$

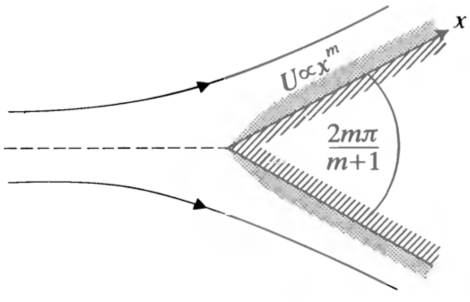

Figure 1: Irrotational flow past a wedge ($0 \le m \le 1$).

(a) Show that the mainstream condition,

$$u \to U(x) \quad \text{as} \quad \xi \to \infty,$$

demands that

$$F(x) \propto U(x)\delta(x).$$

(b) The irrotational flow past a wedge shown in Fig. 1 sets the mainstream flow $U(x)$ of the form

$$U(x) = Ax^m, \qquad A > 0. \tag{3}$$

Show that the thickness $\delta(x)$ of the boundary layer behaves like

$$\delta(x) \propto x^{\frac{1}{2}(1-m)}.$$

3. A homogeneous and isotropic elastic material is placed on a horizontal surface and surrounded by fixed, vertical, and slippery walls as shown in Fig. 2. The material undergoes downward displacement because of gravity. Calculate the displacement and stress fields. What happens if we remove the vertical container walls around the elastic material?

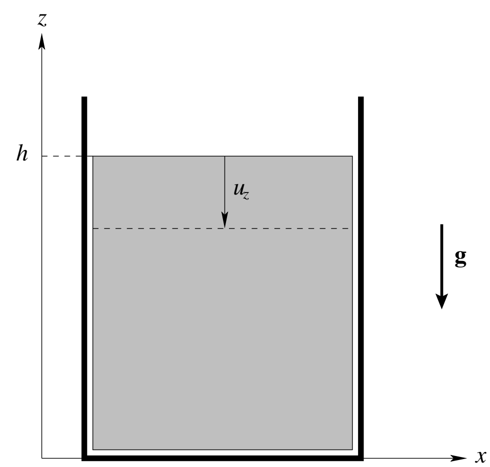

Figure 2: Elastic material in a box undergoing a downward displacement because of gravity. The container has fixed, slippery walls.