

CLOUD PHYSICS - tutorial 3 Activation

Köhler and κ -Köhler curves

1. Kelvin effect

The exact form of equilibrium saturation over a water droplet is given by:

$$S_{ex} = \exp\left(\frac{A(T)}{r}\right) \quad \text{where} \quad A(T) = \frac{2\sigma}{\rho_l R_v T}$$

The approximate form of equilibrium saturation over a water droplet is:

$$S_{approx} = 1 + \frac{A(T)}{r}$$

Plot $S_{ex}(r)$ and $S_{approx}(r)$ for a given temperature (e.g. $T = 0^\circ C$). Calculate the relative error $(S_{ex} - S_{approx})/S_{ex}$. For which values of r the relative error is smaller than 1%, 0.1%?

Assume that the surface tension of water is independent of temperature.

2. Raoult effect

The equilibrium saturation ratio over an aqueous solution droplet is given by:

$$S = a_w \exp\left(\frac{A(T)}{r}\right)$$

where a_w is the activity of water in solution (the Raoult effect) and is given by:

$$a_w = \frac{r^3 - r_d^3}{r^3 - r_d^3(1 - \kappa)}$$

or in approximative form as:

$$a_w = 1 - \kappa \frac{r_d^3}{r^3}$$

r is the radius of a droplet, r_d is the dry radius that describes the amount of solute (CCN), κ is a hygroscopicity parameter.

Plot $a_w(r)$ showing the Raoult effect for NaCl ($\kappa = 1.28$) and NH_4NO_3 (ammonium nitrate, *azotan amonu*, $\kappa = 0.67$), and different values of dry radii ($r_d = 0.02, 0.05, 0.1 \mu\text{m}$).

3. Köhler curve

The κ -Köhler form:

$$S(r, \kappa, r_d, T) = \frac{r^3 - r_d^3}{r^3 - r_d^3(1 - \kappa)} \exp\left(\frac{A(T)}{r}\right)$$

Plot the Köhler curves for NaCl and NH_4NO_3 :

- $r_d = 0.01, 0.03, 0.1 \mu\text{m}$
- $T = 0, 20^\circ \text{C}$

For sufficiently big droplets (let's call them R) the equilibrium saturation converges to 100%. It means that the curvature and solute terms of Köhler equation become unimportant (assume two scenarios: the equilibrium saturation is less than 0.1% or 0.01%) For $\kappa = 1.28$ and $\kappa = 0.2$ plot relations between dry radii and R .

4. Critical radius and critical saturation

Show how critical radii and critical supersaturations (the maximum of the Köhler curve) depend on dry radii, r_d , and hygroscopicity parameter, κ . Assume a constant value of temperature.