Stochastic coalescence in Lagrangian cloud microphysics

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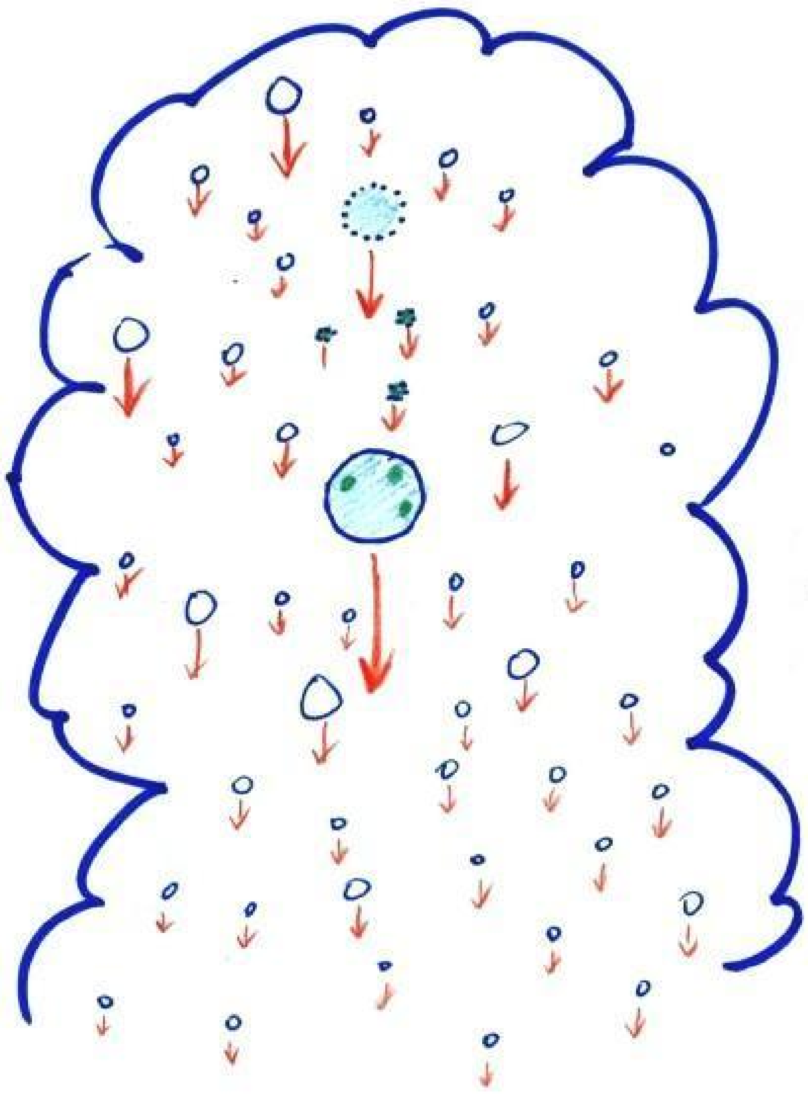
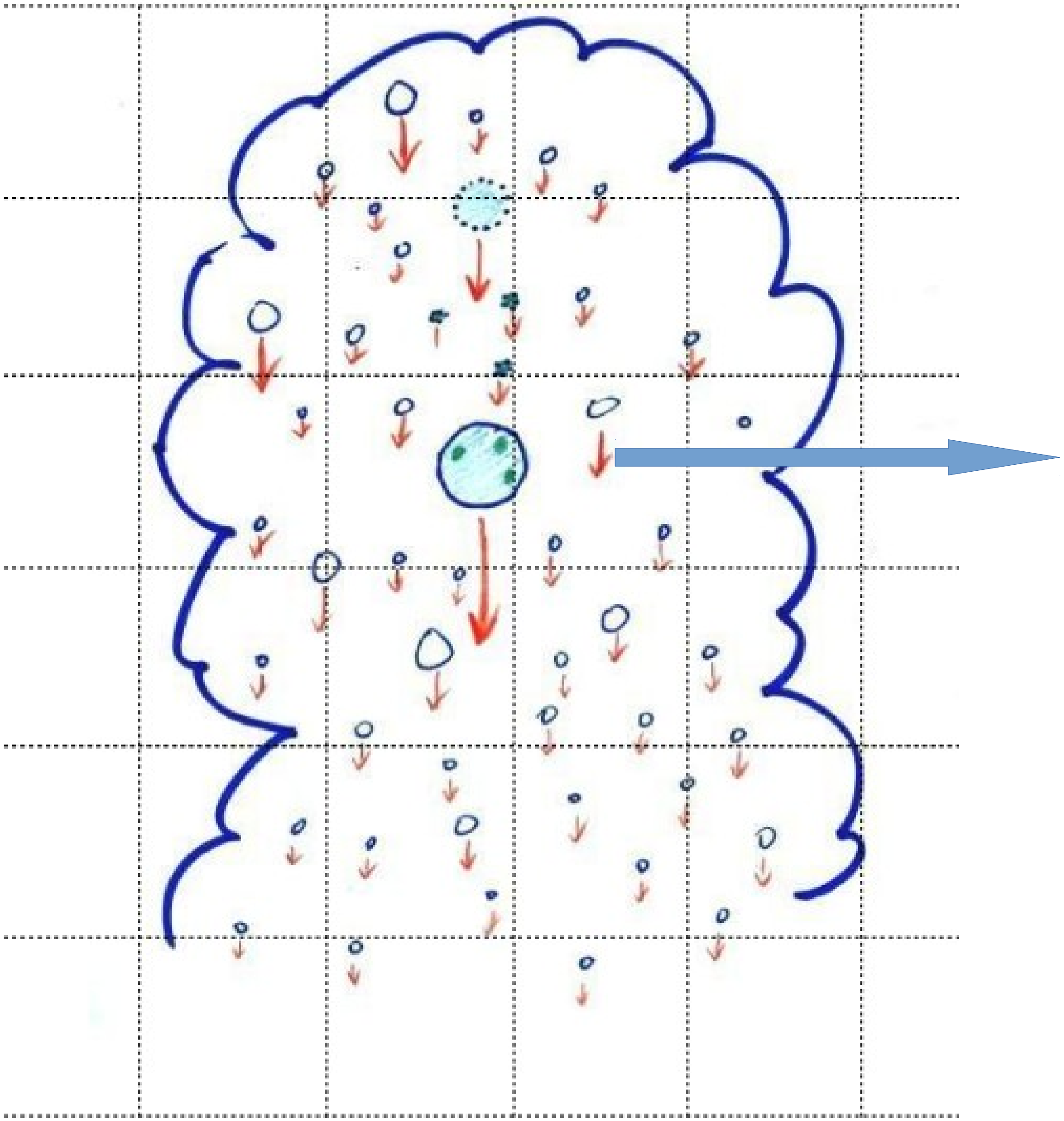
Outline

1. Modelling coalescence
2. Super-droplet method
3. Validity of the Smoluchowski equation
4. Validity of the super-droplet method
5. Conclusions



x(t)

http://www.atmo.arizona.edu/students/courselinks/fall16/atmo170a1s3/lecture_notes/oct27.html



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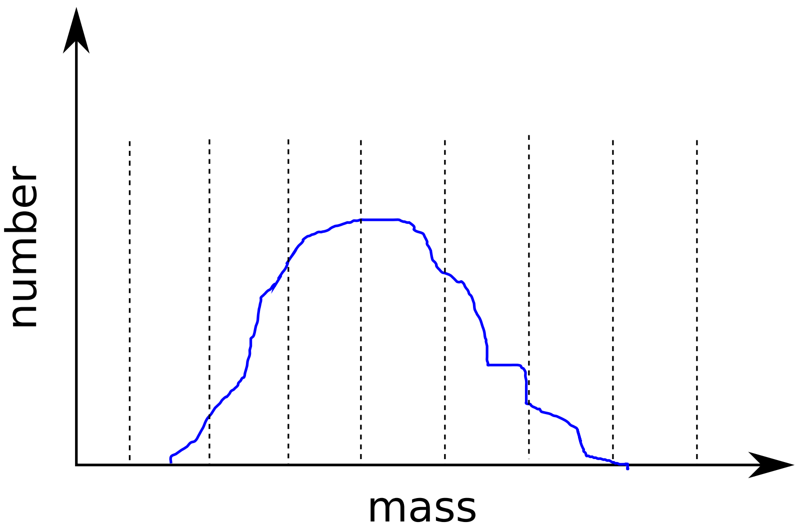
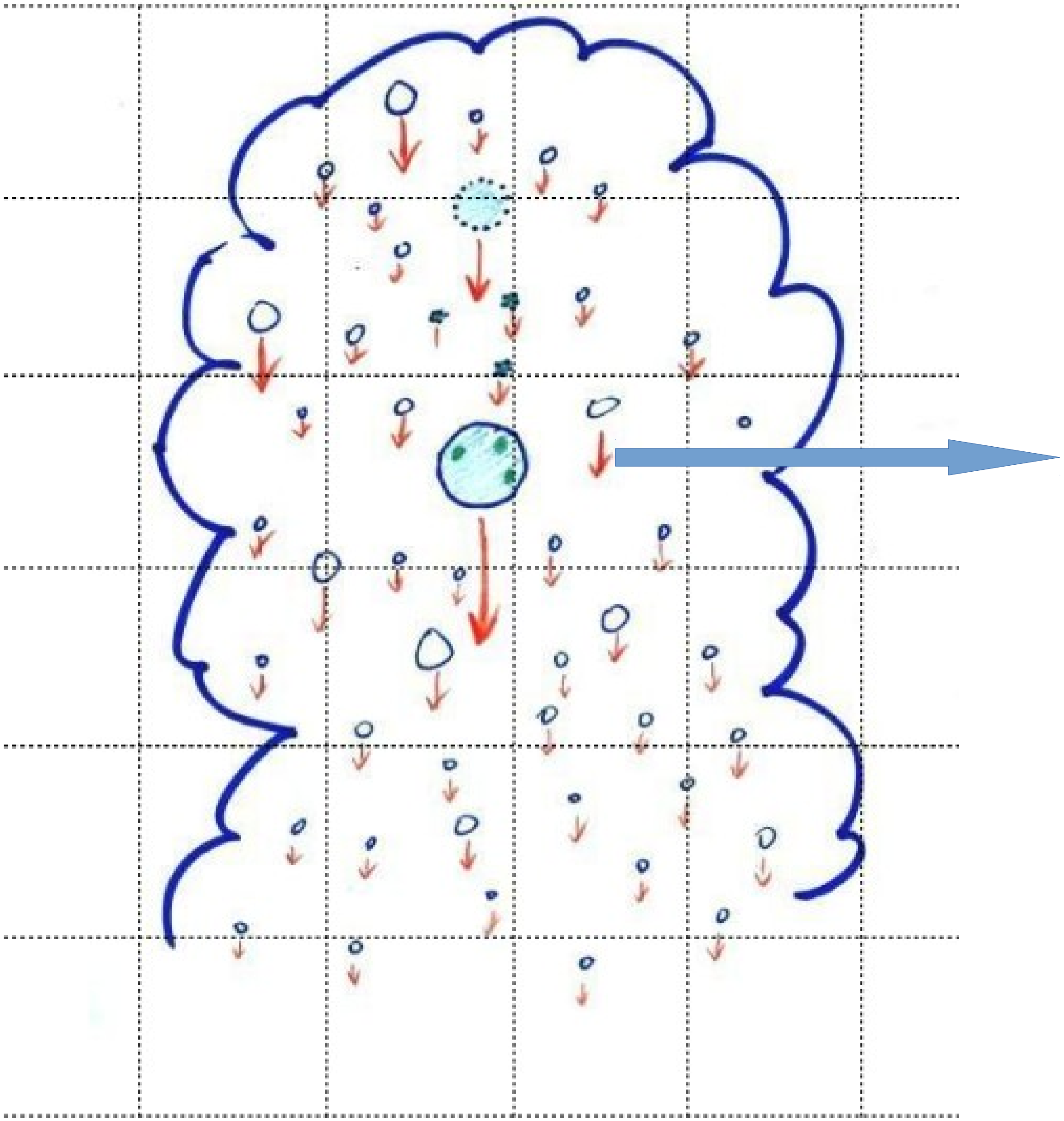


$N_i(t)$

$dN_i/dt = ?$



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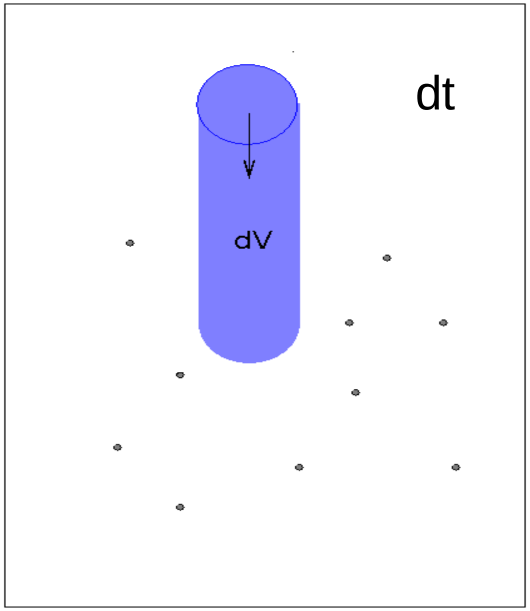

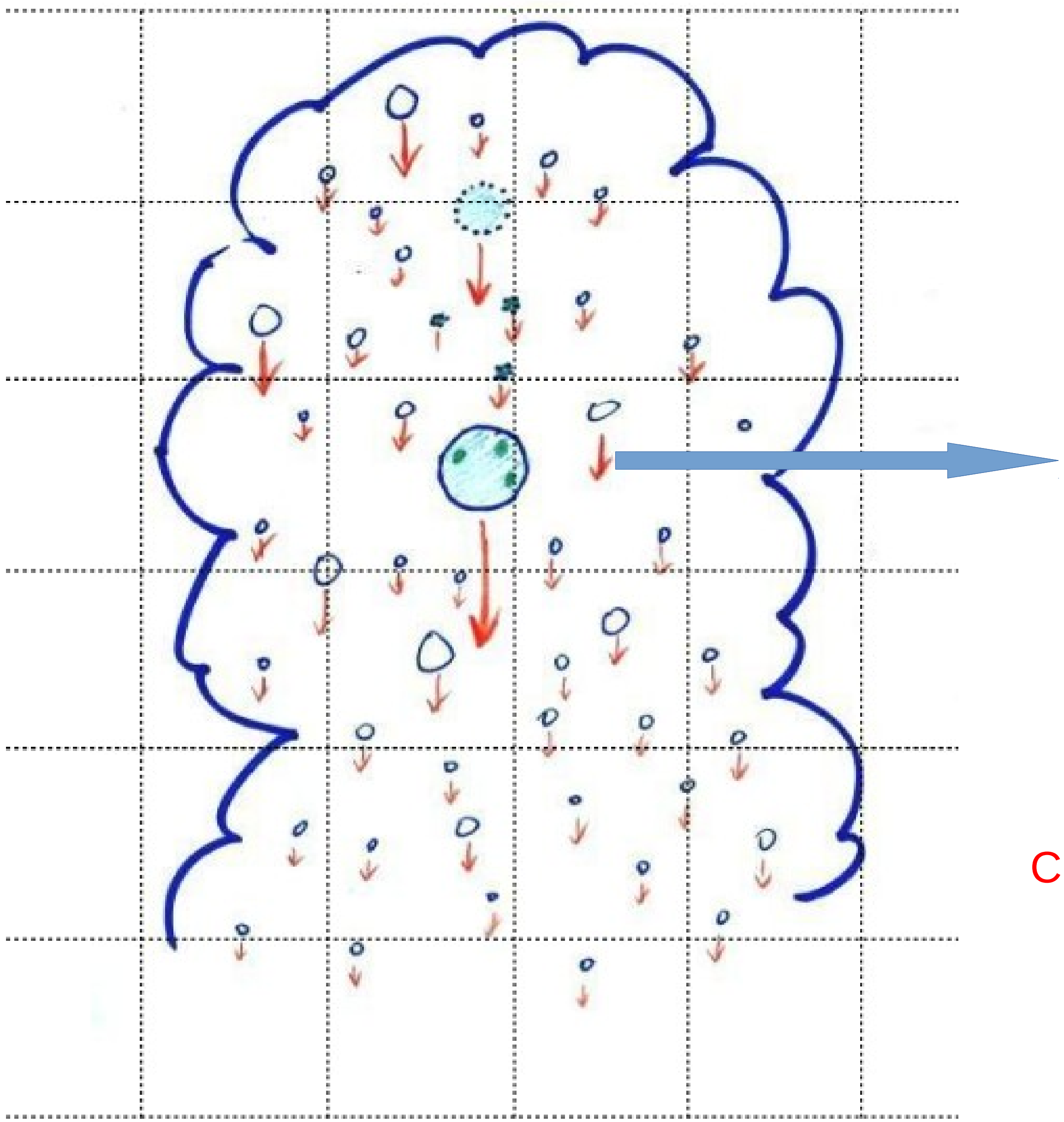

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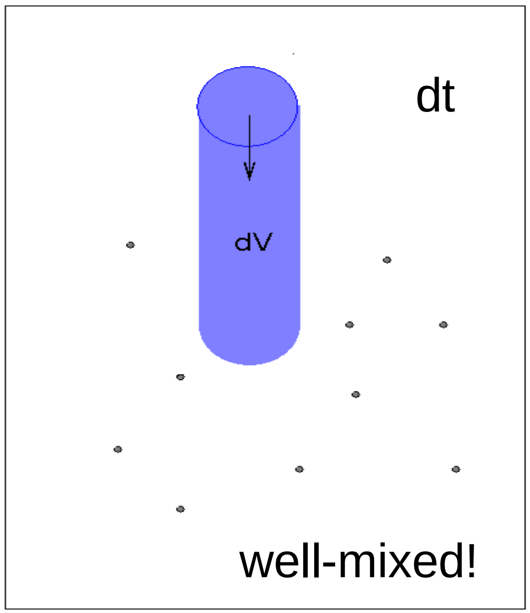
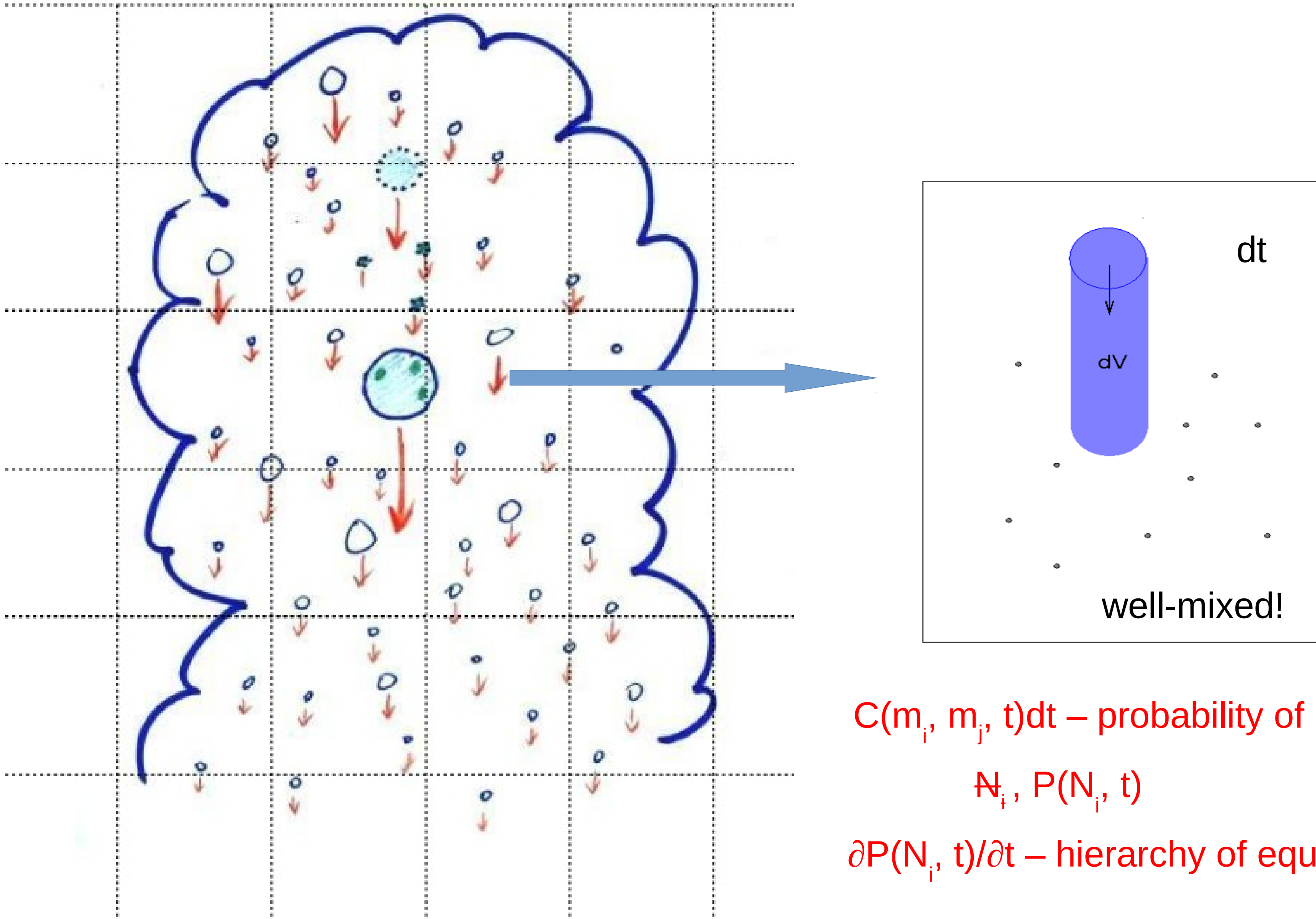


$C(m_i, m_j, t)dt$ – probability of collision



$C(m_i, m_j, t)dt$ – probability of collision

$\cancel{N_i}$, $P(N_i, t)$

$\partial P(N_i, t)/\partial t$ – hierarchy of equations

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$\partial P(N_i, t)/\partial t$ – hierarchy of equations

Stochastic Simulation Algorithm:
1) draw time to the next coalescence event
2) select a single pair to coalesce
3) update state vector

Gives a single history, large ensemble needed

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derive Smoluchowski equation (SCE):

1) neglect correlations:

$$P(N_i \mid N_j, t) = P(N_i, t)$$

2) calculate time evolution of the expected value of N_i.

Assumption 1) is valid as $V \to \infty$, but the cell has to be well-mixed!

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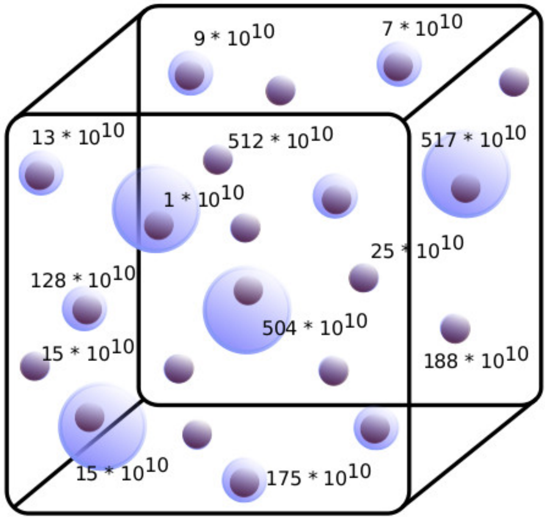
alternative: master equation

$\partial P(N_1, N_2, \ldots, N_{bin}, t) / \partial t$

can be solved only for very simple cases

Coalescence in the super-droplet method



Monte Carlo algorithm for collisions:

- regular “const SD” SDM:
- well-mixed cell
- small number of computational droplets ->
  - unrealistic correlations
  - amplified fluctuations
- linear sampling: $N_{SD}/2$ instead of $N_{SD}(N_{SD}-1)/2$ collision pairs

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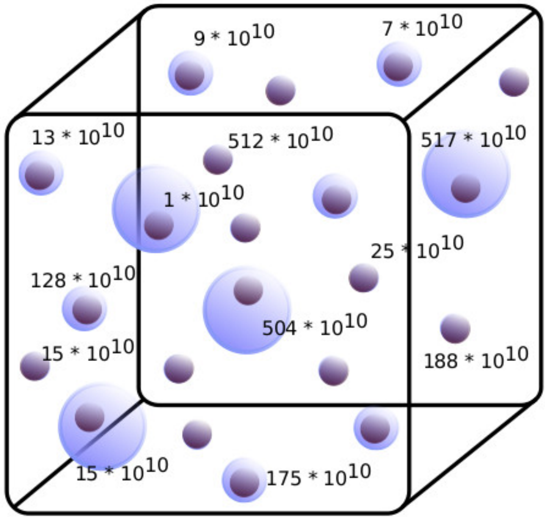

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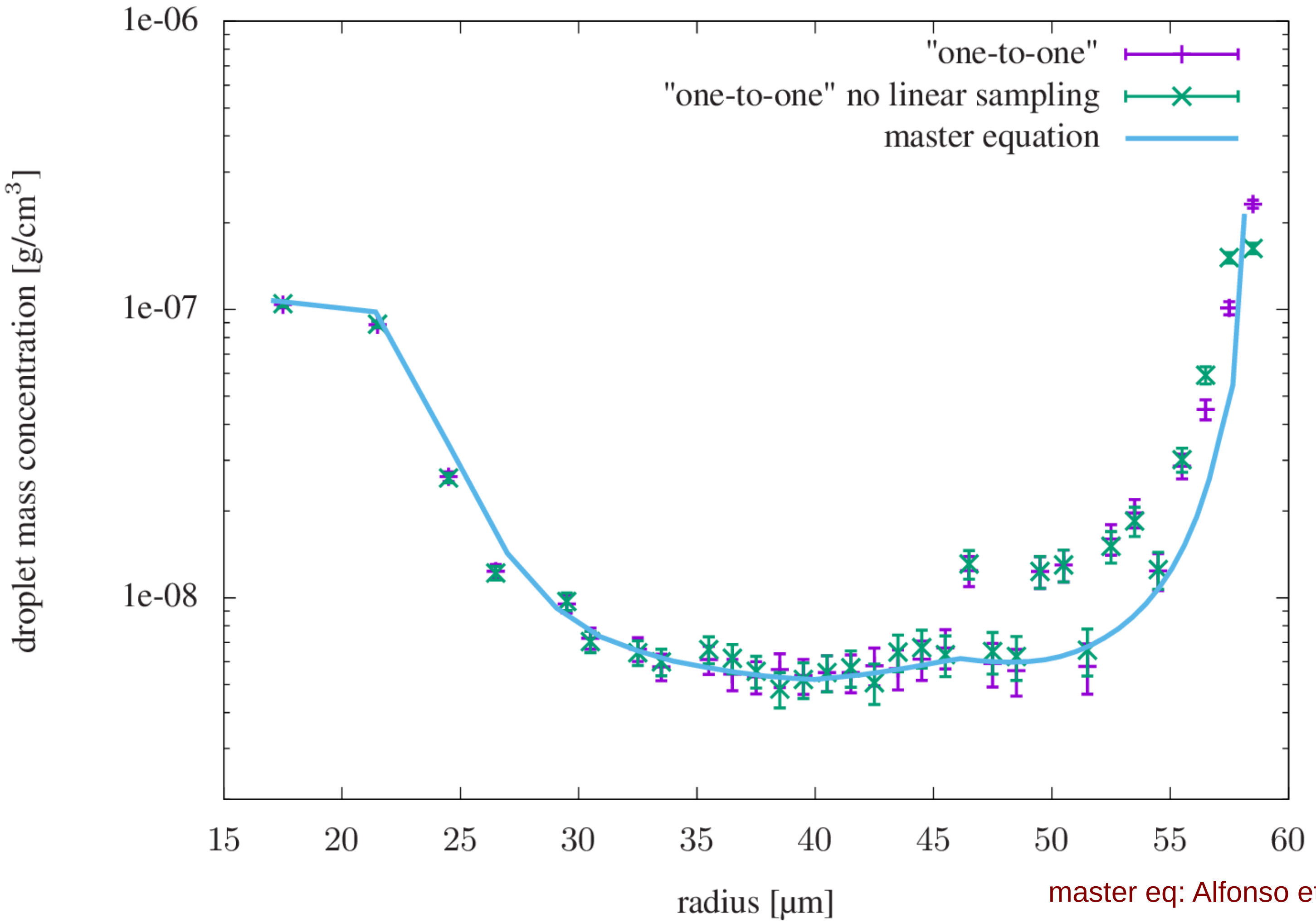
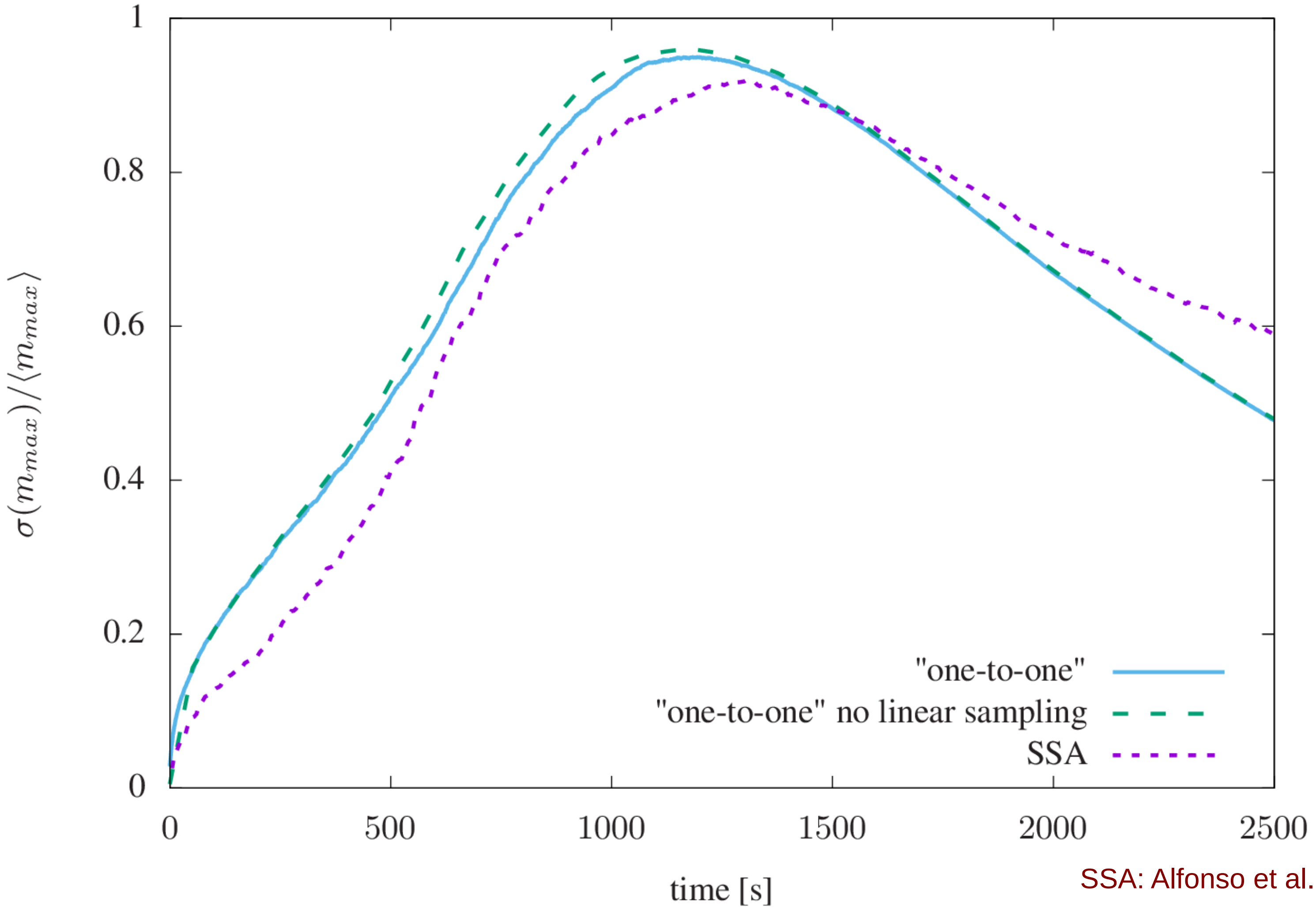
- “one-to-one” SDM:
- well-mixed cell
- each computational droplet represents one real droplet
- linear sampling: $N_{SD}/2$ instead of $N_{SD}(N_{SD}-1)/2$ collision pairs
- should be similar to the SSA, but faster

“one-to-one” vs master equation: average spectrum



master eq: Alfonso et al. 2017

“one-to-one” vs SSA: fluctuations



SSA: Alfonso et al. 2017

“one-to-one” vs master/SSA: summary

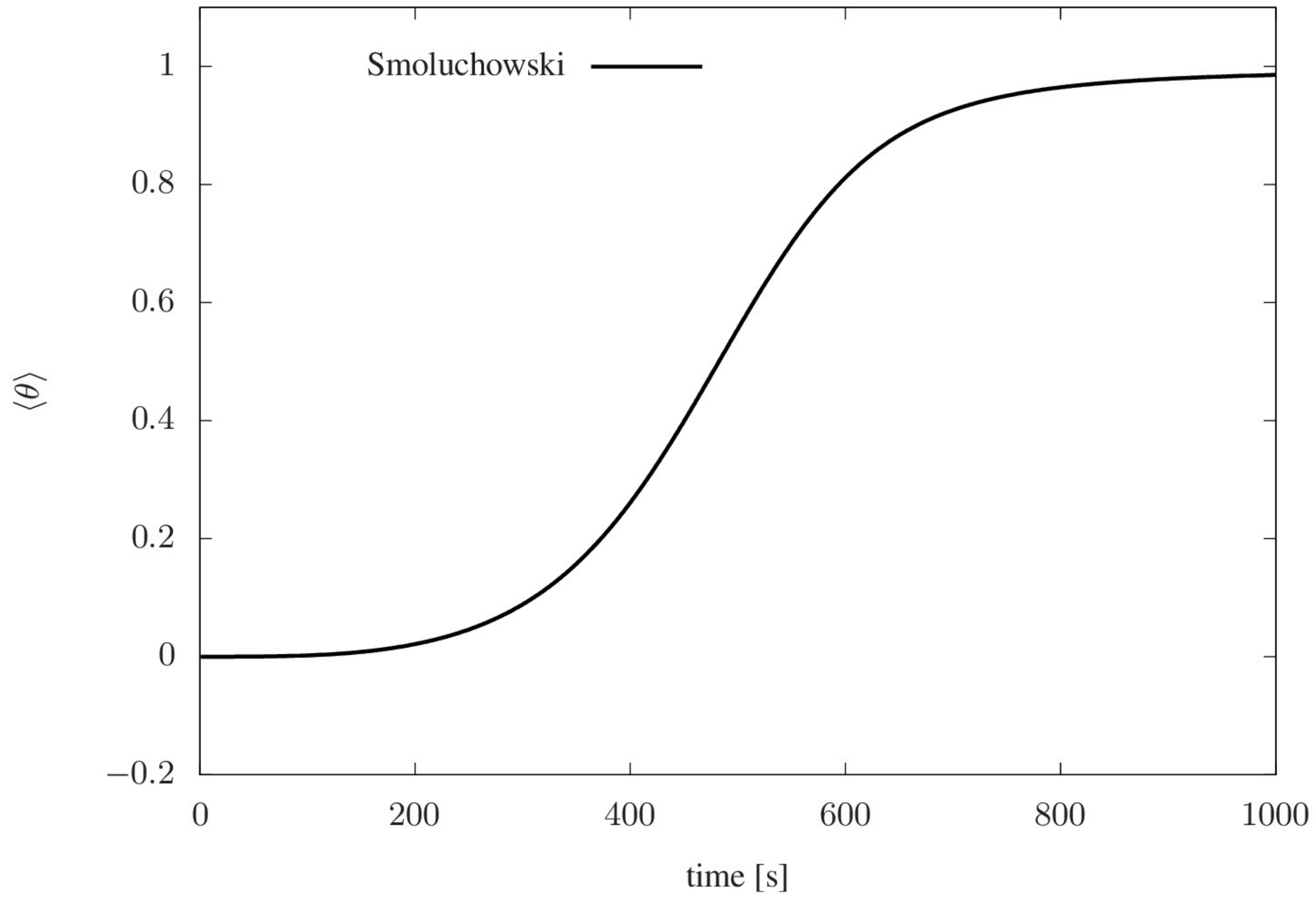
- “One-to-one” SDM at the level of precision of master equation/SSA
- Linear sampling does not affect observed averages and standard deviations, but makes “one-to-one” faster than the SSA

Validity of the Smoluchowski equation

- well-mixed cell
- neglect statistical correlations in the number of droplets of different sizes
- no information about fluctuations
- exact as $V \to \infty$
- what is the smallest cell in which Smoluchowski equation can be used without major errors?
- test against “one-to-one” SDM for two cases:
fast and slow coalescence

Validity of the Smoluchowski equation: fast coalescence

<r>$_{init}$ = 15 µm
$\Theta = m_{rain} / m_{tot}$



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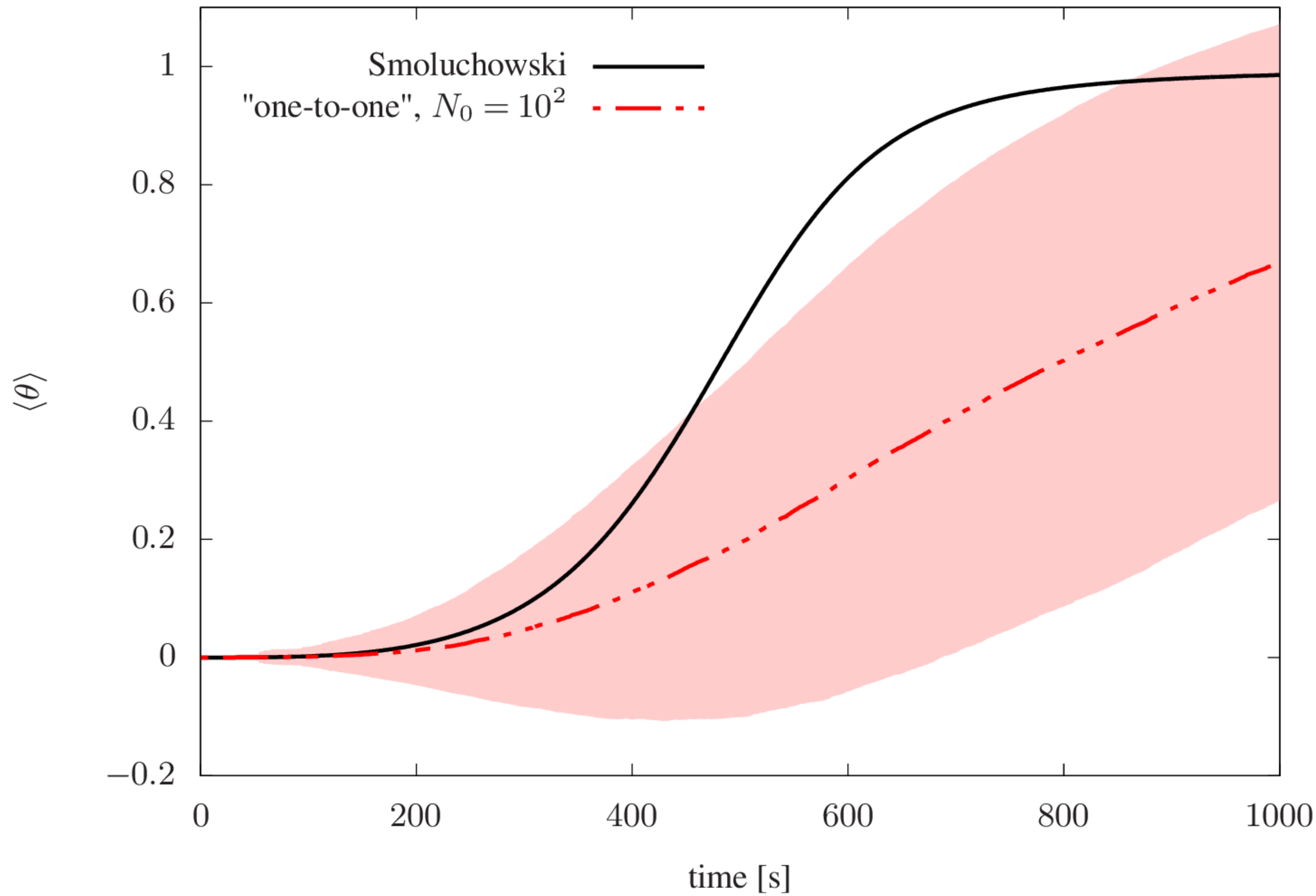

N_0 - initial number of droplets in a colaescence cell

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$\langle r \rangle_{init} = 15\ \mu m$
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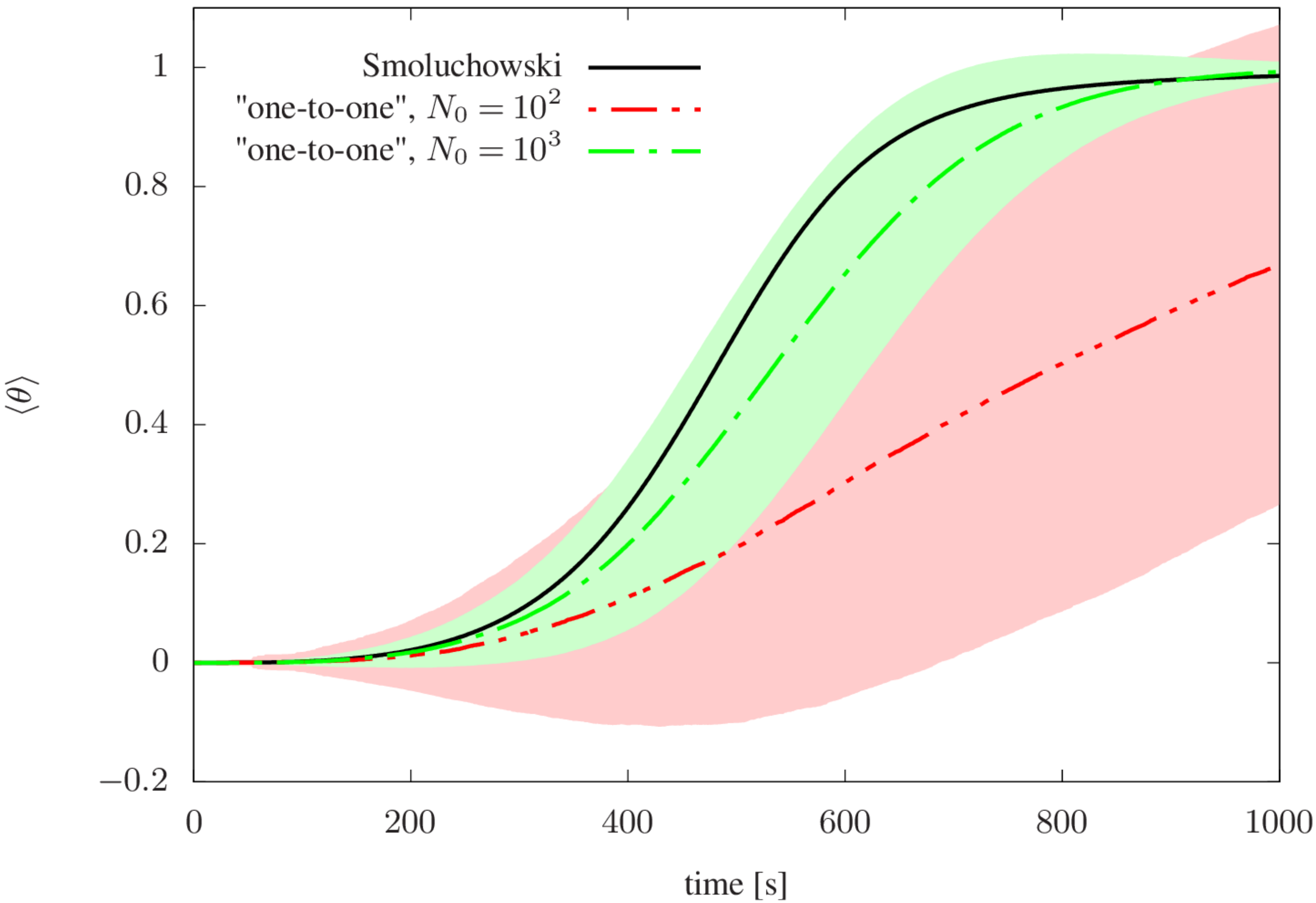

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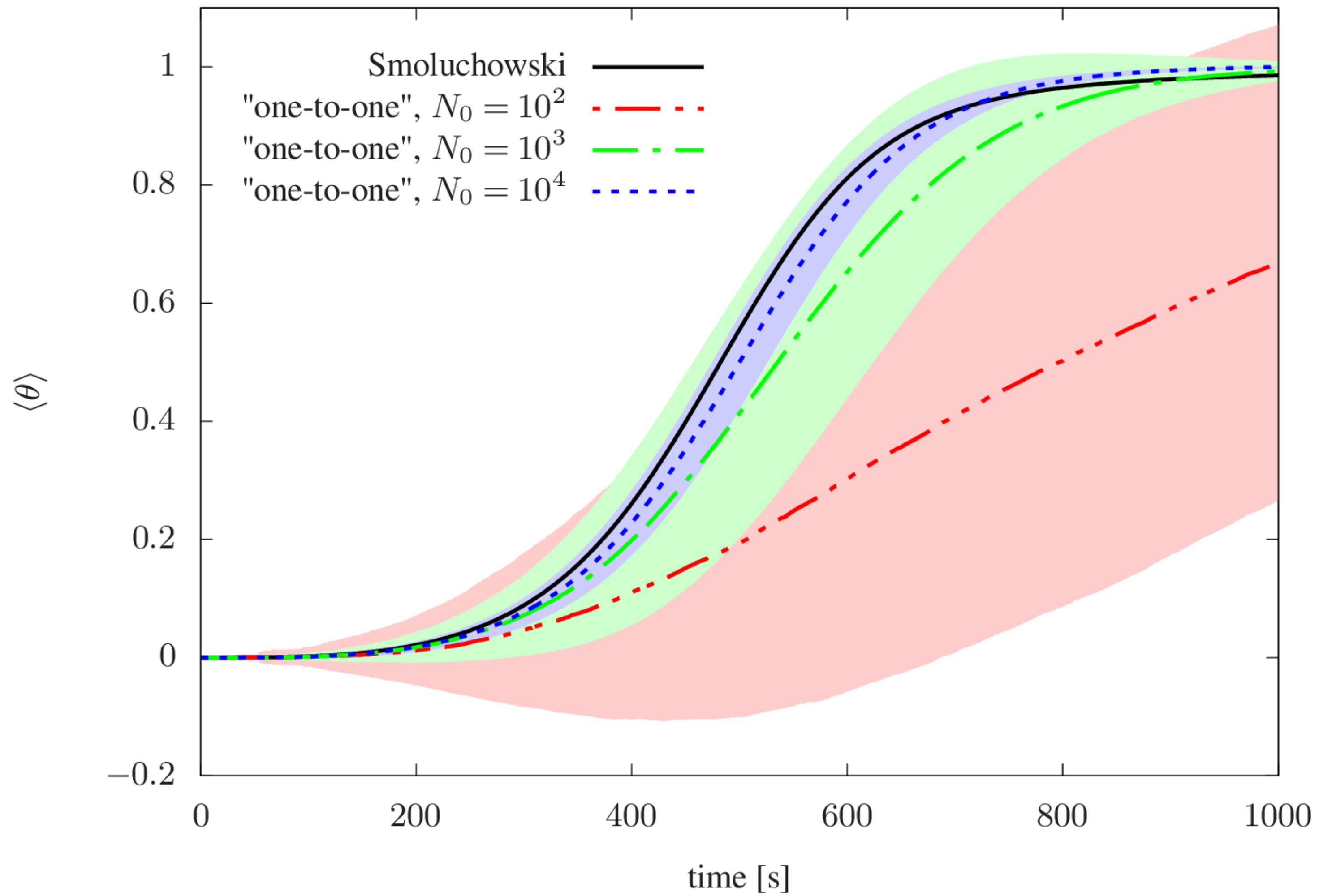

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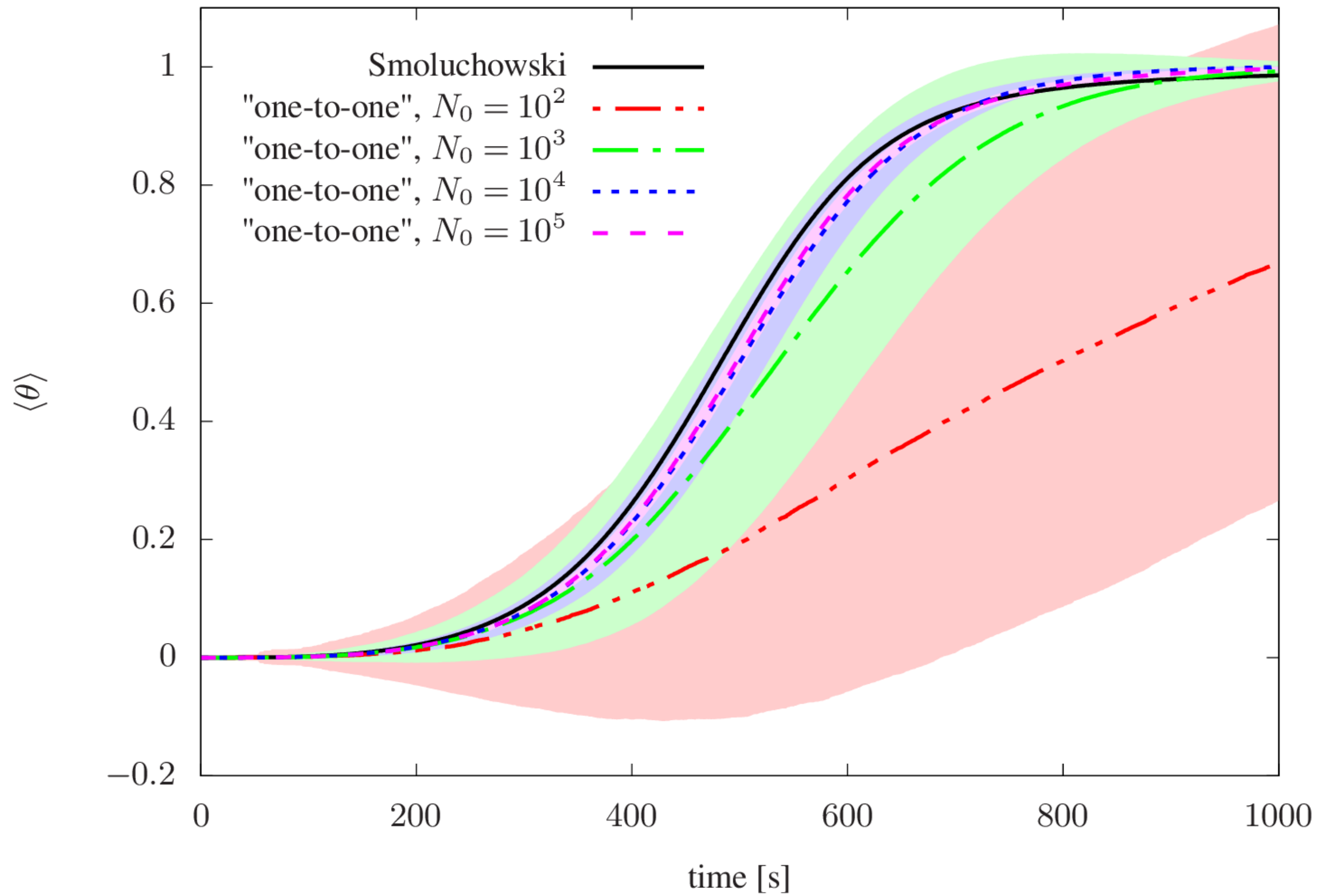
Valid for $N_0 \geq 10^4$



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Validity of the Smoluchowski equation: slow coalescence

$\langle r \rangle_{init}$ = 9.2 µm

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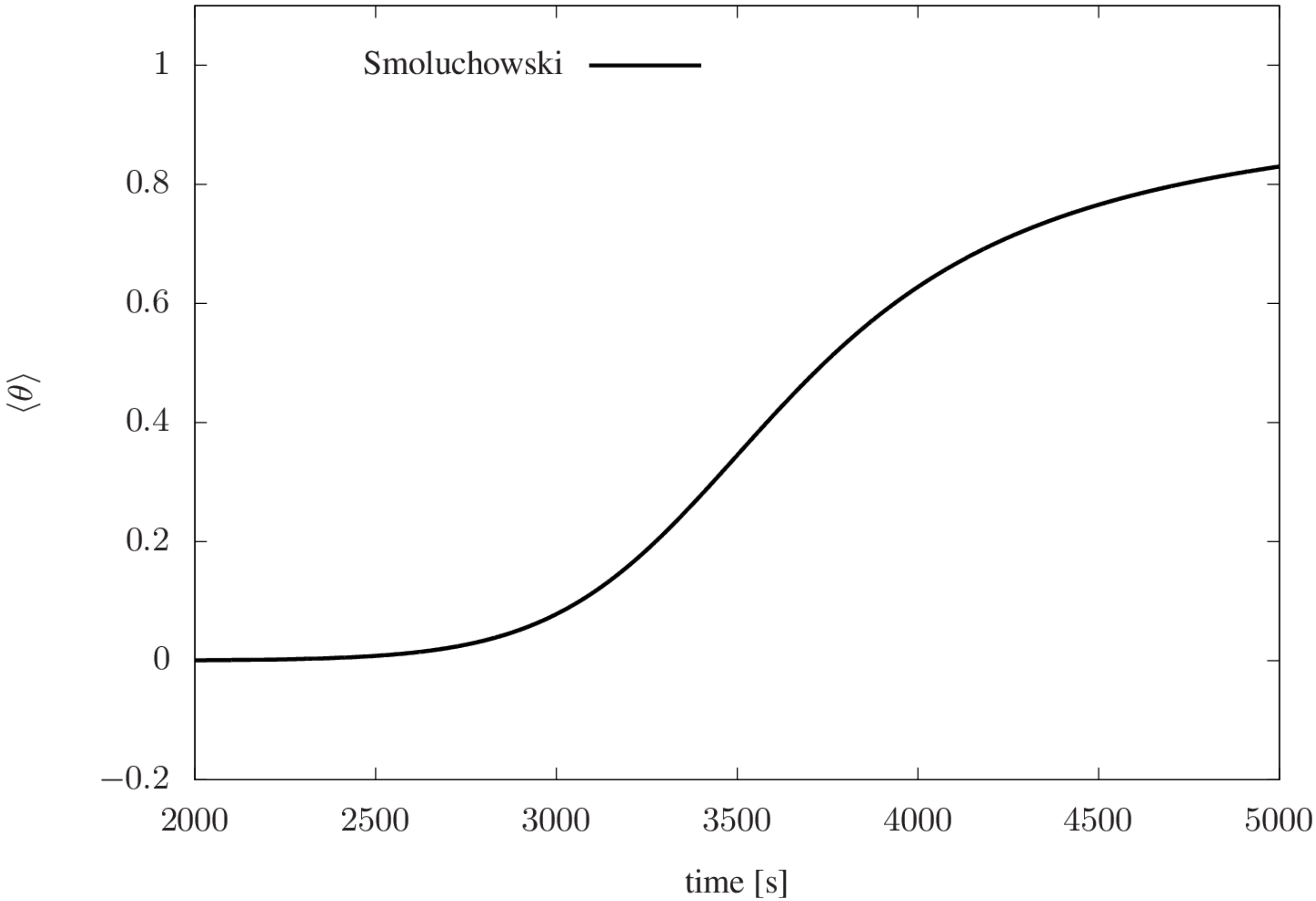

Validity of the Smoluchowski equation: slow coalescence

$\langle r \rangle_{init} = 9.2\ \mu m$
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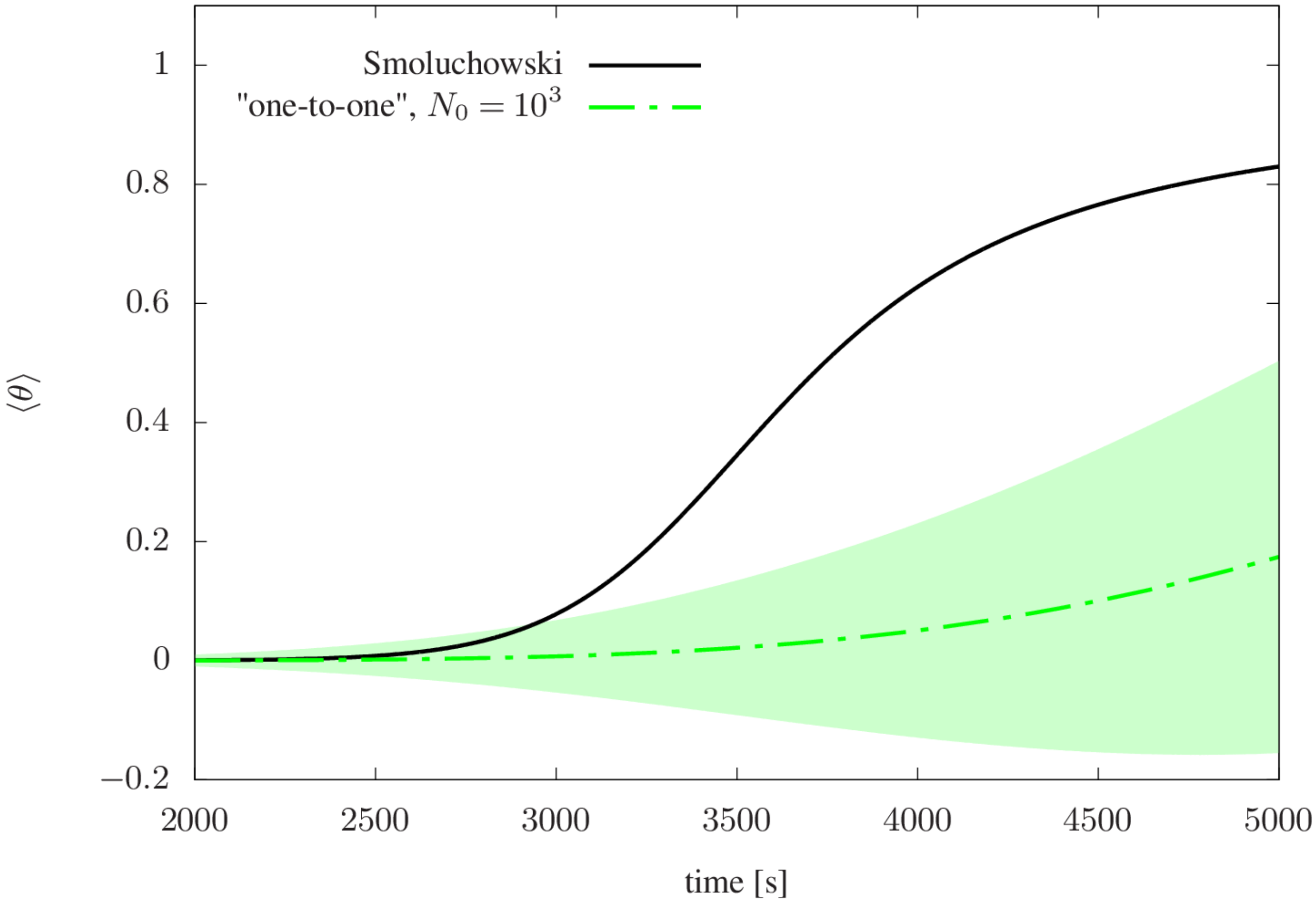

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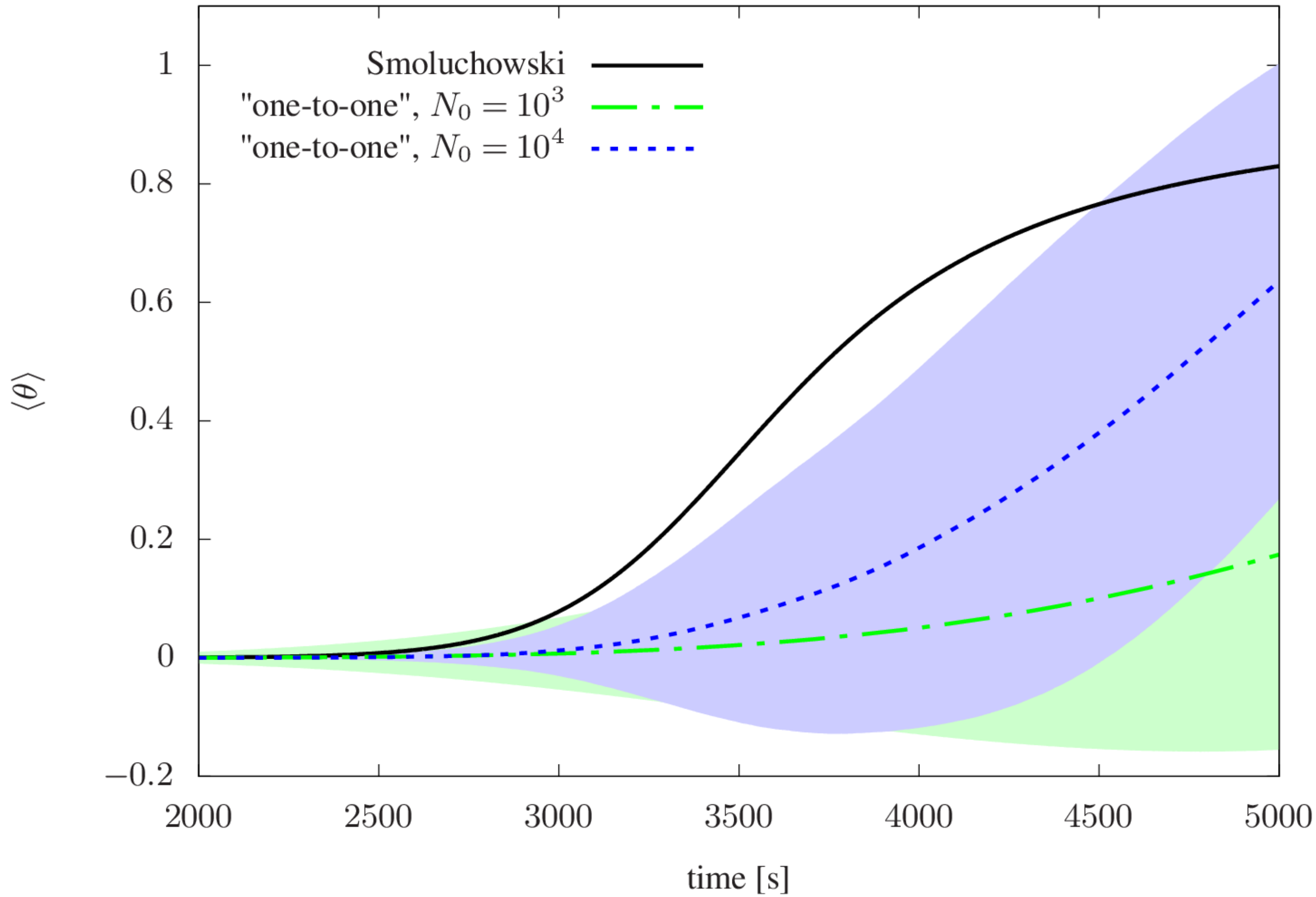

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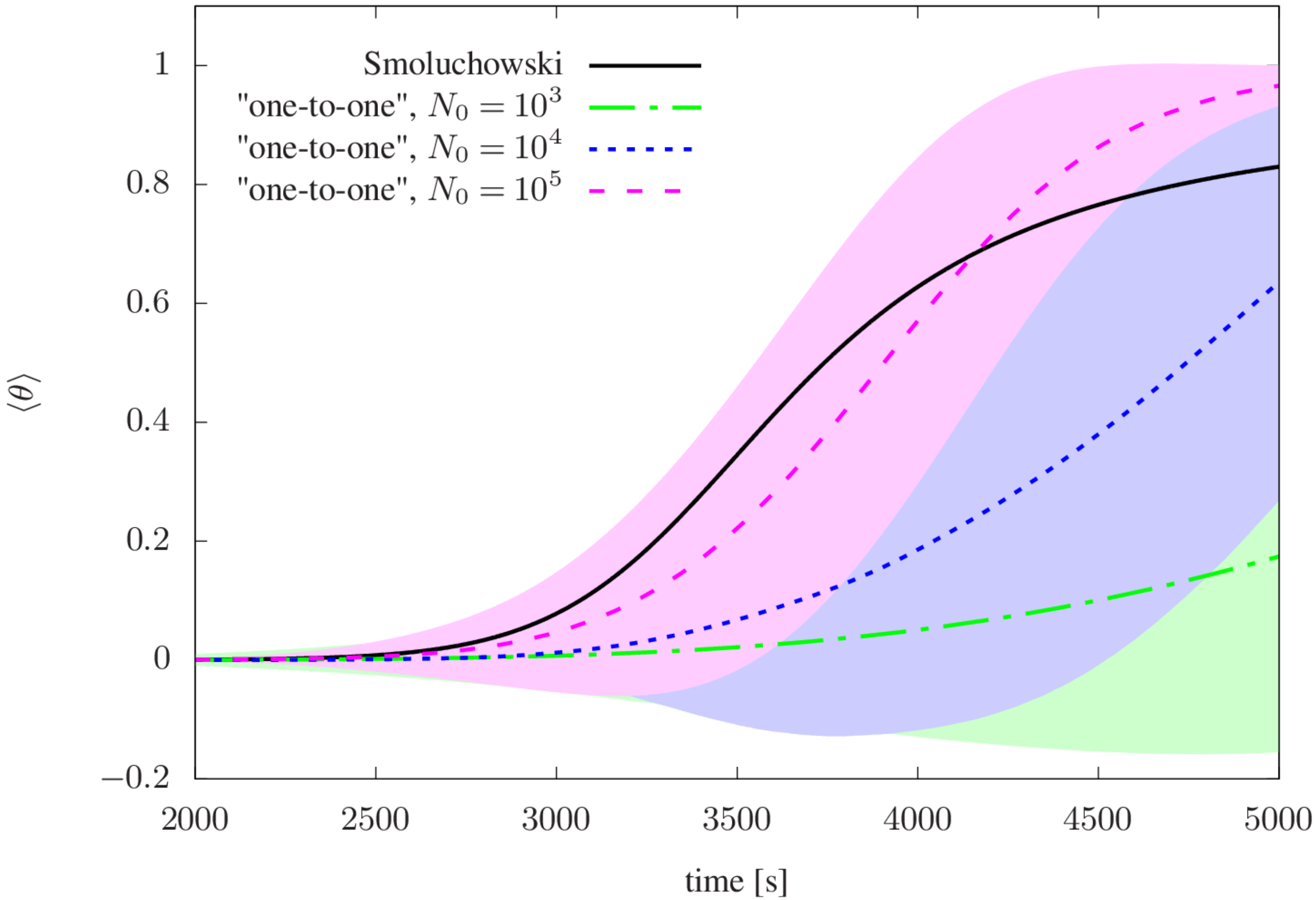

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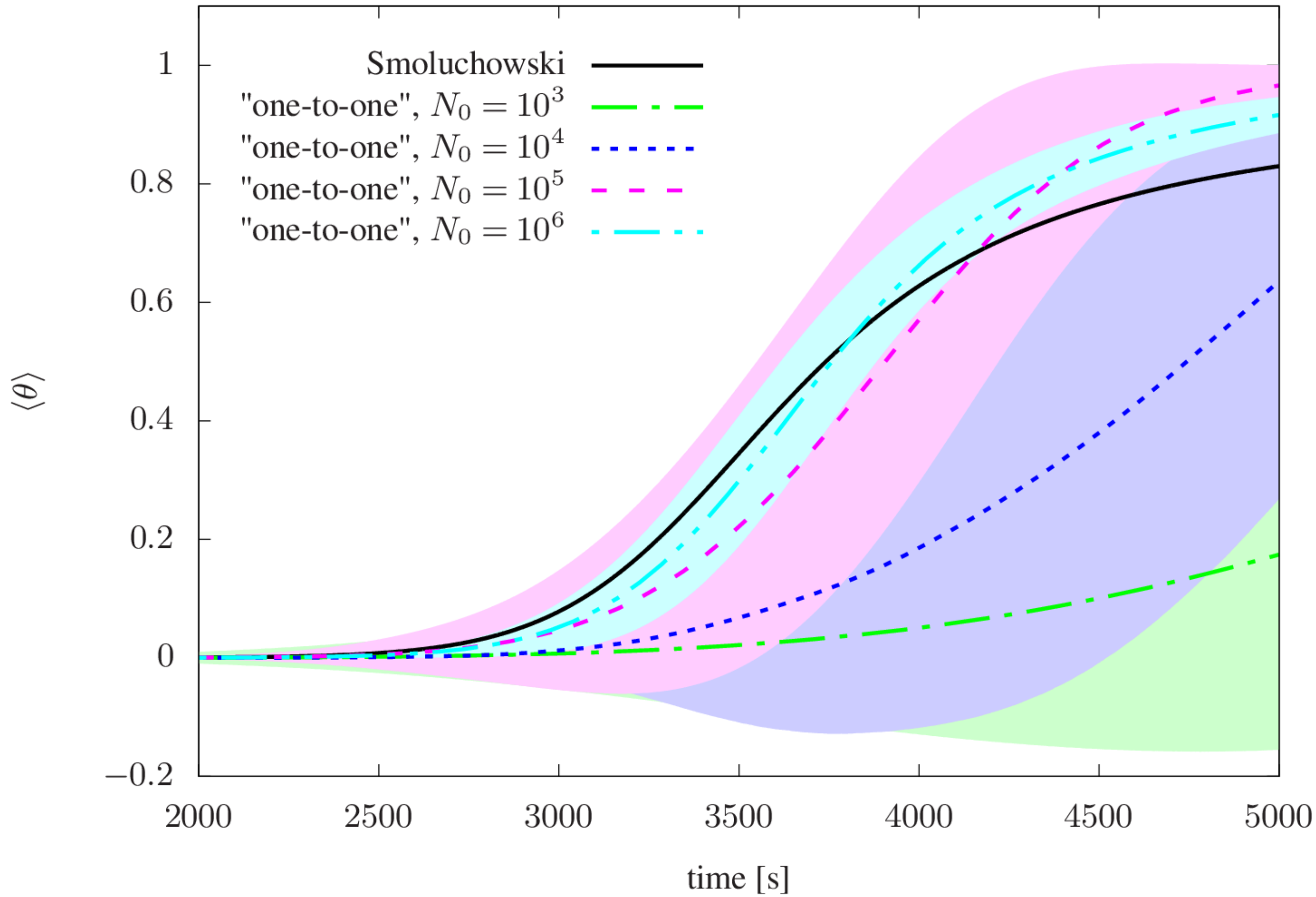

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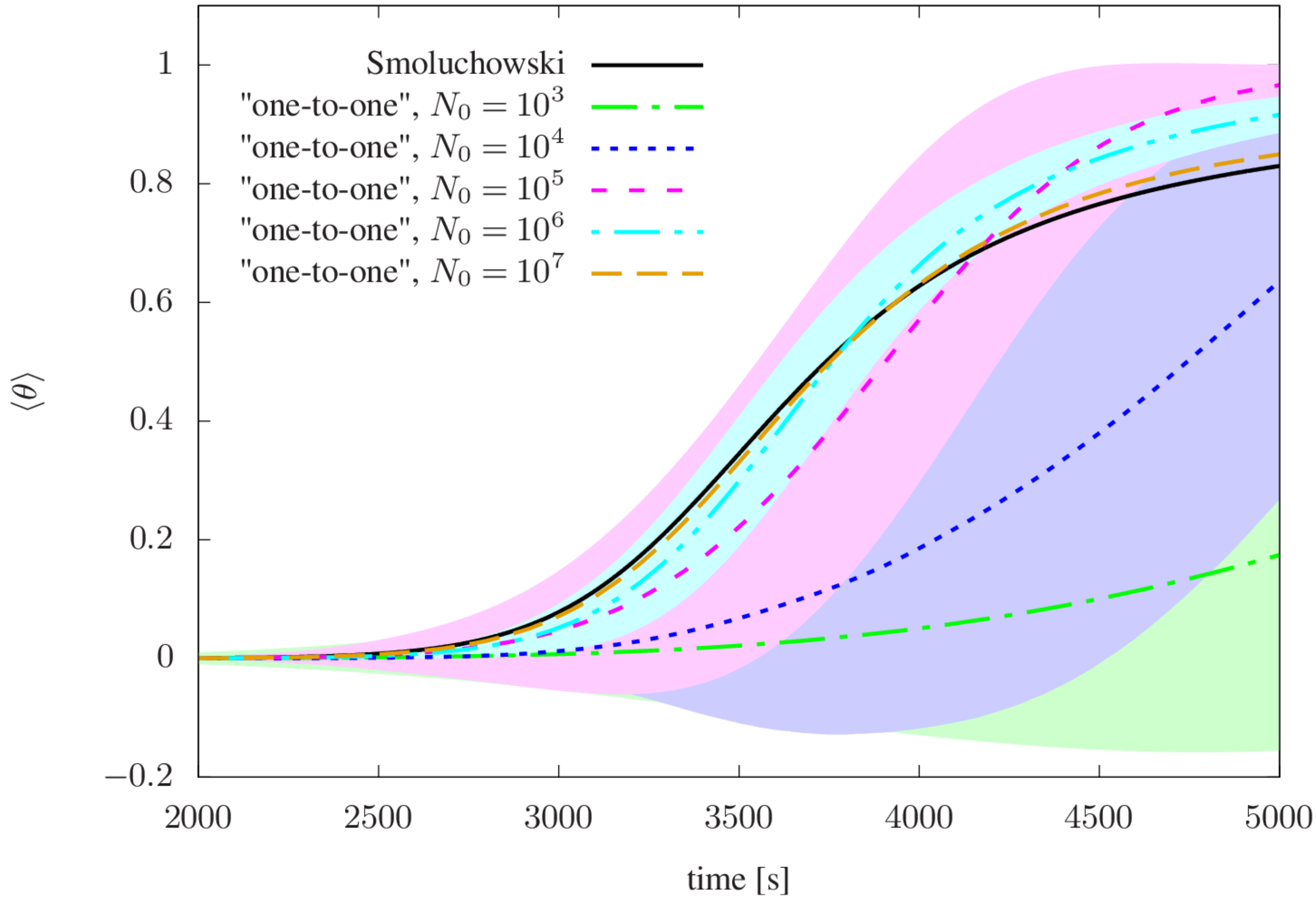
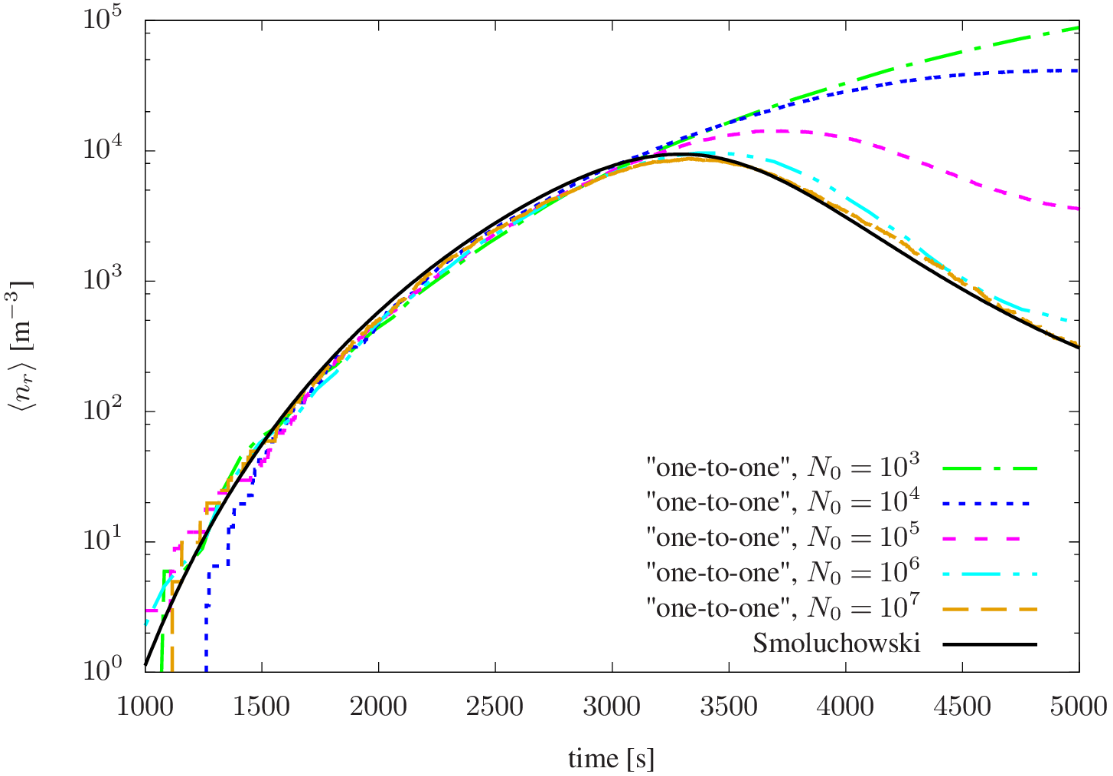
Valid for $N_0 \geq 10^7$



N_0 - initial number of droplets in a colaescence cell

In large cells, rain drop concentration decreases sooner



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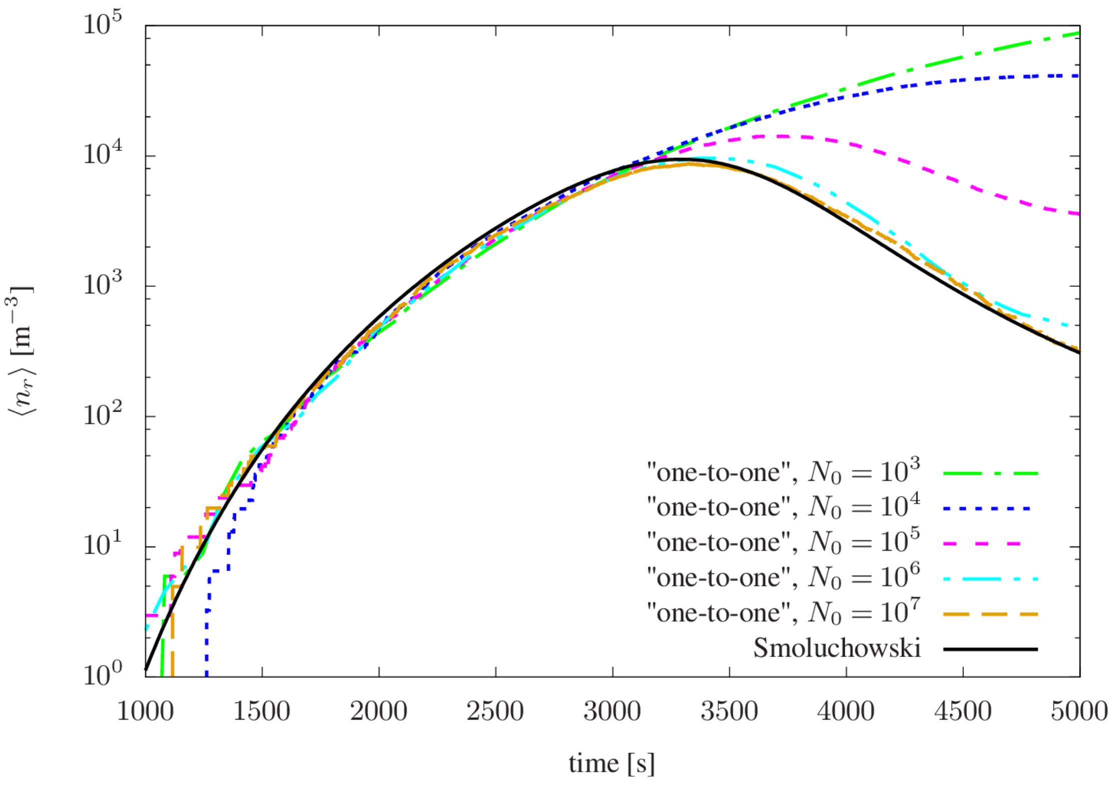
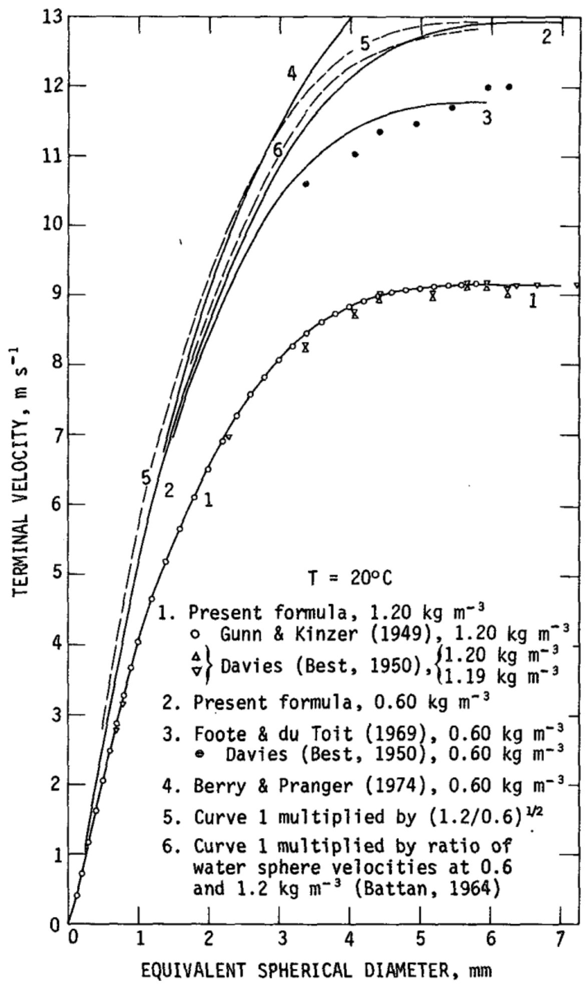




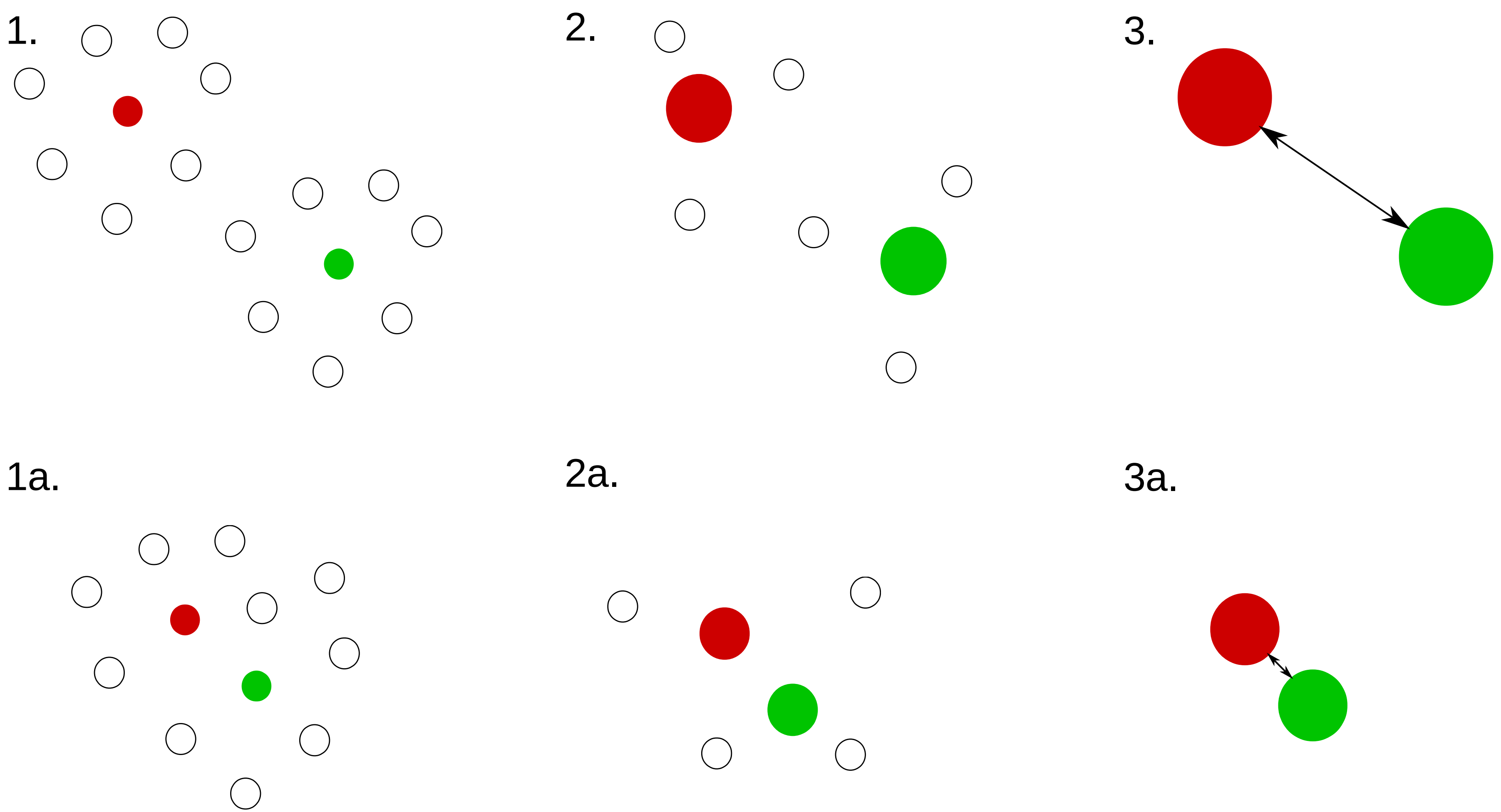
Large rain drops less efficient at scavenging cloud drops:

$V_L = 2V_S$, $r_L^2 = 2^{2/3} r_S^2$

Beard 1976

More rain-rain collisions in large well-mixed cells

- broader spectrum of rain drops
- artificial increase in the rate of collisions between rain drops?:



Validity of the Smoluchowski equation: summary

- Smoluchowski equation overestimates amount of rain in small cells - probably irrelevant due to mixing by sedimentation

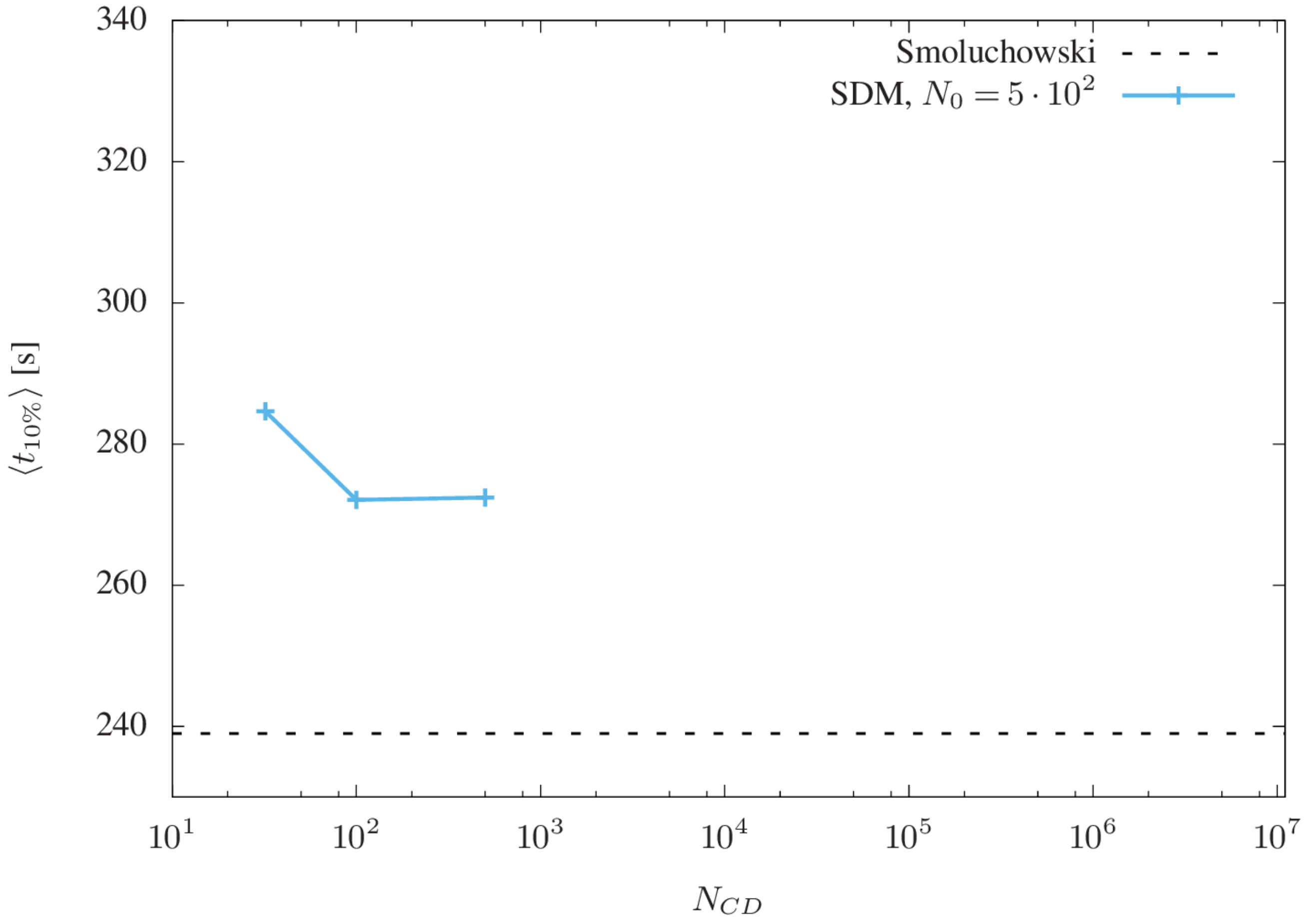
- Smoluchowski equation underestimates amount of rain for cells of intermediate sizes if coalescence is slow - may be irrelevant because of condensational growth

Validity of the super-droplet method

- well-mixed
- small number of computational droplets:
  - unrealistic correlations
  - amplified fluctuations
- check how many computational droplets are needed to obtain correct averages and correct standard deviations

Validity of SDM: average autoconversion time

$m_{rain}(t_{10\%}) = 0.1\ m_{tot}$



$N_{CD} = N_{SD}$

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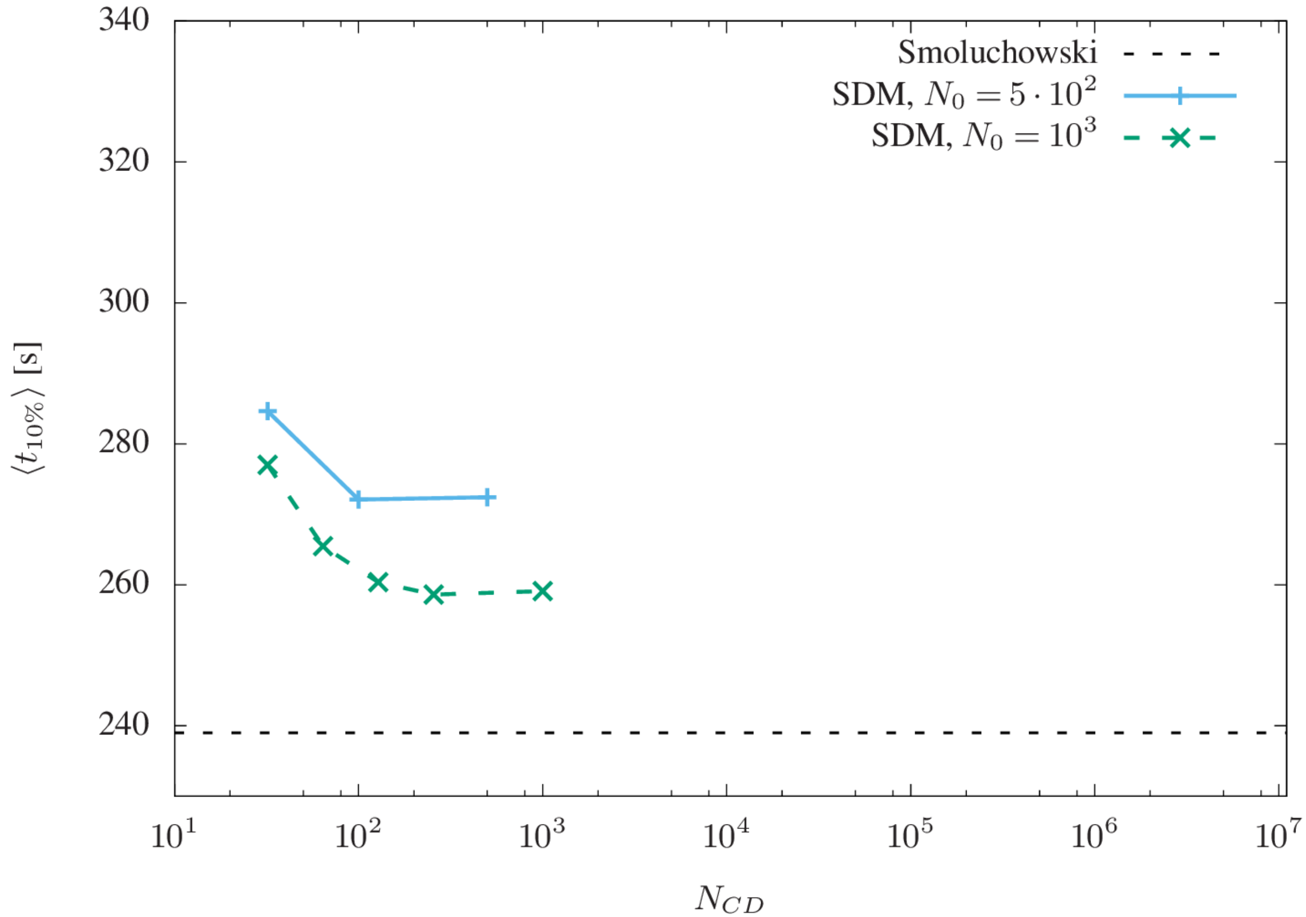

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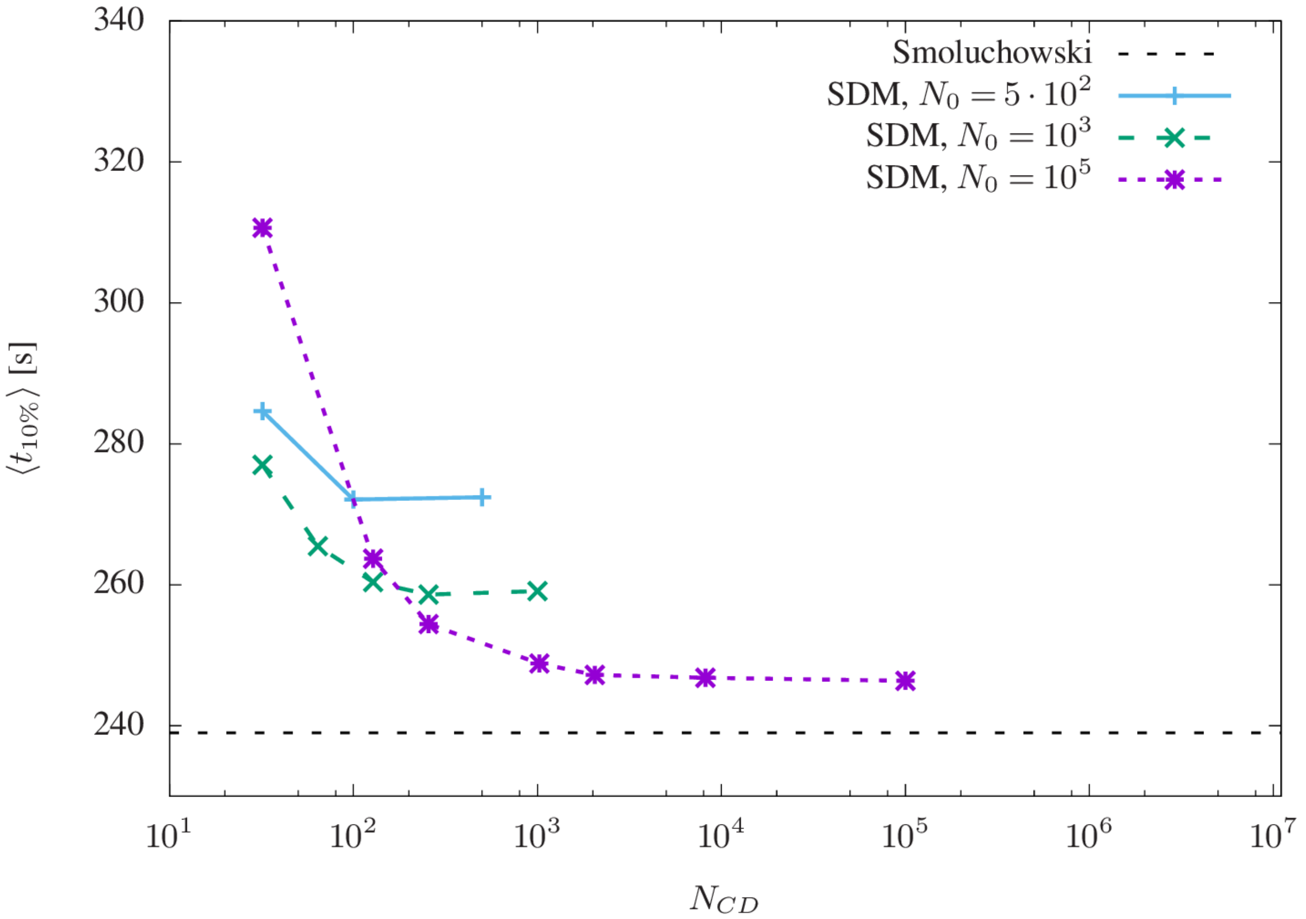

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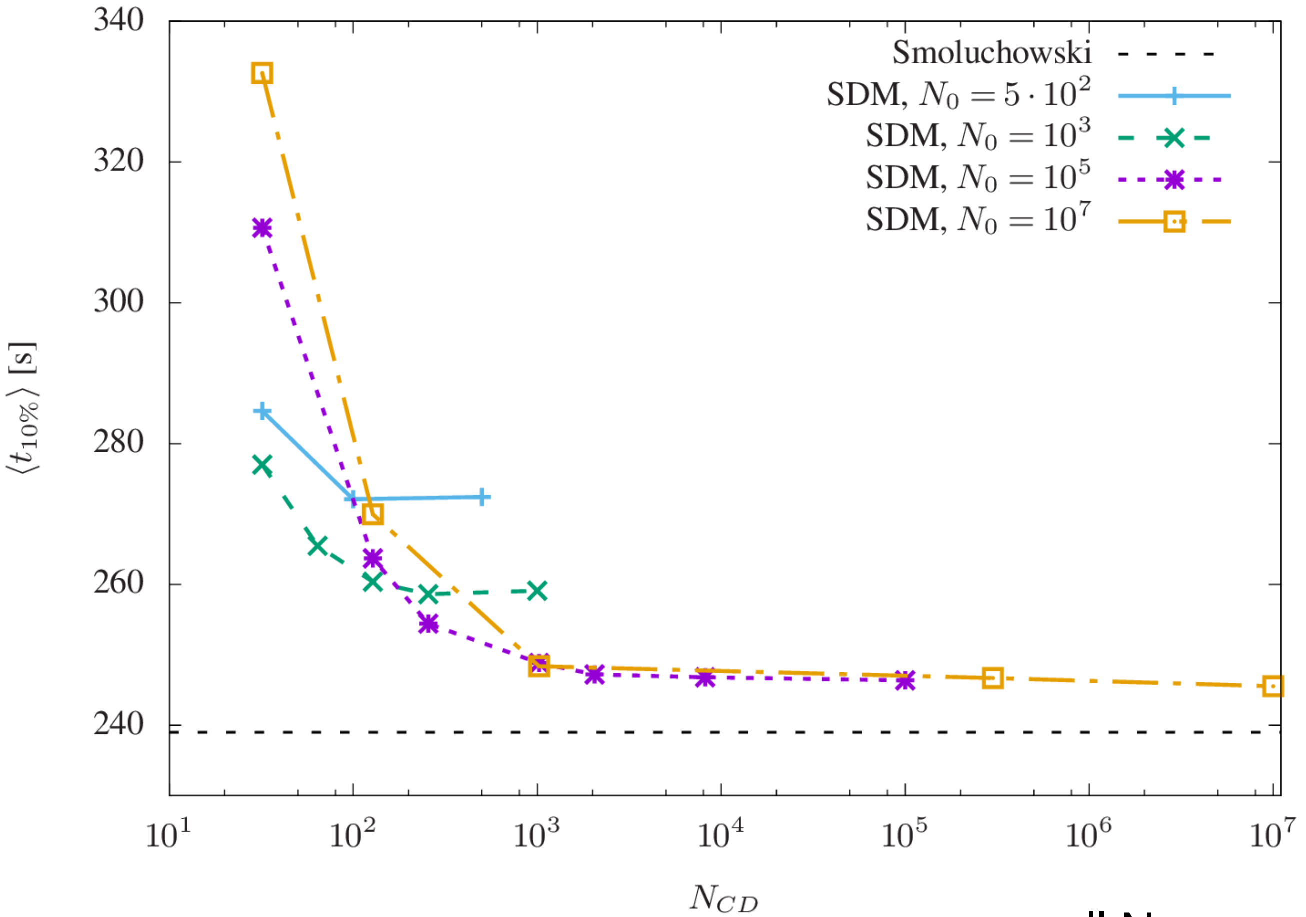

small N_{CD} - correlations

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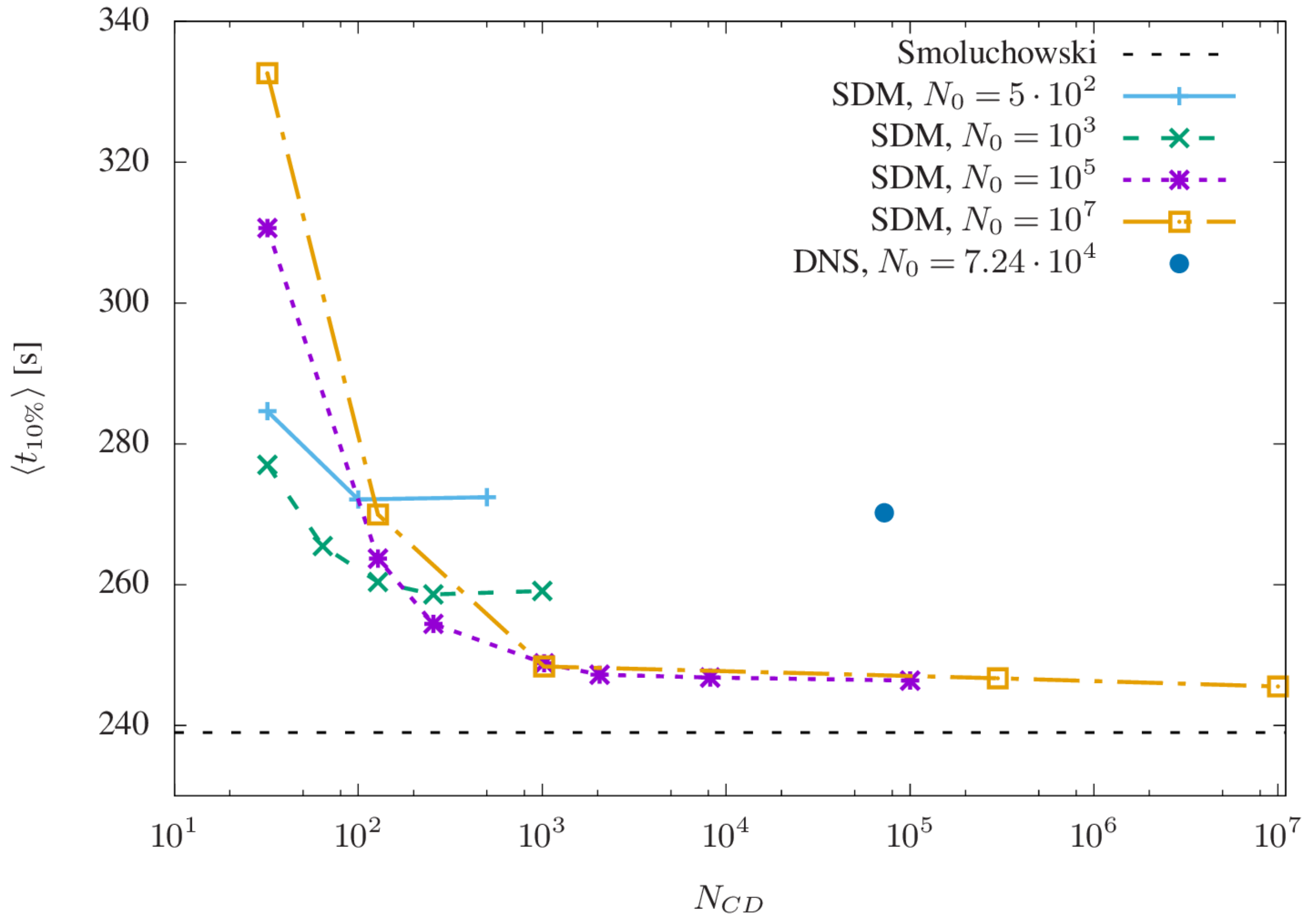

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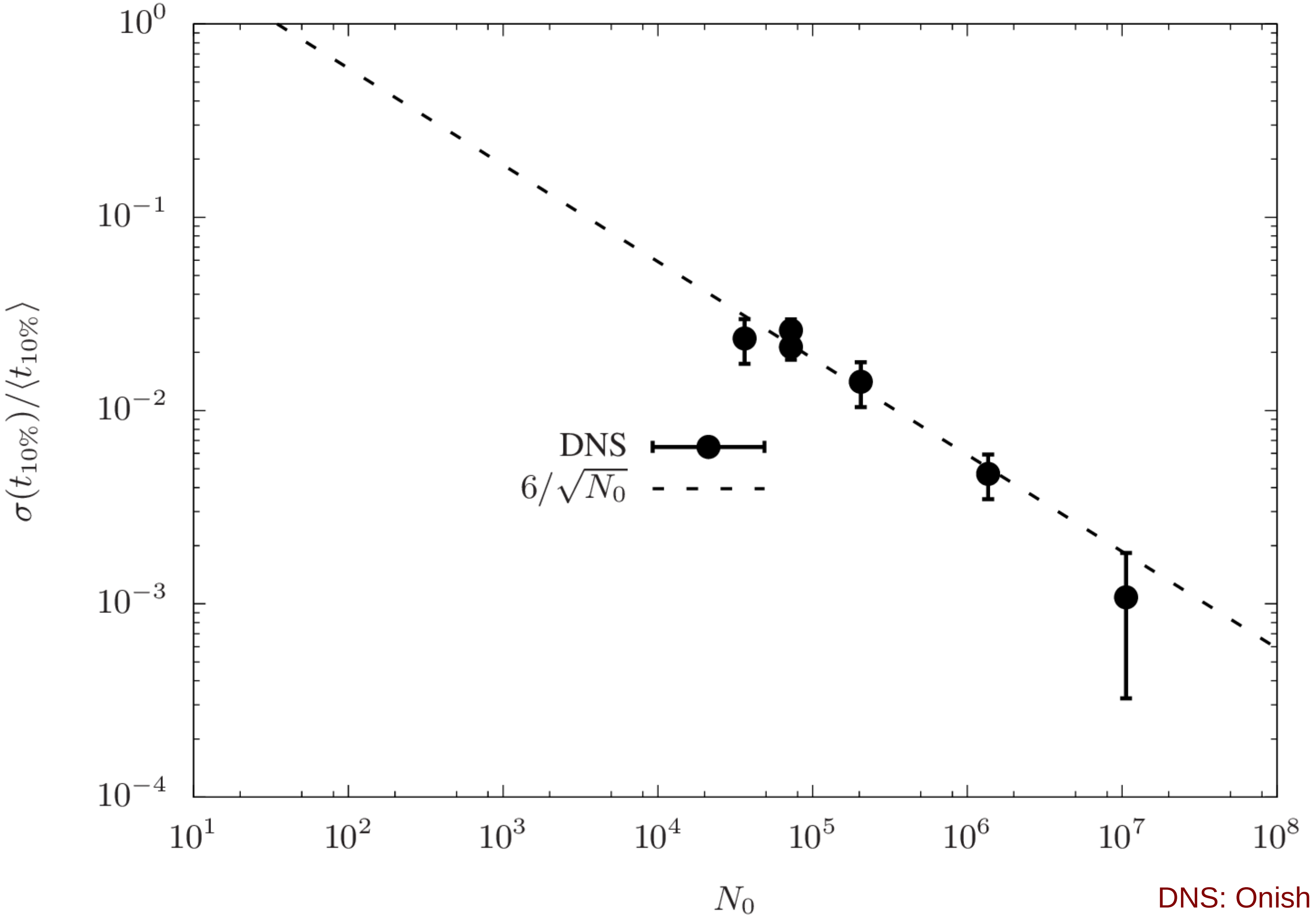
Validity of SDM: fluctuations in autoconversion time



DNS: Onishi et al. 2015

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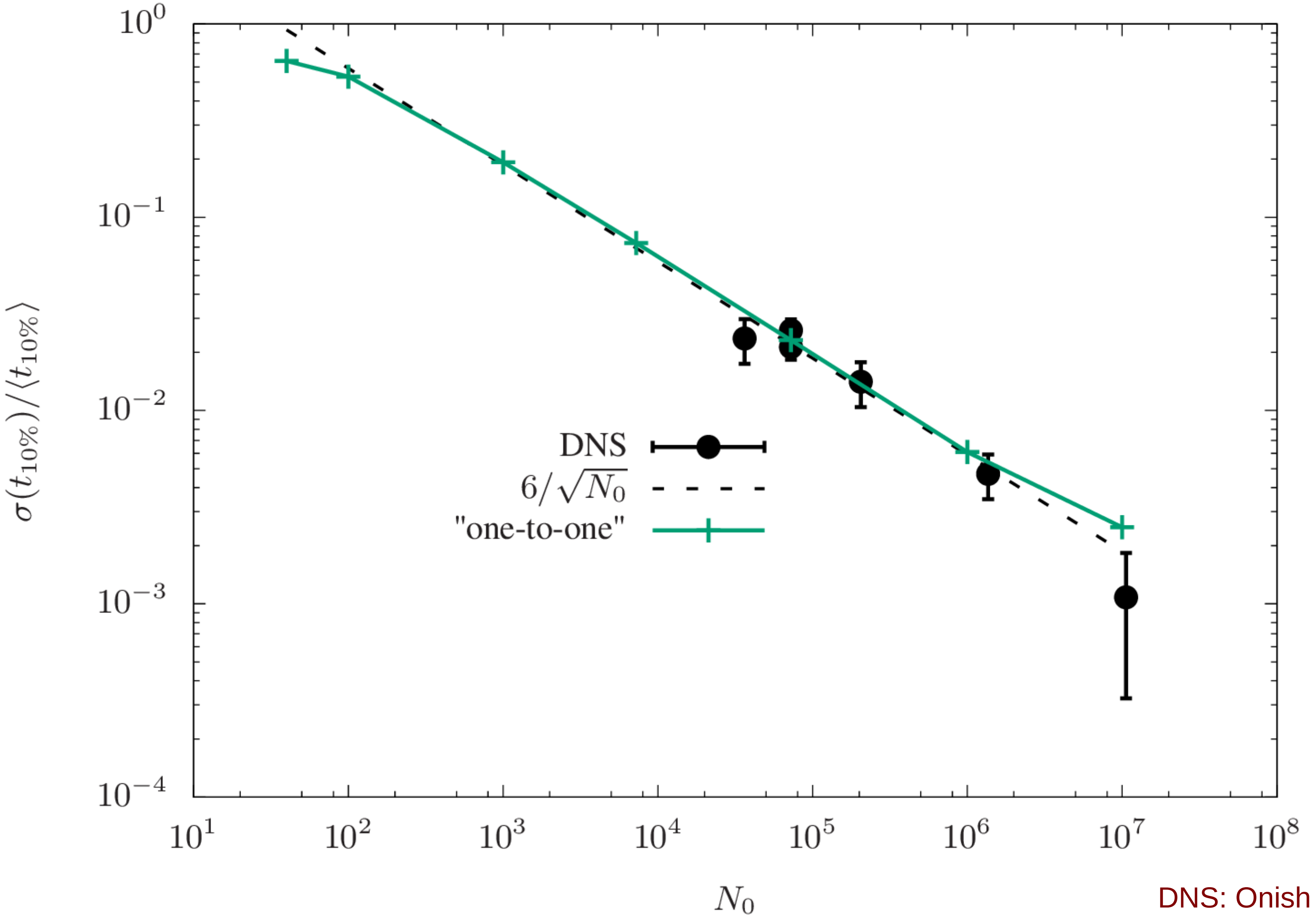

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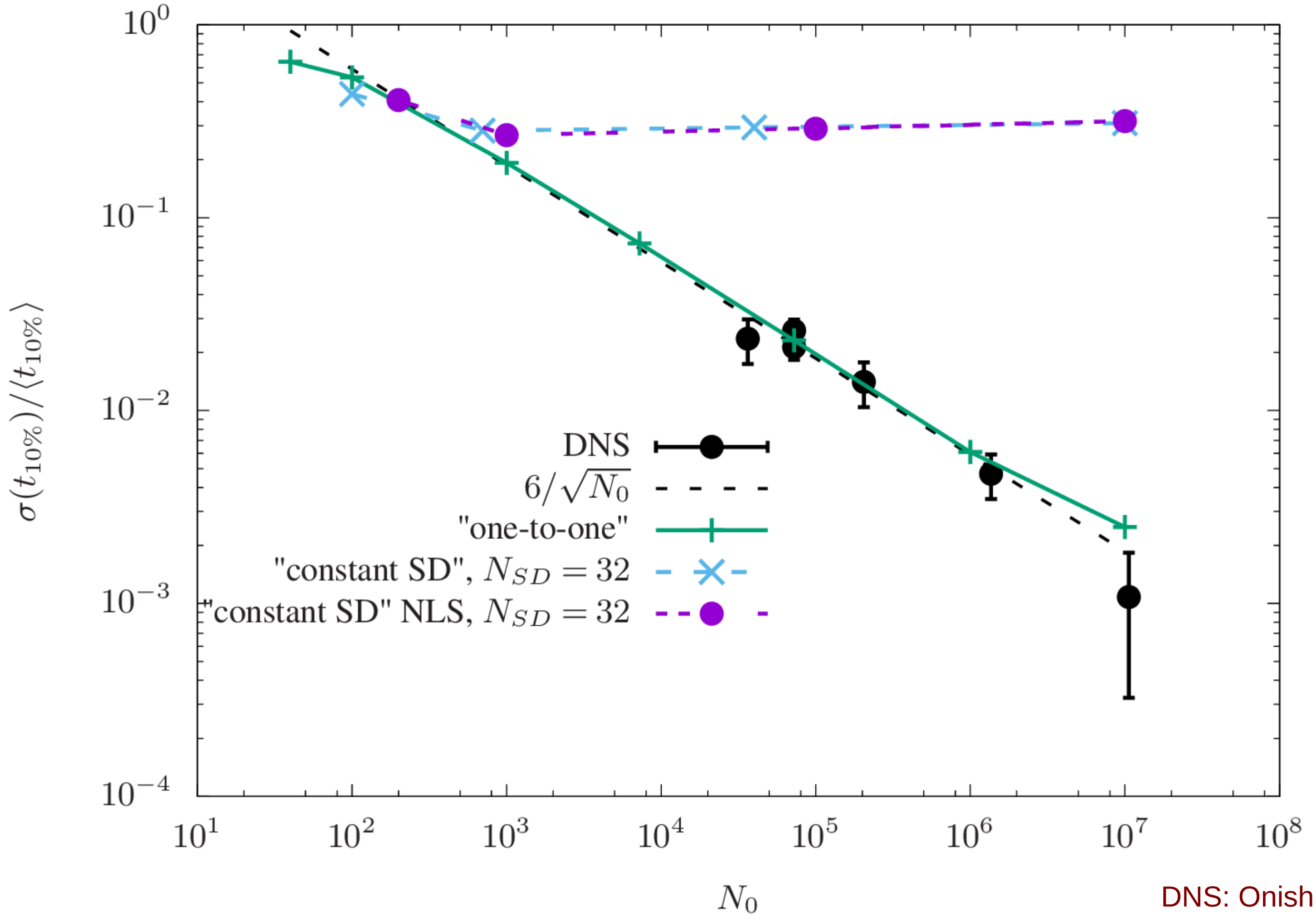

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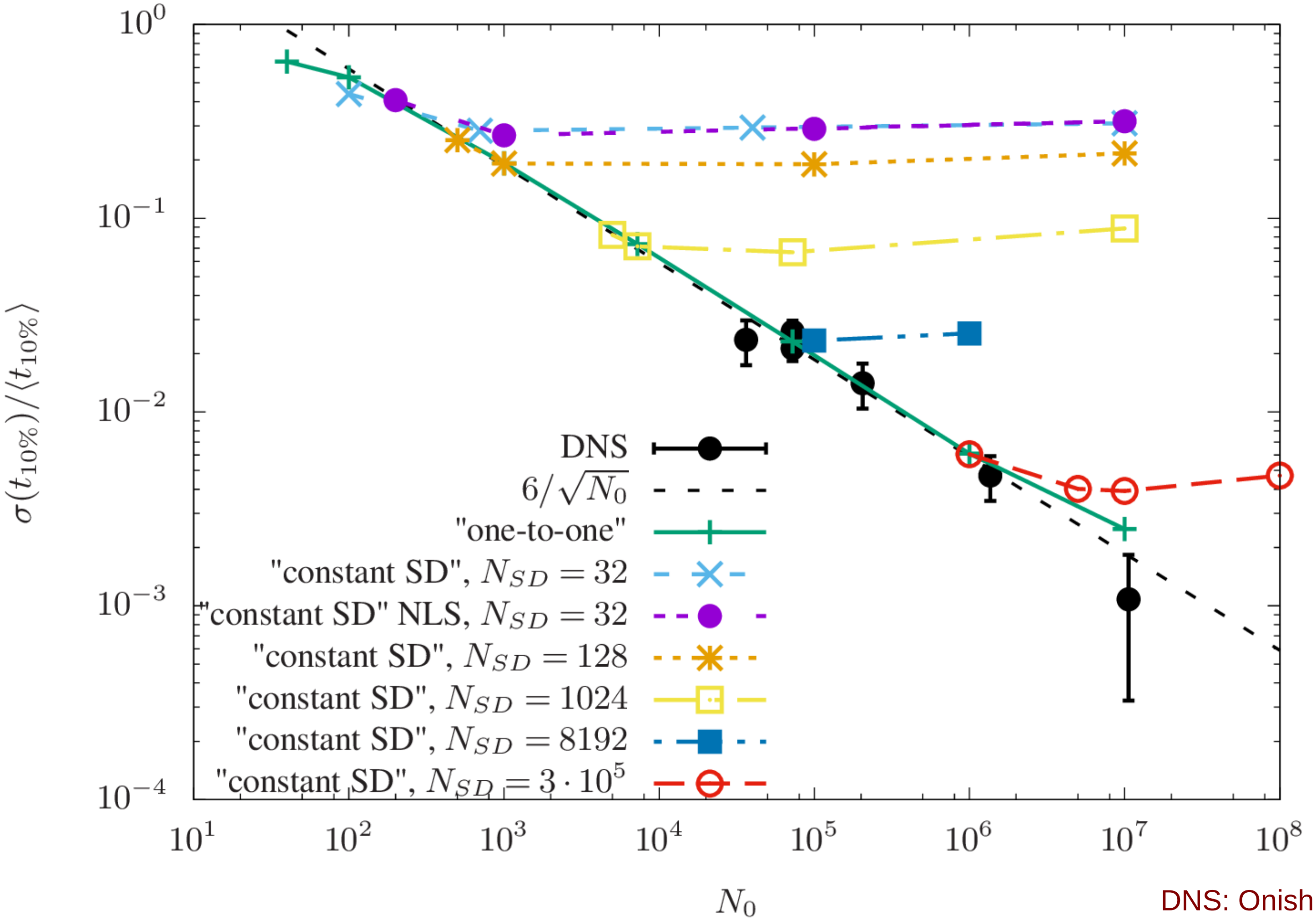

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small N_{SD} - large fluctuations due to a small number of trials

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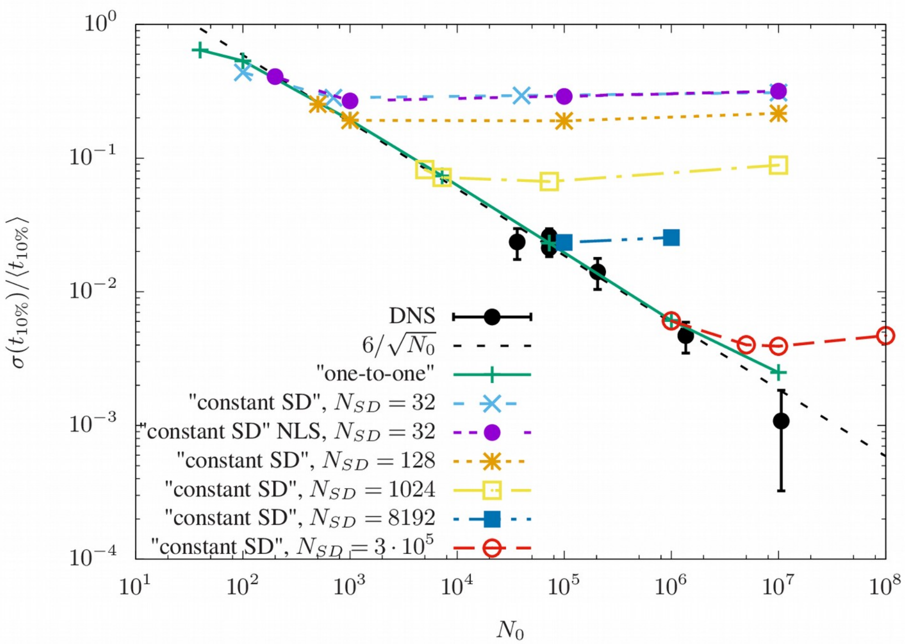

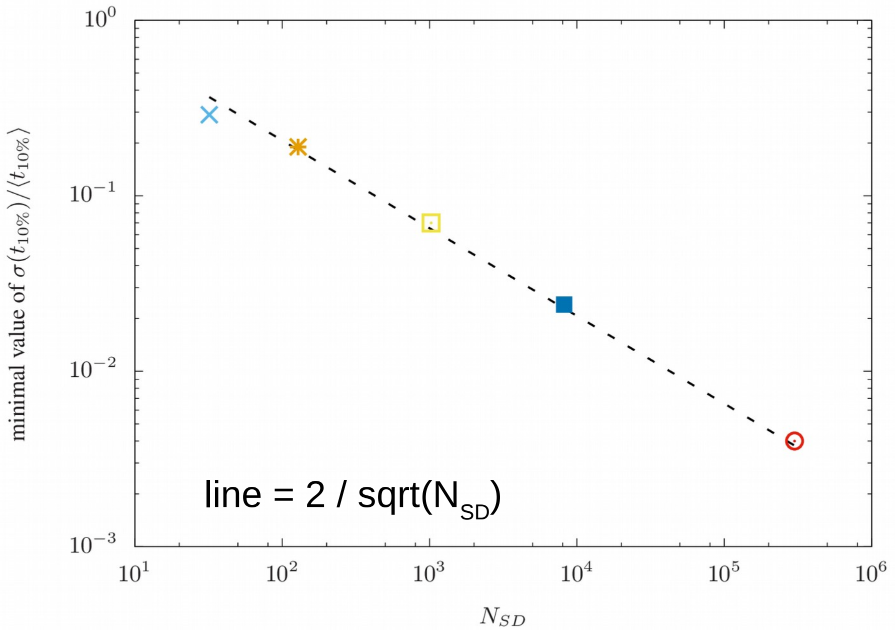

$N_{SD} \geq N_0 / 9$

Validity of SDM: summary

- 1000 super-droplets per cell are needed to get the correct mean autoconversion time
- N_0 / 9 super-droplets per cell are needed to get the correct standard deviation of autoconversion time

Conclusions

- Smoluchowski equation overestimates amount of rain in small cells - probably not a problem because of mixing by sedimentation
- Using a large well-mixed cell (e.g. Smoluchowski equation, SDM) may underestimate amount of rain if coalescence is slow - may not be a problem because of condensational growth
- Super-droplet method: easy to get the right average, very hard to get the right standard deviation
- All methods except DNS assume that the cell is well-mixed - we need DNS simulations to check what is the maximum size of a well-mixed cell

Well-mixed cell

- well-mixed: droplets of each size are scattered randomly and uniformly within the cell
- well-mixed with respect to coalescence: droplets are randomly redistributed between coalescence events:

$$\tau_{mix} << \tau_{coal}\,, \quad \tau_{coal} \approx 0.1 \text{ s}$$

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- mixing by turbulence:

$$\tau^{t}_{mix} = (V^{2/3}/\varepsilon)^{1/3}\ , \ \varepsilon = 10^{3}\ \mathrm{cm}^{2}\ /\ \mathrm{s}^{3}\ , \ V \approx 10^{-2}\ \mathrm{cm}^{3}$$

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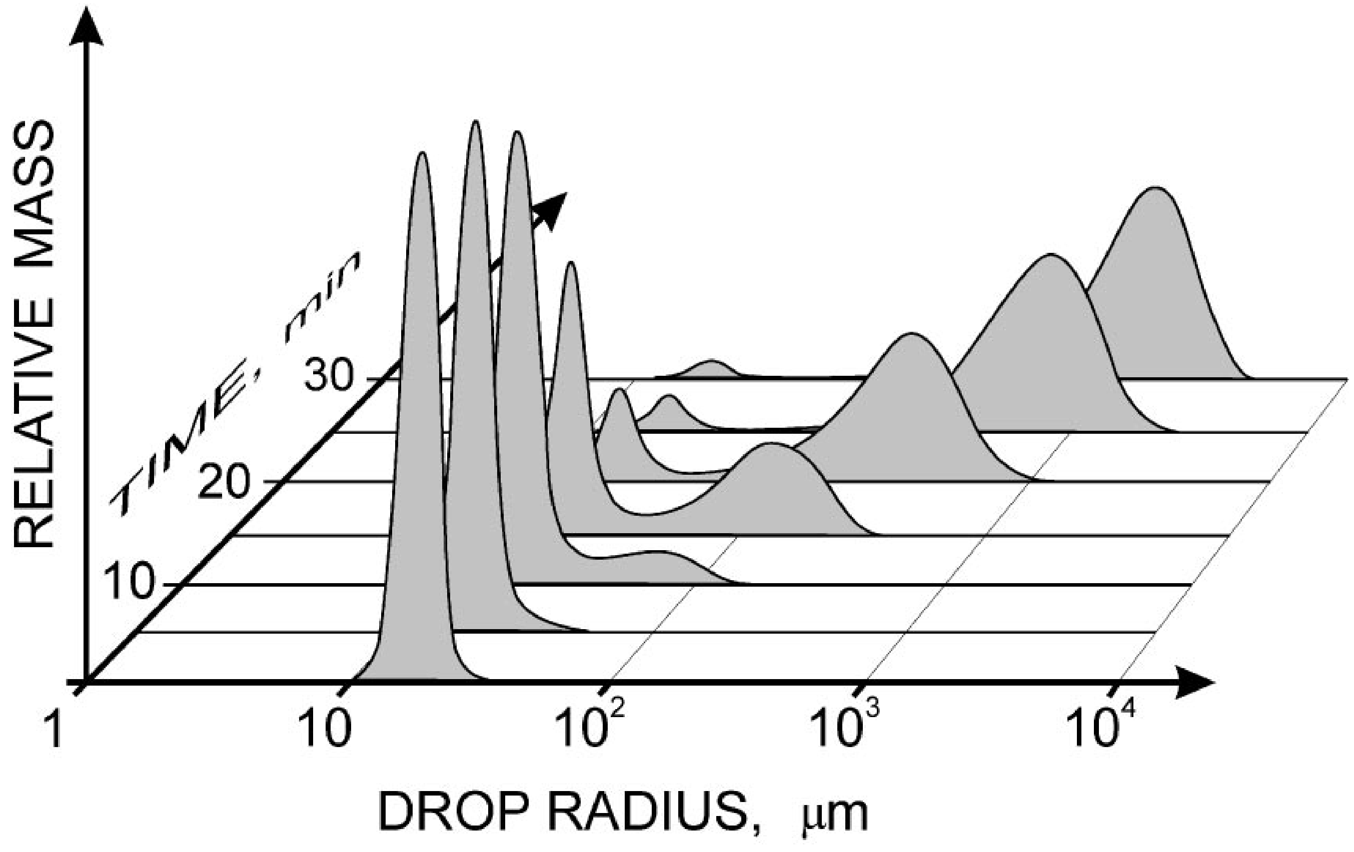
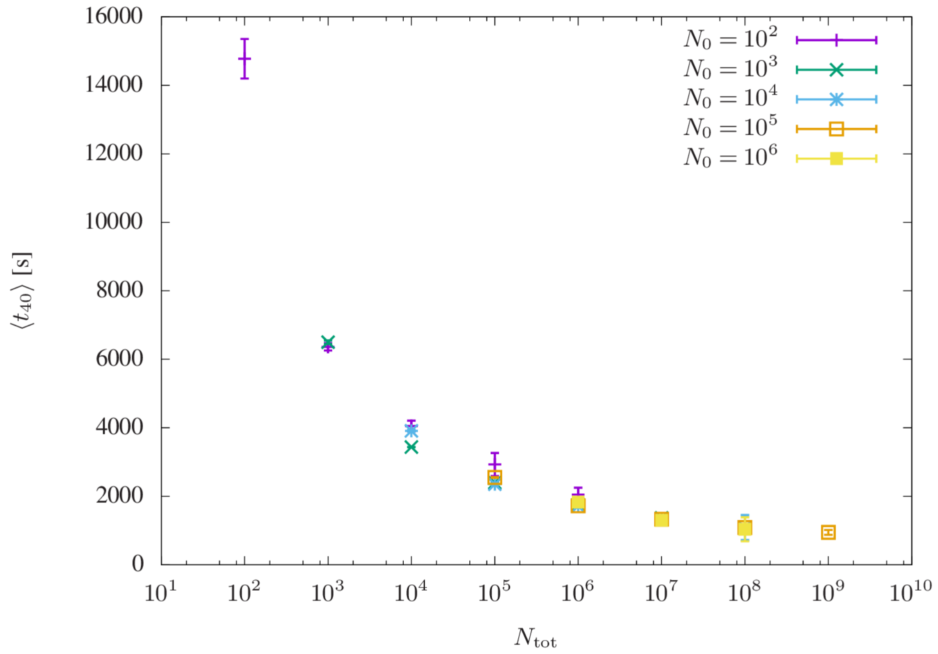
- mixing by sedimentation: time scale? works only in vertical
- typical LES cell: 10^3 m^3 - ensemble of well-mixed cells?
- consequences of assuming a large well-mixed cell - studied here
- size of an approximately well-mixed cell - future research

lucky droplets



	$\gamma = 10^{-4}$			$\gamma = 10^{-3}$			$\gamma = 10^{-2}$			$\gamma = 10^{-1}$			$\gamma = 1$		
N_0	$\langle t_{40} \rangle_\gamma$	$\sigma(t_{40})_\gamma$	$\gamma\Omega$	$\langle t_{40} \rangle_\gamma$	$\sigma(t_{40})_\gamma$	$\gamma\Omega$	$\langle t_{40} \rangle_\gamma$	$\sigma(t_{40})_\gamma$	$\gamma\Omega$	$\langle t_{40} \rangle_\gamma$	$\sigma(t_{40})_\gamma$	$\gamma\Omega$	$\langle t_{40} \rangle_\gamma$	$\sigma(t_{40})_\gamma$	$\gamma\Omega$
10^2	2052	212	10	2930	356	10	4053	517	10^2	6365	1158	10^3	14777	6099	10^3
10^3	1366	120	10^2	1762	170	10^3	2400	267	10^4	3440	505	10^5	6500	1700	10^6
10^4	1089	173	3	1336	103	10	1717	176	10^2	2354	276	10^3	3912	764	10^4
10^5	946	33	2	1090	60	20	1334	85	200	1721	169	2000	2552	415	10^4
10^6							1038	165	2	1301	176	20	1831	277	10^2



16000
14000
12000
10000
8000
6000
4000
2000
0
$\langle t_{40} \rangle$ [s]
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
10^{10}
$N_{\rm tot}$
$N_0 = 10^2$
$N_0 = 10^3$
$N_0 = 10^4$
$N_0 = 10^5$
$N_0 = 10^6$