Fourier Domain Regularized Inversion (FDRI) MATLAB code guide

Krzysztof M. Czajkowski, Anna Pastuszczak and Rafał Kotyński
University of Warsaw, Faculty of Physics

August 7, 2018

1 Introduction

This is a quick-start guide for Fourier Domain Regularized Inversion (FDRI) Matlab code which may be downloaded from https://www.igf.fuw.edu.pl/fdri. FDRI is a reconstruction method for compressive imaging based on the generalized inverse of the measurement matrix. The solution is regularized by minimizing norms of convolutions of the solution with a set of spatial filters. The method requires matrix inversion in order to obtain the regularized inverse matrix, but once the matrix is stored in the memory the reconstruction requires only single matrix by vector multiplication, which outperforms l1 minimization based methods in terms of reconstruction time. At the same time, it provides similar reconstruction quality.

If you find the code useful please cite the following paper:

- K. M. Czajkowski, A. Pastuszczak and R. Kotyński, *"Real-time single-pixel video imaging with Fourier domain regularization,"* Opt. Express Vol. 26, Issue 16, pp. 20009-20022 (2018), DOI:10.1364/OE.26.020009.

In the paper you can also find more details and examples of applications. Also included is a visualisation of the optical single pixel imaging with FDRI.

2 Theory

The problem that the code solves is image reconstruction from undersampled measurements. A typical single-pixel imaging scheme can be written in form of $y = Mx$, where x is vectorized image, M is a measurement matrix with vectorized sampling patterns as rows. Rather than minimizing l1 norm we consider the minimization of the squared l2 norm of convolution of x with a filter h, which in Fourier domain can be expressed in quadratic form

$$E(x) = x^T \cdot C \cdot x = \hat{x}^T \cdot \hat{C} \cdot \hat{x}, \tag{1}$$

with carret denoting Fourier transform and $\hat{C}_{i,j} = \delta_{i,j}|\hat{h}_i|^2$ The minimization is constrained by $y = Mx$.

We propose to use a set of filters $\hat{h}_i$ that lead to the following diagonal form of matrix C expressed in the Fourier space:

$$diag(\hat{C}) = (1-\mu)^2 \left(\sin^2(\omega_x) + \sin^2(\omega_y)\right) + \mu^2 \frac{\omega_x^2 + \omega_y^2}{2\pi^2} + \varepsilon, \tag{2}$$

with μ and ε as parameters and $\omega_{x,y}$ as spatial frequencies.

In our method the image is reconstructed by multiplying the measurement vector y by reconstruction matrix P defined as

$$P = C^{-1} \cdot M^* \cdot (M \cdot C^{-1} \cdot M^*)^{-1}, \tag{3}$$

which can be also cast to Moore-Penrose pseudoinverse using Fourier transform matrix (denoted as F)

$$P = F^* \cdot \hat{\Gamma} \cdot F \cdot (M \cdot F^* \cdot \hat{\Gamma} \cdot F)^+, \tag{4}$$

where $\hat{\Gamma} = \hat{C}^{-2}$.

3 Prerequisites and contents

The code requires Matlab with Image Processing Toolbox or Octave with "image" package (can be downloaded from octave-forge) and an image database (example of which is provided in "images" folder). 8 GB RAM workstation should be sufficient for operation of the example script, although 16 GB RAM or more is recommended. The provided package contains the following Matlab files:

- example_fdri.m - example script, which tests the method at 3% compression ratio and measurement utilizing binarized DCT basis functions.
- fdri.m - main function that calculates the generalized inverse of the measurement matrix using the FDRI method.
- AvgDCTSpectrum.m - calculates the magnitude of the DCT spectrum averaged over an image database with user-specified path to images and images are called 'n.gif' with n=1..49. Images are first resized to user provided size. If the image database is not found, a simple model is assumed for the distribution of the DCT spectrum instead.
- SelectionMatrix.m - a function that creates a (logical) selection matrix SM for the DCT basis with k true elements at the selected DCT basis. Selection is random with with the Bernoulli probabilities proportional to the previously found average DCT spectrum (AvgDCT) or can be internally switched for deterministic choice of the highest DCT coefficients.
- MeasurementMatrix.m - a function that returns the measurement matrix M with rows containing DCT basis selected by the elements of selection matrix SM.

4 Quick start

A basic application example is provided in example_fdri.m script. The user provides image resolution (square image is assumed), number of basis functions, parameter μ that controls the filter type ($\mu = 1$ corresponds to high-frequency penalizing filter, while $\mu = 0$ corresponds to gradient filter), binarization flag and paths to image database and test image. We find that for 256x256 image resolution as little as 2000 DCT basis functions are sufficient for a reasonable image reconstruction quality. Please note, that when the number of basis of function is large, the method will be infeasible due to the increasing memory requirements. We have tested that the optimal μ parameter value for natural images is approximately 0.5.

One does not have to use DCT basis functions for measurement. The main function fdri returns FDRI reconstruction matrix given the following parameters:

- M - measurement matrix with vectorized sampling patterns as rows
- Nx - image width in pixels
- Ny - image height in pixels
- mi - μ parameter in Eq. 2 (default value - 0.5)
- eps - ε parameter in Eq. 2 (default value - 10^{-5})
- method - calculation method: for 0 matrix inverse is used, for 1 FDRI matrix is calculated with pinv, for 2 SVD is used (default value - 0).

5 GPL Licence

The software is licensed upon the GPLv3 license:

6 Contact

In case of queries please contact:

- Krzysztof M. Czajkowski - krzysztof.czajkowski@fuw.edu.pl
- Anna Pastuszczak - apast@igf.fuw.edu.pl
- Rafał Kotyński - rafal.kotynski@fuw.edu.pl