

LECTURE OUTLINE

1. Ways of reaching saturation

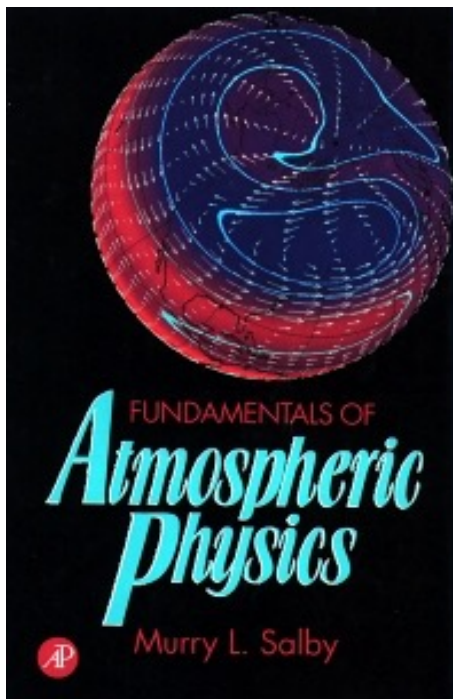
- vertical motion



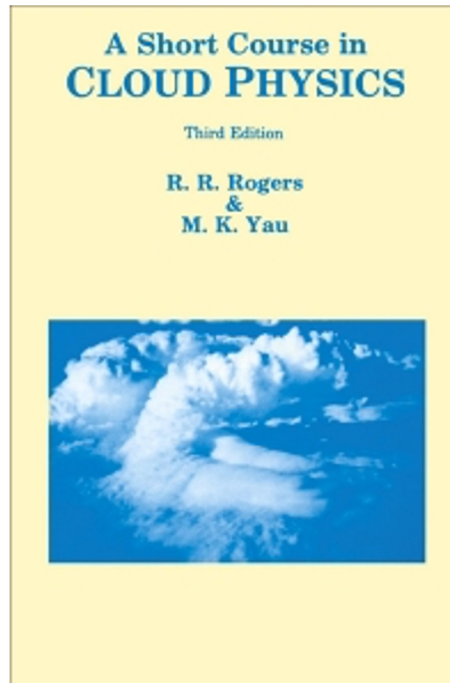
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R&Y, Chapter 4

Salby, Chapter 5

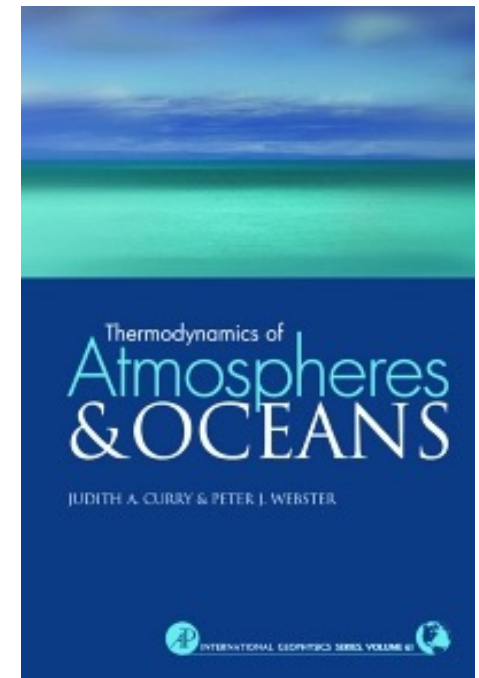


Fundamentals of Atmospheric Physics,
M.L. Salby; [Salby](#)

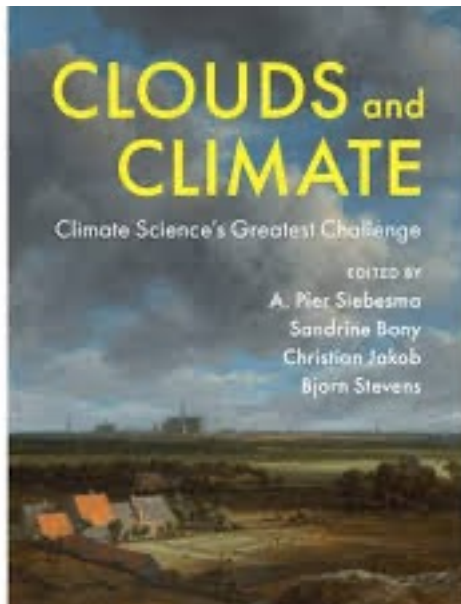


A Short Course in Cloud Physics,
R.R. Rogers and M.K. Yau; [R&Y](#)

C&W, Chapter 6



Thermodynamics of Atmospheres
and Oceans,
J.A. Curry and P.J. Webster; [C&W](#)



Chapter 2: Clouds as Fluids

Siebesma, A., Bony, S., Jakob, C., & Stevens, B. (Eds.). (2020). *Clouds and Climate: Climate Science's Greatest Challenge*. Cambridge: Cambridge University Press. doi:10.1017/9781107447738

THERMODYNAMICS IN VERTICAL MOTION

1. Lifting condensation level (LCL)
2. Dew-point temperature variation in vertical motion
3. Pseudo-adiabatic process
 - saturated adiabatic lapse rate
 - water condensed in pseudo-adiabatic process



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A parcel that moves vertically expands or contracts to preserve its mechanical equilibrium (adjusts its pressure to the environmental pressure).

It results in work being performed. Compensating this work is a change of internal energy, which alters the temperature and hence the saturation vapor pressure.

Saturation vapor pressure depends on temperature (assuming $L_{lv}=\text{const}$):

$$\ln\left(\frac{e_s}{e_{s0}}\right) = -\frac{L_{lv}}{R_v}\left(\frac{1}{T} - \frac{1}{T_0}\right)$$

The saturation specific humidity varies with pressure and temperature:

$$\frac{q_s}{q_{s0}} = \frac{e_s/e_{s0}}{p/p_0} = \frac{\exp\left[-\frac{L_{lv}}{R_v}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]}{\left(\frac{p}{p_0}\right)}$$

$$q_s = \varepsilon \frac{e_s}{p}$$
$$q_{s0} = \varepsilon \frac{e_{s0}}{p_0}$$

$$\frac{q_s}{q_{s0}} = \frac{e_s/e_{s0}}{p/p_0} = \frac{\exp \left[-\frac{L_{lv}}{R_v} \left(\frac{1}{T} - \frac{1}{T_0} \right) \right]}{\left(\frac{p}{p_0} \right)}$$

The saturation specific humidity in a parcel:

- increases with decreasing pressure (increase of altitude)
- decreases sharply with decreasing temperature, which likewise accompanies upward motion.

Even though an ascending parcel's pressure decreases exponentially with altitude, the temperature dependence prevails, so its saturation specific humidity decreases monotonically with altitude.

LIFTING CONDENSATION LEVEL



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Consider a moist (unsaturated) air parcel ascending in thermal convection.

Under unsaturated conditions, the parcel's specific humidity and saturation specific humidity satisfy $q_v < q_s$.

As it rises, the parcel performs work at the expense of its internal energy.

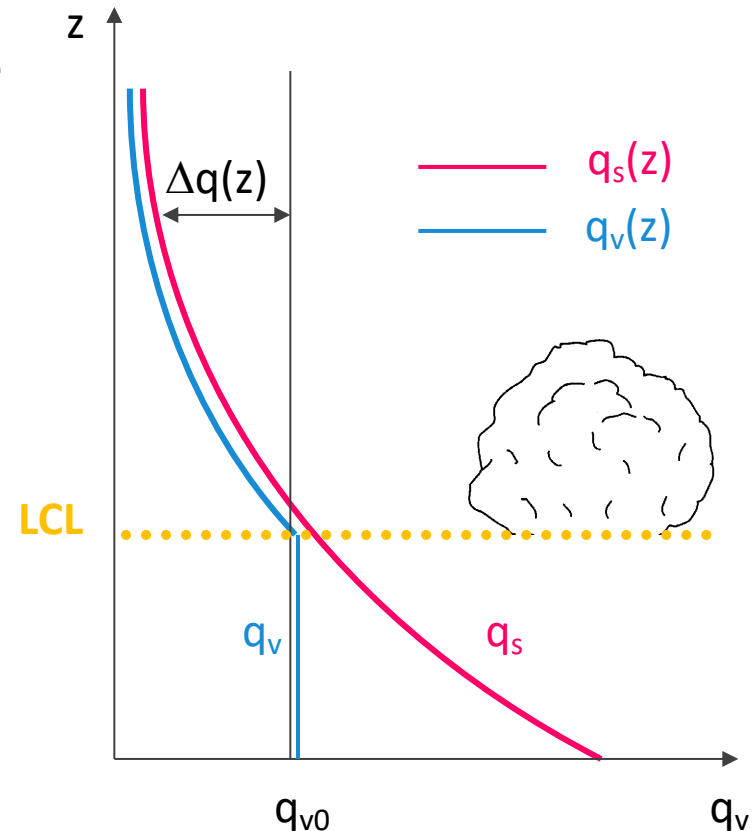
The parcel's temperature decreases at the dry adiabatic lapse rate Γ_d .

The decrease of temperature is attended by a reduction of saturation specific humidity q_s .

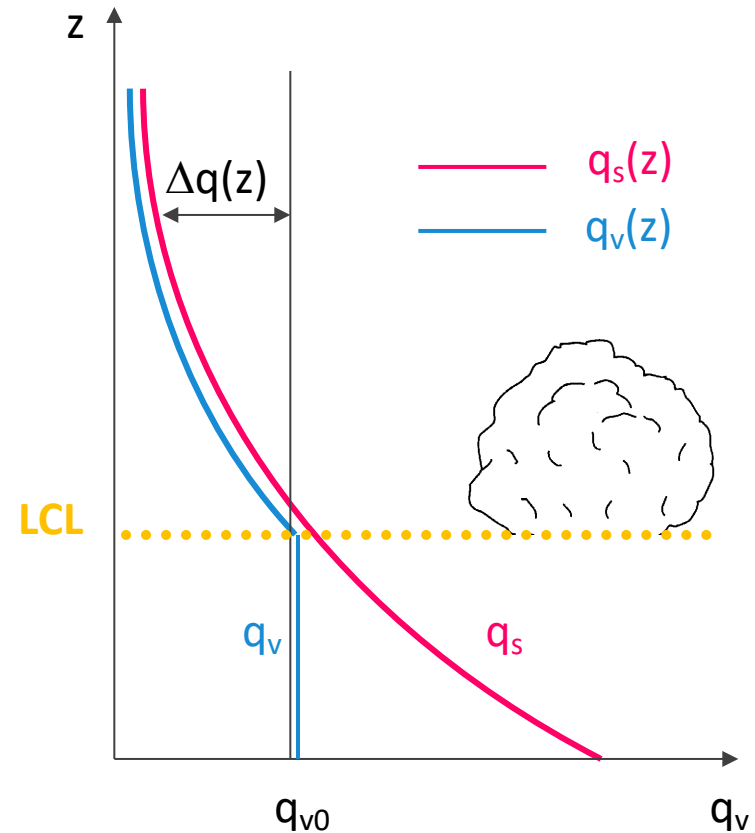
The parcel's actual specific humidity q_v and potential temperature θ remain constant.

Sufficient upward displacement will reduce the saturation specific humidity, q_s , to the actual specific humidity, q_v .

The elevation where $q_v = q_s$ for the first time is referred to as the **lifting condensation level (LCL)**.



The **lifting condensation level** defines the base of cumulus clouds that are fueled by air originating at the surface.



Below the LCL, the parcel's thermodynamic behavior can be regarded as **adiabatic** because the timescale for vertical motion (from minutes in cumulus convection to 1 day in sloping convection) is small compared to the characteristic timescale for heat transfer.

POSITION OF THE LCL

As air expands adiabatically and cools, the relative humidity (f) increases as the temperature and saturation mixing ratio decrease.

We will find coordinates of the LCL (T_{LCL}, p_{LCL}) – the point where the air becomes saturated.

The change of relative humidity fulfils the following equation: $f = \frac{e}{e_s}$

$$d(\ln f) = d(\ln e) - d(\ln e_s)$$

We will find how e and e_s depend on temperature in adiabatic ascent.

Using Dalton's law of partial pressure, that states that the total pressure exerted by a mixture of gases is equal to the sum of the partial pressures that would be exerted by each constituents alone if it filled the entire volume at the temperature of the mixture.

$$p_i = \frac{m_i R_i T}{V}$$
$$\frac{dp_i}{p_i} = \frac{dT}{T}$$

Therefore: $d(\ln p) = d(\ln e)$.

The First Law of thermodynamics for an adiabatic process in enthalpy form:

$$c_p dT = v dp \rightarrow c_p dT = \frac{RT}{p} dp$$

$$d(\ln p) = \frac{c_p}{R} d(\ln T)$$

Because $d(\ln p) = d(\ln e)$:

$$d(\ln e) = \frac{c_p}{R} d(\ln T)$$

Using the Clausius-Clapeyron equation:

$$d(\ln e_s) = \frac{L_{lv}}{R_v T} d(\ln T)$$

The change of relative humidity fulfils the following equation:

$$d(\ln f) = d(\ln e) - d(\ln e_s)$$

$$d(\ln f) = \frac{c_p}{R} d(\ln T) - \frac{L_{lv}}{R_v T} d(\ln T)$$

We will integrate the equation from initial conditions to conditions where saturation is attained, indicated by $f=1$ and $T = T_{LCL}$, where T_{LCL} . We will assume that $L_{lv}=\text{const}$.

$$\int_f^1 d(\ln f') = \int_T^{T_{LCL}} \left(\frac{c_p}{R} - \frac{\varepsilon L_{lv}}{RT} \right) d(\ln T')$$
$$-\ln f = \frac{c_p}{R} \ln \frac{T_{LCL}}{T} + \frac{\varepsilon L_{lv}}{R} \left(\frac{1}{T_{LCL}} - \frac{1}{T} \right)$$

Equation can be solved numerically to obtain T_{LCL} .

The saturation pressure can be obtained from the dry adiabat equation:

$$p_{LCL} = p \left(\frac{T_{LCL}}{T} \right)^{c_p/R}$$

An approximate equation for T_{LCL} given initial values of T and f, is given by Bolton (1980).

f	T (°C)	T_{LCL} (°C)	p_{LCL} (hPa)
0,1	0	-16	815
0,1	10	-7	806
0,1	20	2	797
0,1	30	10	788
0,3	0	-8	896
0,3	10	1	891
0,3	20	10	886
0,3	30	19	880
0,5	0	-5	938
0,5	10	5	935
0,5	20	14	932
0,5	30	24	929
0,7	0	-3	968
0,7	10	7	966
0,7	20	17	964
0,7	30	27	962

$$T_{LCL} = \frac{1}{\frac{1}{T - 55} - \frac{\ln f}{2840}} + 55, \quad [T] = K$$

VARIATION OF DEW-POINT TEMPERATURE WITH ALTITUDE



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During adiabatic ascent, the water vapor specific humidity, q_v , remains constant until saturation occurs.

The dew-point temperature decreases slightly during the ascent as pressure decreases.

We will calculate how the dew-point temperature changes during ascent of non-saturated adiabatic parcel.

The dew-point temperature fullfils the equation: $d \ln e = \frac{\varepsilon L_{lv}}{RT_d^2} dT_d$

The hydrostatic equation: $dp = -\rho g dz \rightarrow d \ln p = -\frac{g}{RT} dz$ $p = RT\rho$

From the Dalton's law: $d \ln p = d \ln e$ $d \ln e = -\frac{g}{RT} dz$

$$\frac{\varepsilon L_{lv}}{RT_d^2} dT_d = -\frac{g}{RT} dz$$

$$\frac{dT_d}{dz} = -\frac{T_d^2 g}{\varepsilon L_{lv} T} \cdot \frac{c_p}{c_p} \cong -\frac{T_d^2 c_p}{\varepsilon L_{lv} T} \cdot \Gamma_d$$

$$\Gamma_d = \frac{g}{c_{pd}} \approx \frac{g}{c_p}$$

For typical atmospheric values dT_d/dz is approximately 1/6 of the dry adiabatic lapse rate.

$$\frac{T_d^2 c_p}{\varepsilon L_{lv} T} \cong \frac{280^2 \cdot 1004}{0.622 \cdot 2.5 \cdot 10^6 \cdot 300} = 0.166 \approx 1/6$$

At saturation level, T becomes equal to T_d (and to T_{LCL}).

The **lifting condensation level (LCL)**, z_{LCL} corresponds to the level where water vapor becomes saturated.

Equation of change of the dew-point temperature can be written in form of a change of dew-point deficit: $T - T_d$

$$\frac{dT_d}{dz} = -\frac{T_d^2 c_p}{\varepsilon L_{lv} T} \cdot \Gamma_d \quad \rightarrow \quad \frac{dT}{dz} - \frac{dT_d}{dz} = -\Gamma_d + \frac{T_d^2 c_p}{\varepsilon L_{lv} T} \cdot \Gamma_d$$
$$\frac{d(T - T_d)}{dz} = -\left(1 - \frac{T_d^2 c_p}{\varepsilon L_{lv} T}\right) \cdot \Gamma_d$$

When $T = T_d$, the saturation level has been reached, and a value of z_{LCL} can be determined by integrating from initial values $(0, T_0 - T_{d0})$ to the saturation state $(z_{LCL}, 0)$. $T_0 - T_{d0}$ is the dew-point depression at the surface.

$$\int_{T_0 - T_{d0}}^0 d(T - T_d) = - \int_0^{z_{LCL}} \left(1 - \frac{T_d^2 c_p}{\varepsilon L_{lv} T}\right) \Gamma_d dz$$

For a parcel lifted from the surface, the value of z_{LCL} can be estimated (assuming $dT_d/dz = -1/6 \Gamma_d$):

$$z_{LCL} \approx 0.12(T_0 - T_{d0}) \quad [\text{km}]$$

Calculation of the lifting condensation level provides a good estimate of the cloud base height for clouds that form by adiabatic ascent.

ADIABATIC AND PSEUDO-ADIABATIC PROCESSES

1. Wet adiabatic lapse rate / saturated adiabatic lapse rate
2. Pseudo-adiabatic process
3. Water condensed in pseudo-adiabatic process



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WET ADIABATIC LAPSE RATE

If expansion work occurs fast enough the heat transfer with the environment remains negligible.

If no moisture precipitates out, the parcel is closed and its behavior above the LCL is described by a reversible saturated adiabatic process.

$$dh = c_p dT + L_{lv} dq_v$$

$$c_p dT + L_{lv} dq_v - v dp = 0$$

$$dq_v = dq_s = \frac{\partial q_s}{\partial T} dT + \frac{\partial q_s}{\partial p} dp$$

First Law of thermodynamics for adiabatic processes

$$dh = \delta q + v dp$$

Water vapor is saturated

$$q_v = q_s(T, p)$$

It is convenient to express the partial derivatives of q_s as logarithmic partial derivatives:

$$\frac{\partial q_s}{\partial T} = \frac{q_s}{T} \frac{\partial \ln q_s}{\partial \ln T} \quad \frac{\partial q_s}{\partial p} = \frac{q_s}{p} \frac{\partial \ln q_s}{\partial \ln p}$$

$$\beta_T = \frac{\partial \ln q_s}{\partial \ln T} \quad \beta_p = -\frac{\partial \ln q_s}{\partial \ln p}$$

$$dq_s = \frac{q_s}{T} \beta_T dT - \frac{q_s}{p} \beta_p dp$$

$$c_p dT + L_{lv} \frac{q_s}{T} \beta_T dT - L_{lv} \frac{q_s}{p} \beta_p dp - v dp = 0$$

$$\left(c_p + L_{lv} \frac{q_s}{T} \beta_T \right) dT + \left(L_{lv} \frac{q_s}{RT} \beta_p + 1 \right) g dz = 0$$

$$dp = -\rho g dz$$

$$v dp = -g dz$$

$$dp = -\frac{p}{RT} g dz$$

Saturated moist adiabatic lapse rate:

$$\Gamma_s = -\frac{dT}{dz} = g \frac{1 + q_s \beta_p \frac{L_{lv}}{RT}}{c_p + q_s \beta_T \frac{L_{lv}}{T}}$$

We will calculate β_p and β_T .

$$q_s = \frac{\varepsilon e_s}{p - (1 - \varepsilon)e_s} = \frac{\varepsilon \frac{e_s}{p}}{1 - (1 - \varepsilon) \frac{e_s}{p}}$$

$$\beta_p \quad \frac{\partial q_s}{\partial p} = -\frac{q_s}{p} \left(1 + \frac{1 - \varepsilon}{\varepsilon} q_s \right)$$

$$\beta_p = -\frac{\partial \ln q_s}{\partial \ln p} = \left[1 + \left(\frac{1}{\varepsilon} - 1 \right) q_s \right] \approx 1$$

$$\beta_T \quad \frac{\partial q_s}{\partial T} = \frac{1}{e_s} \frac{de_s}{dT} q_s + \frac{1 - \varepsilon}{\varepsilon} \frac{1}{e_s} \frac{de_s}{dT} q_s^2$$

$$\beta_T = \frac{\partial \ln q_s}{\partial \ln T} = T \frac{d \ln e_s}{dT} \left(1 + \frac{1 - \varepsilon}{\varepsilon} q_s \right) = T \frac{d \ln e_s}{dT} \beta_p$$

$$\beta_T = \frac{L_{lv}}{R_v T} \beta_p \approx \frac{5\,400\text{ K}}{T}$$

Clausius-Clapeyron equation

$$\frac{d \ln e_s}{dT} = \frac{L_{lv}}{R_v T^2}$$

SATURATED MOIST ADIABATIC LAPSE RATE

$$\Gamma_s = g \frac{1 + q_s \beta_T \frac{R_v}{R}}{c_p + q_s \beta_T \frac{L_{lv}}{T}}$$

$$\Gamma_s = \Gamma_d \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \frac{R_v}{R}}{1 + q_s \beta_T \frac{L_{lv}}{c_p T}}$$

$$\Gamma_d = \frac{g}{c_{pd}}$$

$$\Gamma_s \equiv -\frac{dT}{dz} = \gamma \Gamma_d$$

$$\gamma = \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \frac{R_v}{R}}{1 + q_s \beta_T \frac{L_{lv}}{c_p T}}$$

$$\gamma \leq 1$$

$$c_p = q_d c_{pd} + q_s c_{pv} + q_l c_l$$

$$R = q_d R_d + q_s R_v$$

$$\beta_T = \frac{L_{lv}}{R_v T} \beta_p \approx \frac{5\,400\text{ K}}{T}$$

If no moisture precipitates out, the parcel is closed and its behavior above the LCL is described by a reversible saturated adiabatic process.

The process depends weakly on the abundance of condensate present (e.g., on how much the system's enthalpy is represented by condensate).

Because the condensate is present only in trace abundance, the variation of condensate unnecessarily complicates the parcel's description under saturated conditions.

A simplification is proposed

Pseudo-adiabatic process: the system is treated as open and condensate is removed (added) immediately after (before) it is produced (destroyed).

A pseudo-adiabatic change of state may be constructed in two legs:

1. Reversible saturated adiabatic expansion (compression), which results in the production (destruction) of condensate of mass dm_c and a commensurate release (absorption) of latent heat to (from) the gas phase
2. Removal (addition) of condensate of mass dm_c

WET ADIABATIC VS PSEUDO-ADIABATIC LAPSE RATE

$$\Gamma_s = \gamma \Gamma_d \quad \gamma = \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \frac{R_v}{R}}{1 + q_s \beta_T \frac{L_{lv}}{c_p T}}$$

$$c_p = q_d c_{pd} + q_s c_{pv} + q_l c_l$$

$$R = q_d R_d + q_s R_v$$

Wet adiabatic lapse rate:

$$q_d + q_s + q_l = 1 \quad \text{and} \quad q_d = \text{const}, \quad q_s + q_l = \text{const}$$

Pseudo-adiabatic lapse rate:

$$q_l = 0, \quad q_d + q_s = 1$$

SIMPLIFIED VERSION OF Γ_s

In many textbooks a simplified version of Γ_s is presented.

$$R \rightarrow R_d$$

$$c_p \rightarrow c_{pd}$$

$$\Gamma_s = \gamma \Gamma_d \quad \gamma = \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \frac{R_v}{R}}{1 + q_s \beta_T \frac{L_{lv}}{c_p T}} \rightarrow \gamma = \frac{1 + \frac{q_s L_{lv}}{R_d T}}{1 + \frac{q_s L_{lv}^2}{c_{pd} R_v T^2}}$$

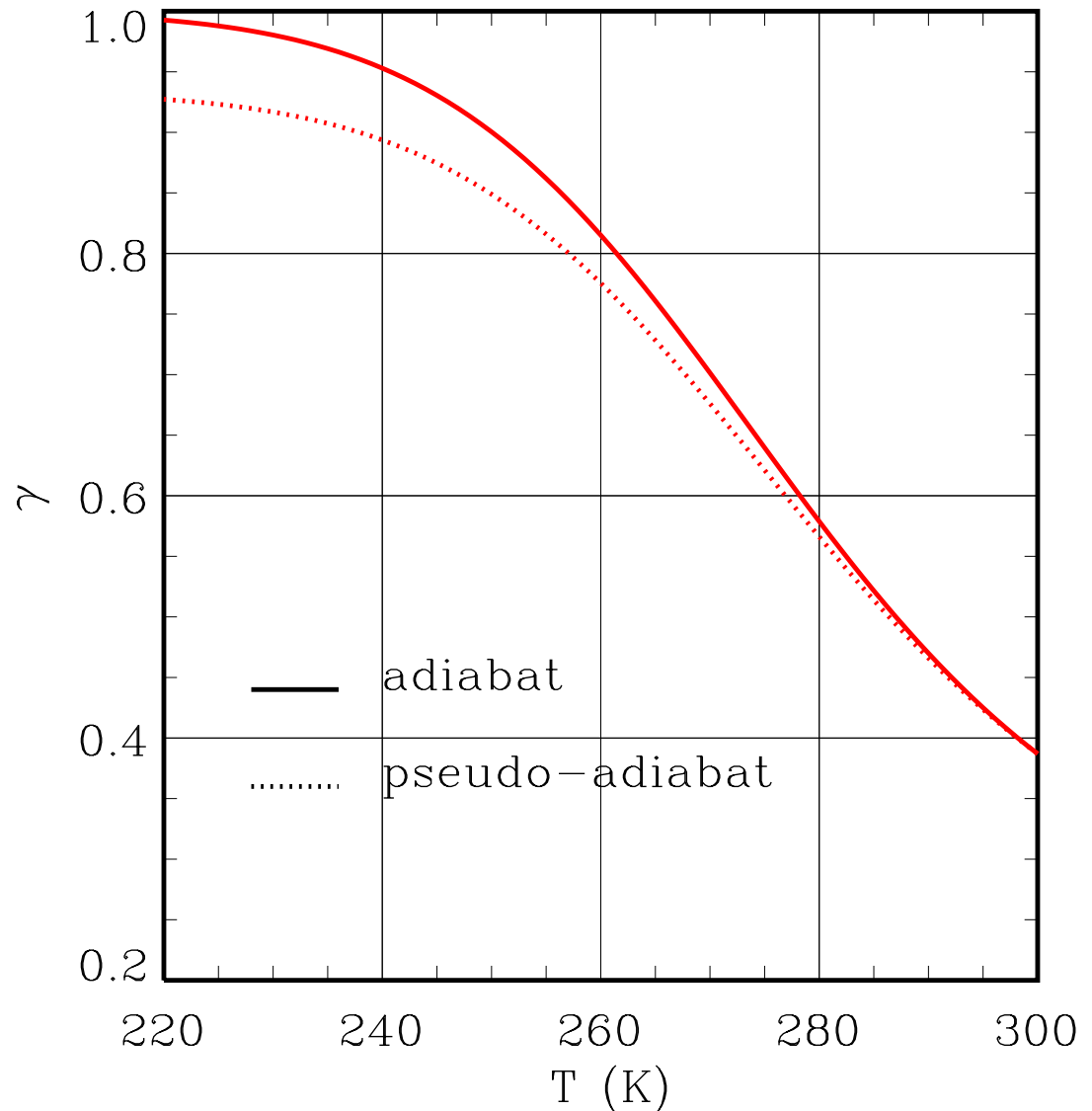
NON-DIMENSIONAL LAPSE RATE γ

- pressure: 1 000 hPa
- the air initially saturated at 300 K

$$\Gamma_s < \Gamma_d$$

$$\Gamma_s = \gamma \Gamma_d$$

$$\gamma = \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \frac{R_v}{R}}{1 + q_s \beta_T \frac{L_{lv}}{c_p T}}$$



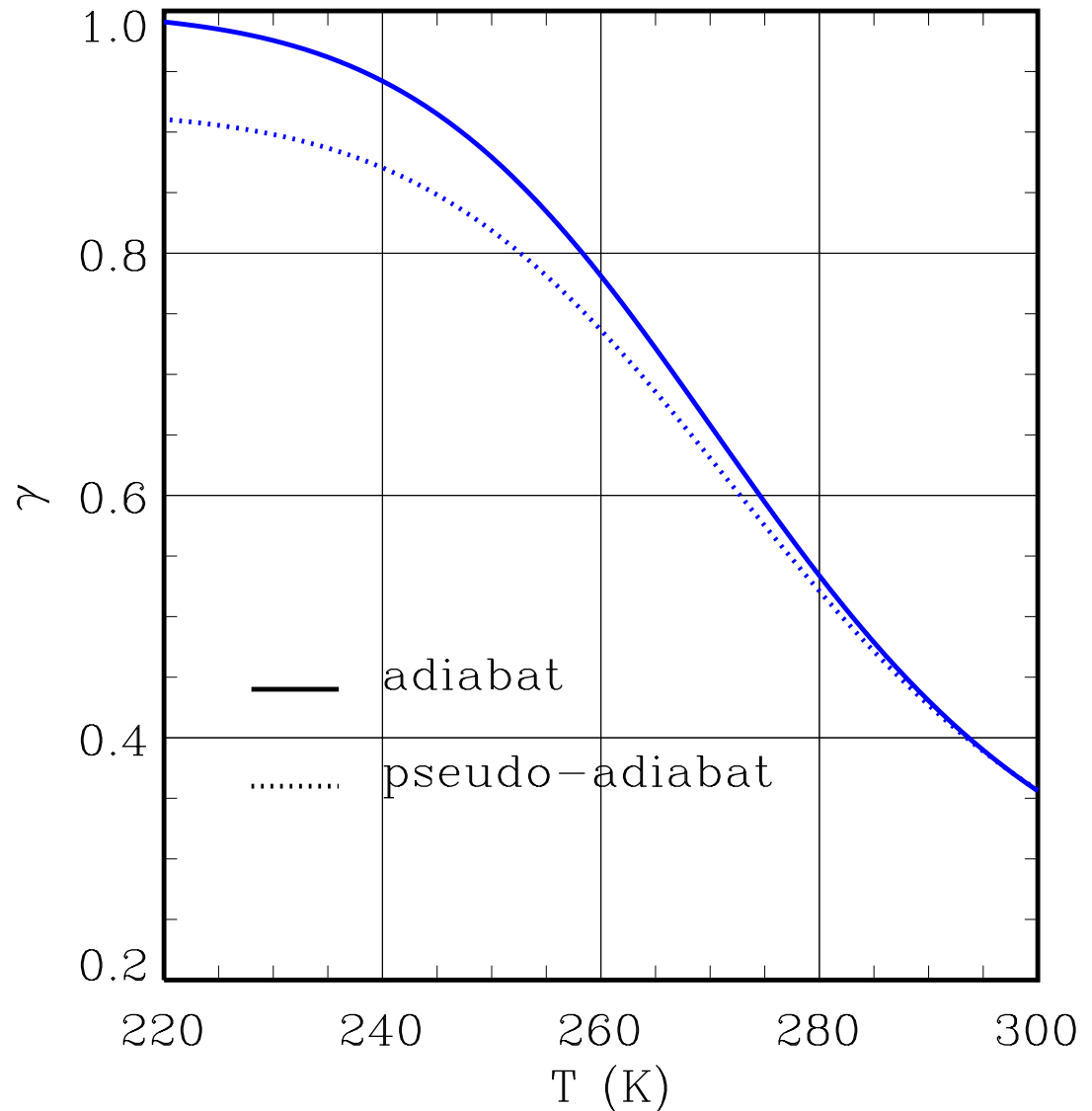
NON-DIMENSIONAL LAPSE RATE γ

- pressure: 800 hPa
- the air initially saturated at 300 K

$$\Gamma_s < \Gamma_d$$

$$\Gamma_s = \gamma \Gamma_d$$

$$\gamma = \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \frac{R_v}{R}}{1 + q_s \beta_T \frac{L_{lv}}{c_p T}}$$



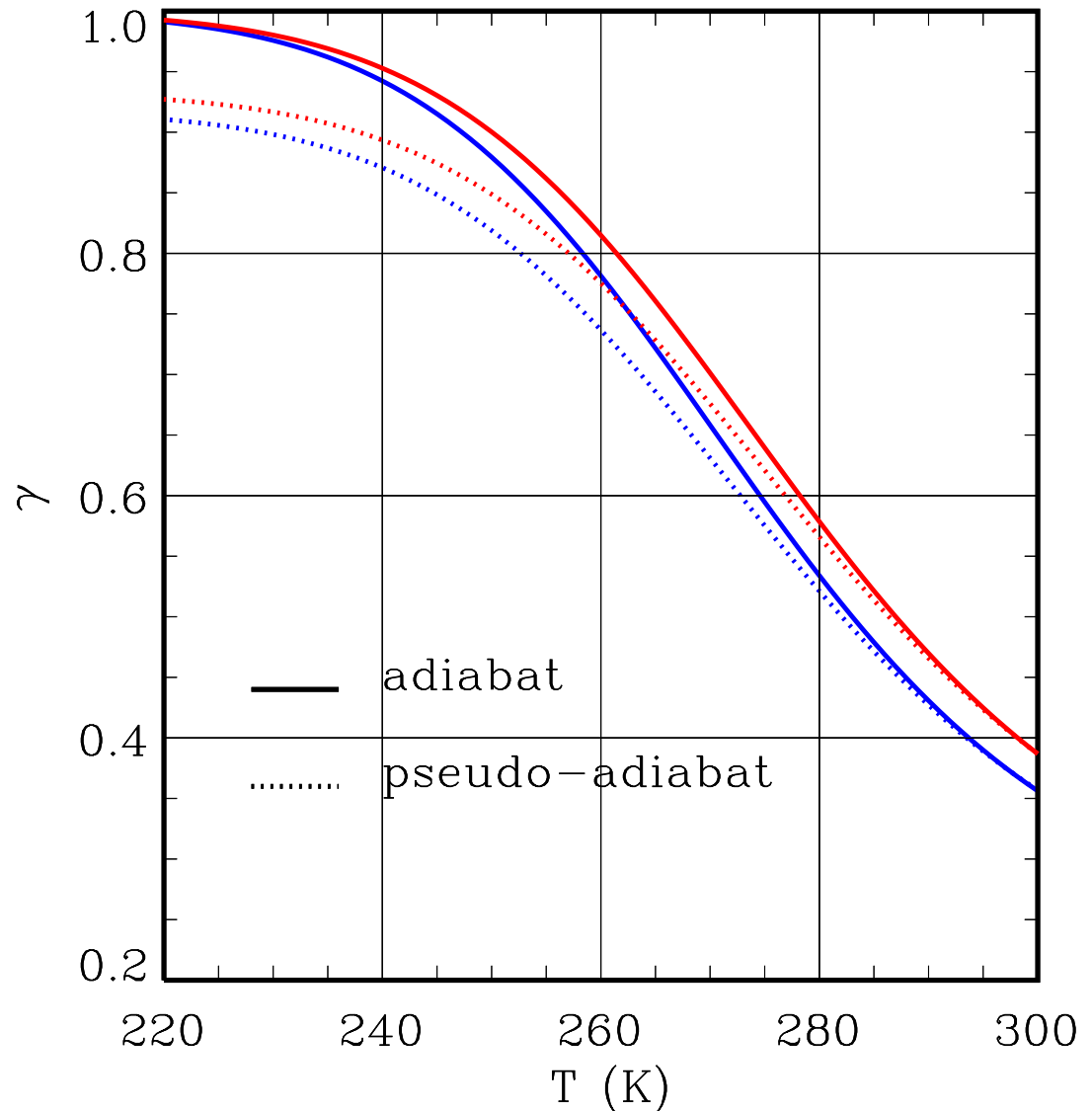
NON-DIMENSIONAL LAPSE RATE γ

- pressure: 1000 hPa
- pressure: 800 hPa
- the air initially saturated at 300 K

$$\Gamma_s < \Gamma_d$$

$$\Gamma_s = \gamma \Gamma_d$$

$$\gamma = \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \frac{R_v}{R}}{1 + q_s \beta_T \frac{L_{lv}}{c_p T}}$$



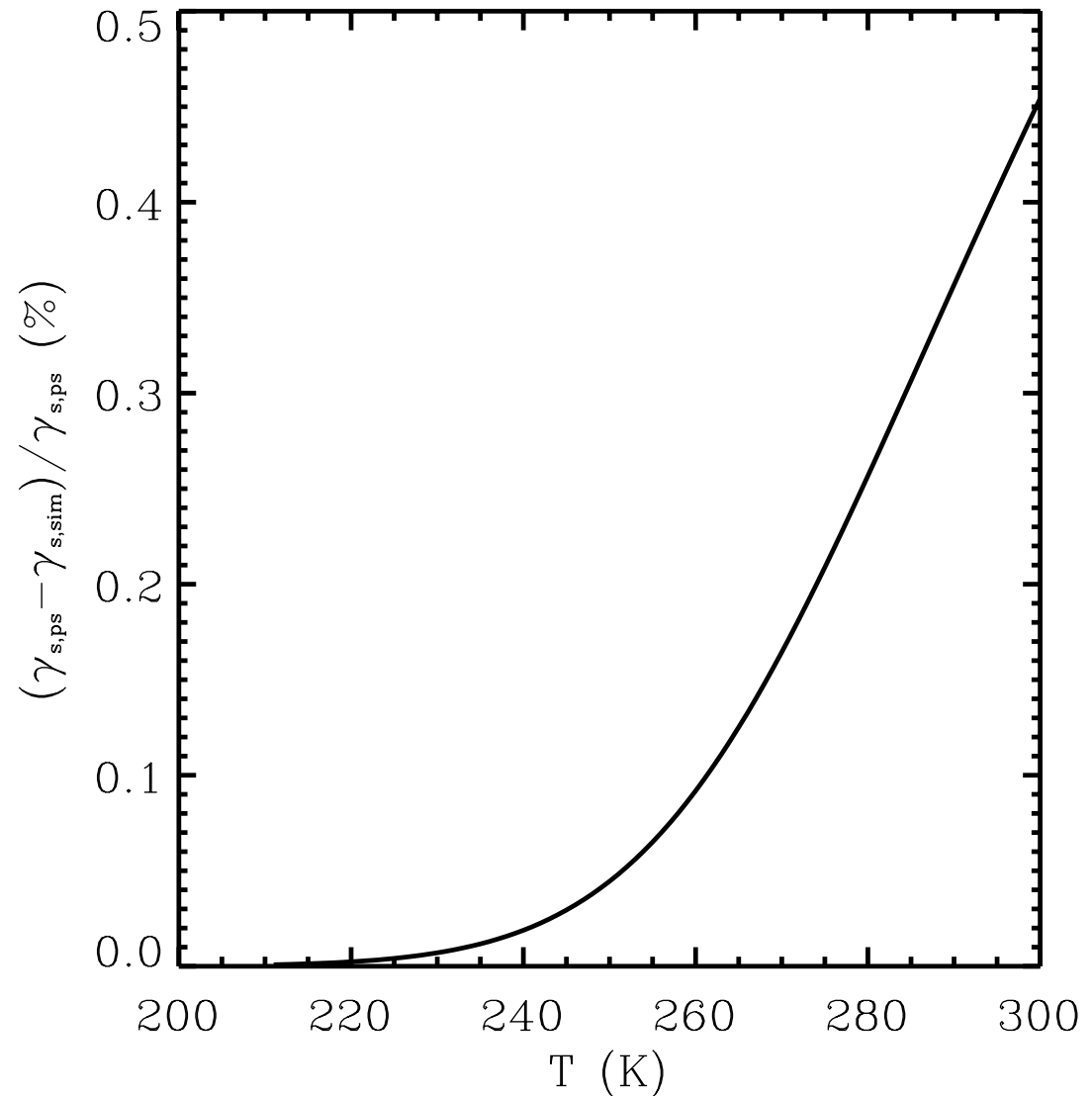
DIFFERENCE BETWEEN PSEUDO-ADIABATIC AND SIMPLIFIED PSEUDO- ADIABATIC NORMALISED LAPSE RATES

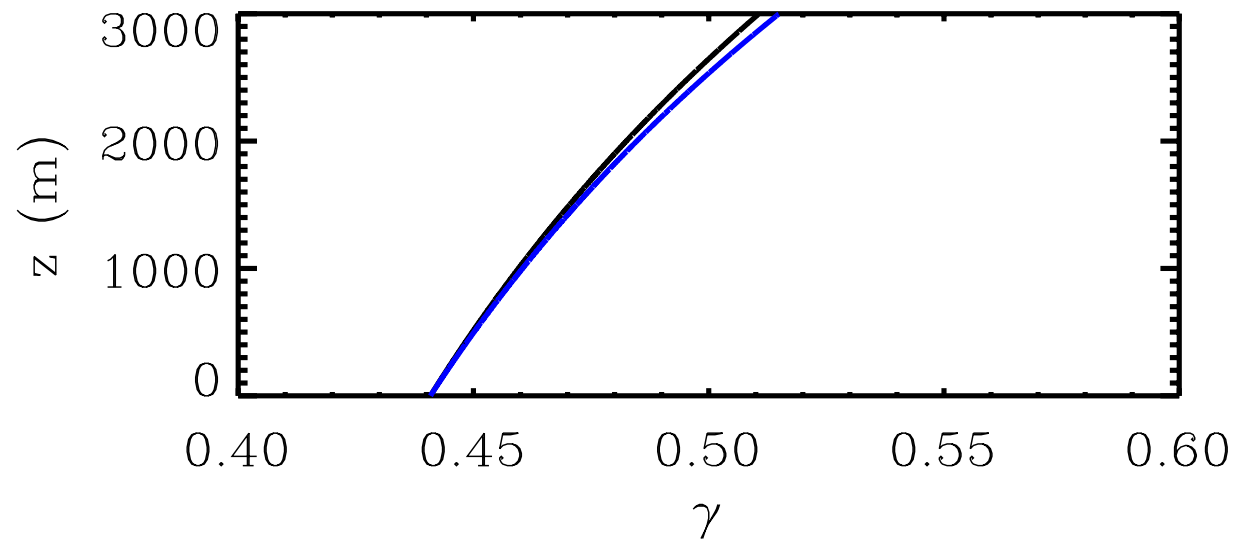
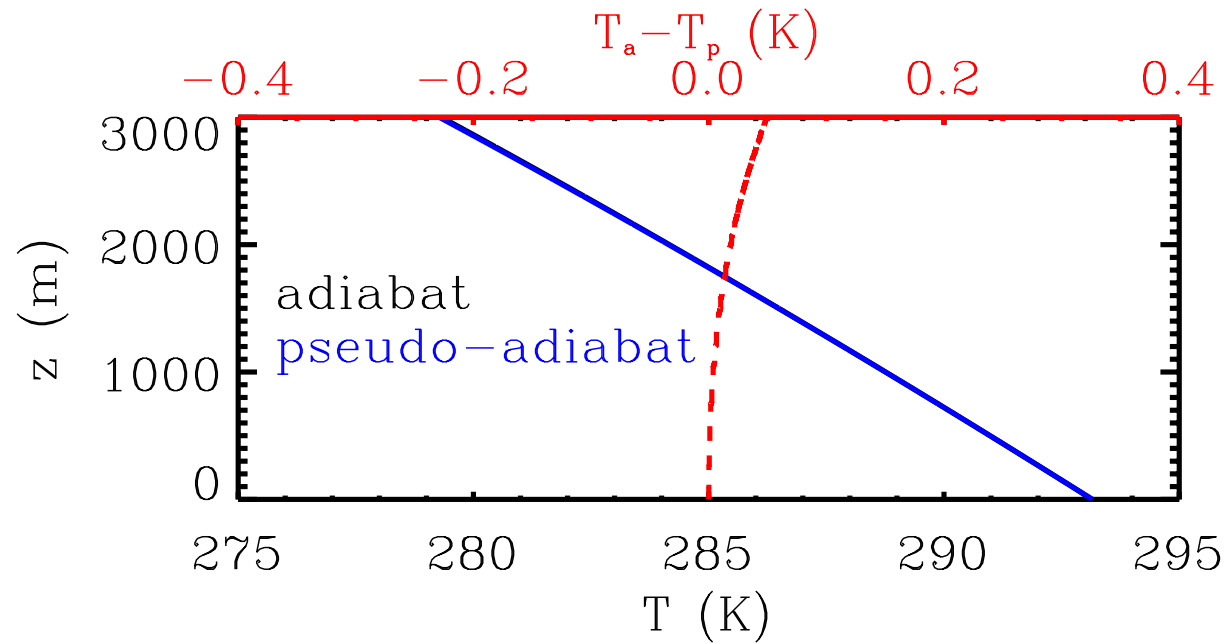
- pressure: 1 000 hPa
- the air initially saturated at 300 K

$$\Gamma_s < \Gamma_d$$

$$\Gamma_s = \gamma \Gamma_d$$

$$\gamma = \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \frac{R_v}{R}}{1 + q_s \beta_T \frac{L_{lv}}{c_p T}}$$





WATER CONDENSED IN ADIABATIC PROCESS

Adiabatic enthalpy equation of a closed system:
dry air, water vapor, condensed water:

$$dh = c_p dT + L_{lv} dq_s ; \quad dh = \delta q + v dp$$

$$0 = c_p dT + L_{lv} dq_s - v dp$$

$$dq_l = -dq_s = \frac{c_p}{L_{lv}} dT - \frac{v}{L_{lv}} dp$$

$$dq_l = \frac{c_p}{L_{lv}} dT + \frac{g}{L_{lv}} dz$$

$$dq_l = \frac{c_p}{L_{lv}} \left(\frac{dT}{dz} + \frac{g}{c_p} \right) dz$$

$$dq_l \cong \frac{c_p}{L_{lv}} (\Gamma_d - \Gamma_s) dz$$

$$dp = -\frac{g}{v} dz$$

$$\Gamma_d = \frac{g}{c_{pd}} \approx \frac{g}{c_p} , \quad \Gamma_s = -\frac{dT}{dz}$$

The amount of water condensed in a rising adiabatic parcel increases with the height above the cloud base and increases with increasing temperature at the cloud base.

For shallow clouds (cloud depth not bigger than ca. 300-500 m, for instance stratocumulus clouds) it can be assumed that the amount of condensed water increases linearly with height above the cloud base (h).

The rate of this increase (c_q) is approximately **constant** and depends on temperature and pressure at the cloud base.

$$q_l(h) = c_q(T, p) \cdot h; \quad c_q = \frac{c_p}{L_{lv}} (\Gamma_d - \Gamma_s) \left[\frac{g}{kg \cdot m} \right]$$

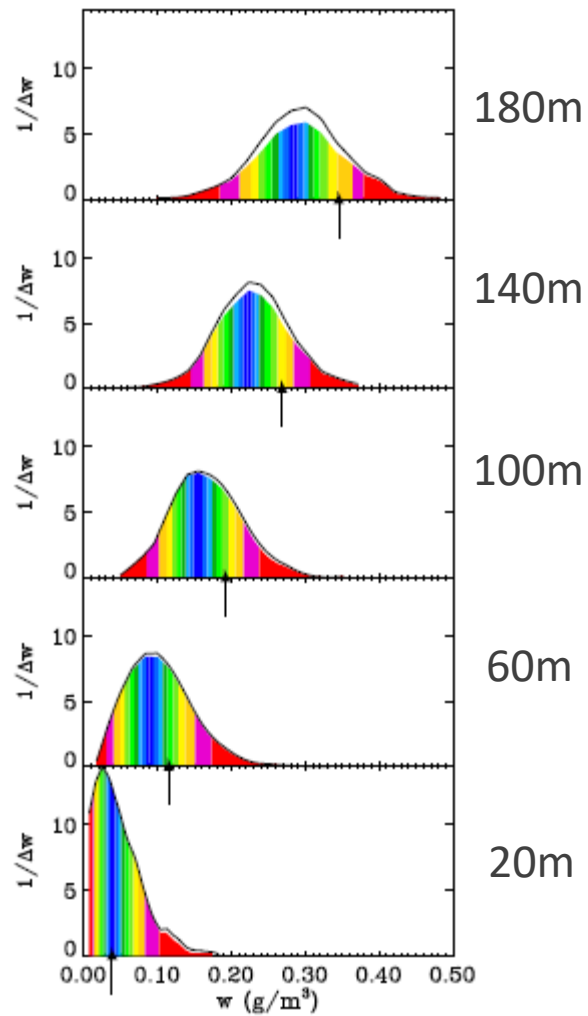
Liquid Water Content (LWC) is the amount of liquid water per unit volume:

$$LWC = q_l \cdot \rho; \quad \rho - \text{density of the air}$$

For shallow clouds one can assume that the air density is constant, therefore:

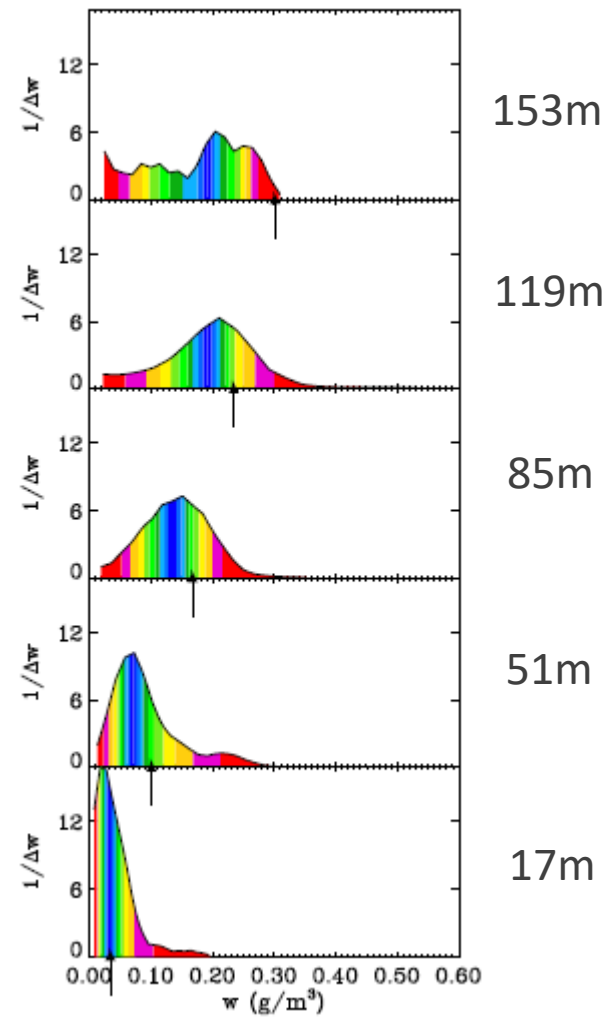
$$LWC = c_w(T, p) \cdot h; \quad c_w = \rho \frac{c_p}{L_{lv}} (\Gamma_d - \Gamma_s) \left[\frac{g}{m^4} \right]$$

$N=50 \text{ cm}^{-3}$
 $H_{\text{base}}=1278 \text{ m}$



$$c_w = 1.9 \cdot 10^{-3} \frac{g}{m^4}$$

$N=255 \text{ cm}^{-3}$
 $H_{\text{base}}=844 \text{ m}$



$$c_w = 2 \cdot 10^{-3} \frac{g}{m^4}$$



Modeling microphysical effects of entrainment in clouds observed during EUCAARI-IMPACT field campaign

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