Extra exercises for midterm exam

1. Derive the hypsometric equation. Derive the thermal wind equation.

2. Find the geopotential profile $\Phi(p)$ in a hydrostatic atmosphere of uniform temperature. What is the geopotential height at the $p = 300$ hPa surface for the temperature $T = 250$ K and surface pressure $p_s = 1000$ hPa? Find the geopotential profile $\Phi(p)$ in a hydrostatic atmosphere of constant potential temperature θ.

3. Assuming that the geostrophic wind is purely zonal ($v_g = 0$) and independent of height, the wind profile in the atmospheric Ekman layer reads

$$\begin{aligned} u &= u_g[1 - e^{-\gamma z}\cos(\gamma z)], \\ v &= u_g e^{-\gamma z}\sin(\gamma z), \end{aligned}$$

   where $\gamma = \sqrt{f/(2A)}$ and A is the eddy diffusivity.

   (a) Find the lowest height at which the wind is parallel to the geostrophic wind. This is called the **gradient wind level**.

   (b) Calculate the profiles of x and y components of the friction force,

$$\tau_x = \rho A \frac{\partial u}{\partial z}, \quad \tau_y = \rho A \frac{\partial v}{\partial z}.$$

   (c) Determine the limiting angle at the ground (i.e., for $\gamma z \ll 1$) between the actual wind and the geostrophic wind.

   (d) Evaluate the frictionally induced **ageostrophic transport** (U, V), defined as

$$U = \int_0^\infty (u - u_g)\,\mathrm{d}z, \quad V = \int_0^\infty (v - v_g)\,\mathrm{d}z.$$

   How is the ageostrophic transport oriented with respect to the friction force (τ_x, τ_y) at the gradient wind level?