Session 6

1. Derive the profile of the wind velocity in the atmospheric Ekman layer assuming the geostrophic wind is independent of height:

$$\begin{aligned} u &= u_g[1 - e^{-\gamma z}\cos\gamma z] - v_g e^{-\gamma z}\sin\gamma z \\ v &= v_g[1 - e^{-\gamma z}\cos\gamma z] + u_g e^{-\gamma z}\sin\gamma z \end{aligned}$$

where $\gamma = \sqrt{\frac{f}{2K}}$ and K is eddy diffusivity. Assuming the geostrophic wind is purely zonal ($v_g = 0$):

(a) Find the lowest height z_{Ek} at which the wind is parallel to the geostrophic wind. It conventionally designates the top of the Ekman layer. Calculate its value at 43^o N using the eddy viscosity coefficient $K = 5$ m^2 s^{-1}.

(b) Calculate the profiles of x and y components of the friction force:

$$\begin{aligned} \tau_x &= \eta\frac{\partial u}{\partial z} \\ \tau_y &= \eta\frac{\partial v}{\partial z} \end{aligned}$$

where η is dynamic viscosity. Find the direction of the friction force at the z_{Ek} level with respect to the geostrophic wind.

(c) Determine the limiting angle at the ground between the actual wind and the geostrophic wind.

(d) Determine the direction of the net mass transport within the Ekman layer.

(e) Show that vertical velocity at the top of atmospheric Ekman layer can be written as

$$w = \frac{1}{2f\gamma}\nabla_z^2\Phi$$