STABILITY IN THE ATMOSPHERE

UNIVERSITY
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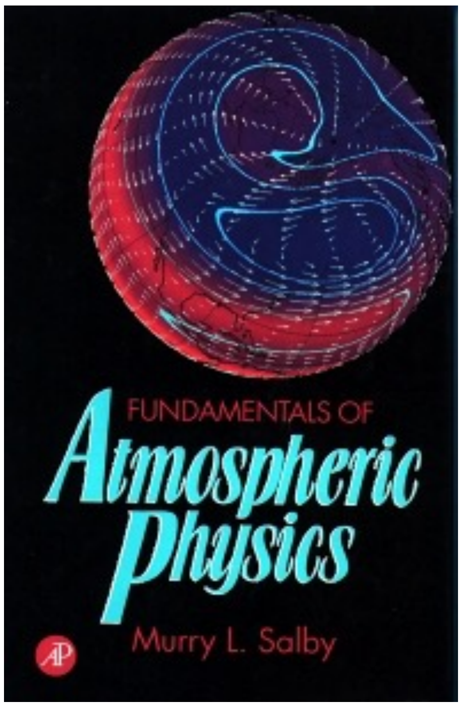
LECTURE OUTLINE

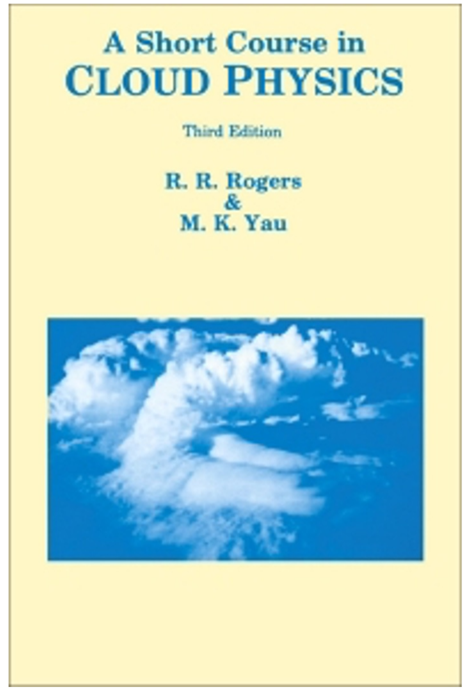
1. Stratifiction
2. Hydrostatic stability; parcel method

Salby, Chapter 6



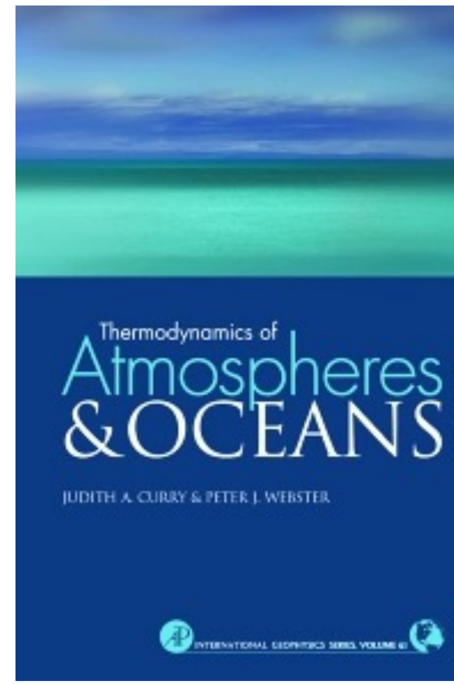
Fundamentals of Atmospheric Physics,
M.L. Salby; Salby

R&Y, Chapter 4



A Short Course in Cloud Physics,
R.R. Rogers and M.K. Yau; R&Y

C&W, Chapter 7



Thermodynamics of Atmospheres and Oceanes,
J.A. Curry and P.J. Webster; C&W

Stratification

The hydrostatic balance can be expressed by: $vdp = -gdz$

The ideal gas law transforms this into: $dz = -\frac{R_d T}{g} d \ln p$

Neither Γ_d nor Γ_s has a direct relationship to the temperature of the surroundings because a displaced parcel is thermally isolated under adiabatic conditions.

Thermal properties of the environment are dictated by the history of air residing at a given location, for example, by where that air has been and what thermodynamic influences have acted on it.

The environmental lapse rate is defined as: $\Gamma = -\frac{dT}{dz}$ where T refers to the ambient temperature

The hydrostatic balance equation can be expressed as:

$$\frac{d \ln T}{d \ln p} = \frac{R_d}{g} \Gamma$$

$$\frac{d \ln T}{d \ln p} = \frac{\Gamma}{\Gamma_d} \kappa$$

$$\Gamma_d = \frac{g}{c_{pd}}$$

$$\kappa = \frac{R_d}{c_{pd}}$$

Lagrangian interpretation of stratification - 1

- Hydrostatic equilibrium applies in the presence of motion as well as under static conditions.
- Interpreting thermal structure in terms of the behavior of individual air parcels provides some insight into mechanisms controlling mean stratification.
- For a layer of constant lapse rate, the relationship between temperature and pressure

$$\frac{d\ln T}{d\ln p} = \frac{\Gamma}{\Gamma_d}\kappa \longrightarrow \frac{T}{T_s} = \left(\frac{p}{p_s}\right)^{\kappa\left(\Gamma/\Gamma_d\right)}$$

resembles one implied by Poisson's relation $Tp^{-\kappa} = const$

Lagrangian interpretation of stratification -2

For polytropic process

$$\delta q = c dT$$

$$(c_v - c)dT + pdv = 0 \qquad c_p \longrightarrow (c_p - c)$$

$$(c_p - c)dT - vdp = 0 \qquad c_p \longrightarrow (c_p - c)$$

Air parcels moving vertically exchange heat with their surroundings in such proportion for their temperatures varies linearly with height.

$$\kappa \longrightarrow \frac{R}{c_p - c} = \frac{g}{c_p}\frac{c_p}{g}\frac{R}{c_p - c} = \kappa\frac{\Gamma}{\Gamma_d} \qquad \longrightarrow \qquad c = c_p\left(1 - \frac{\Gamma_d}{\Gamma}\right)$$

$$\Gamma = const \neq 0$$

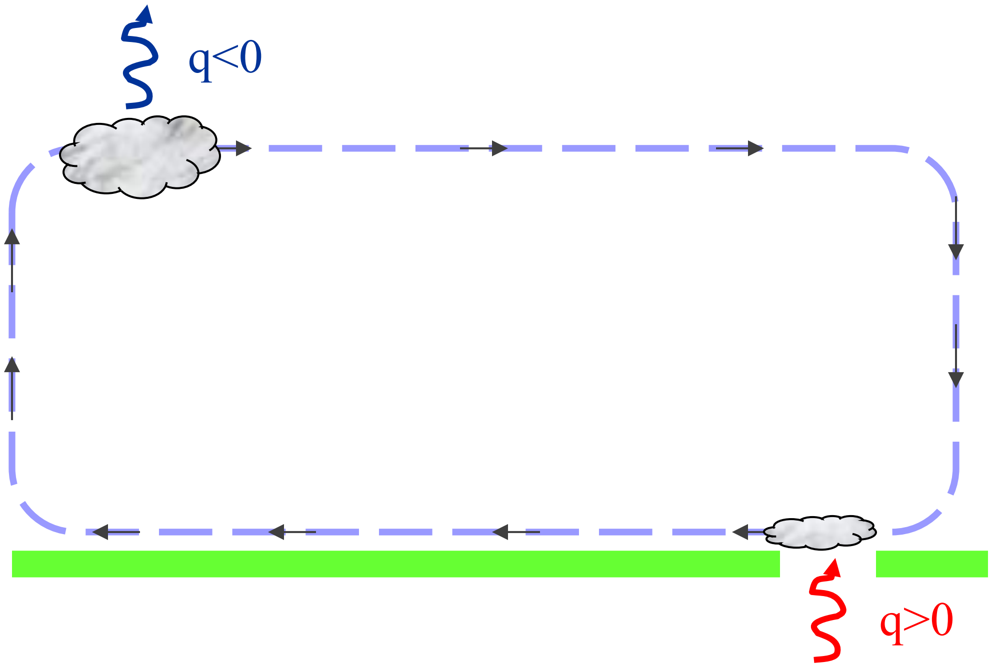
$$\frac{T}{T_s} = \left(\frac{p}{p_s}\right)^{\kappa\left(\Gamma/\Gamma_d\right)}$$

Adiabatic stratification (1)

$$\Gamma = \Gamma_d$$

$$c = c_p\left(1 - \frac{\Gamma_d}{\Gamma}\right) \longrightarrow c = 0$$

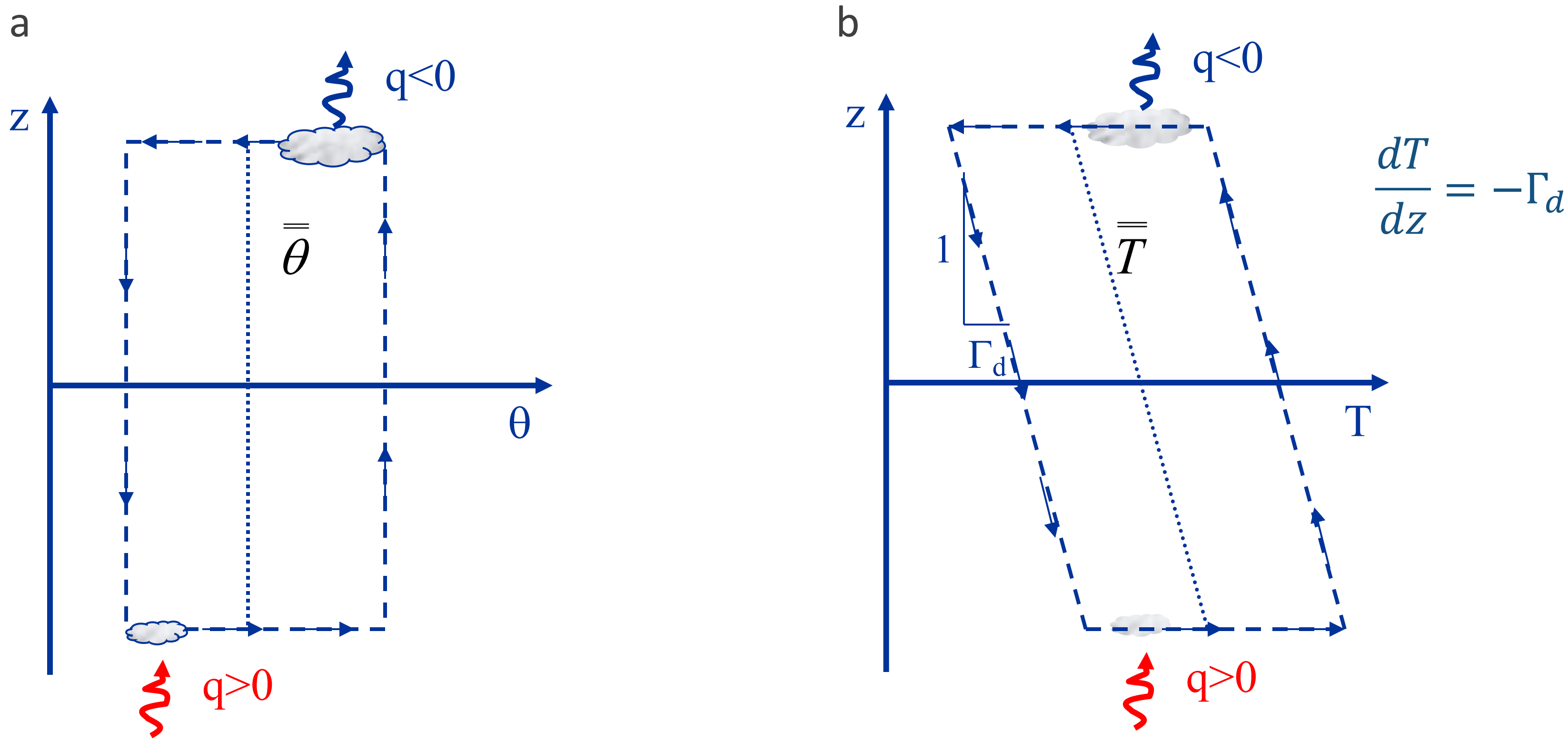
$$\delta q = \delta\theta = 0$$



$$\overline{\theta(z)} = const$$

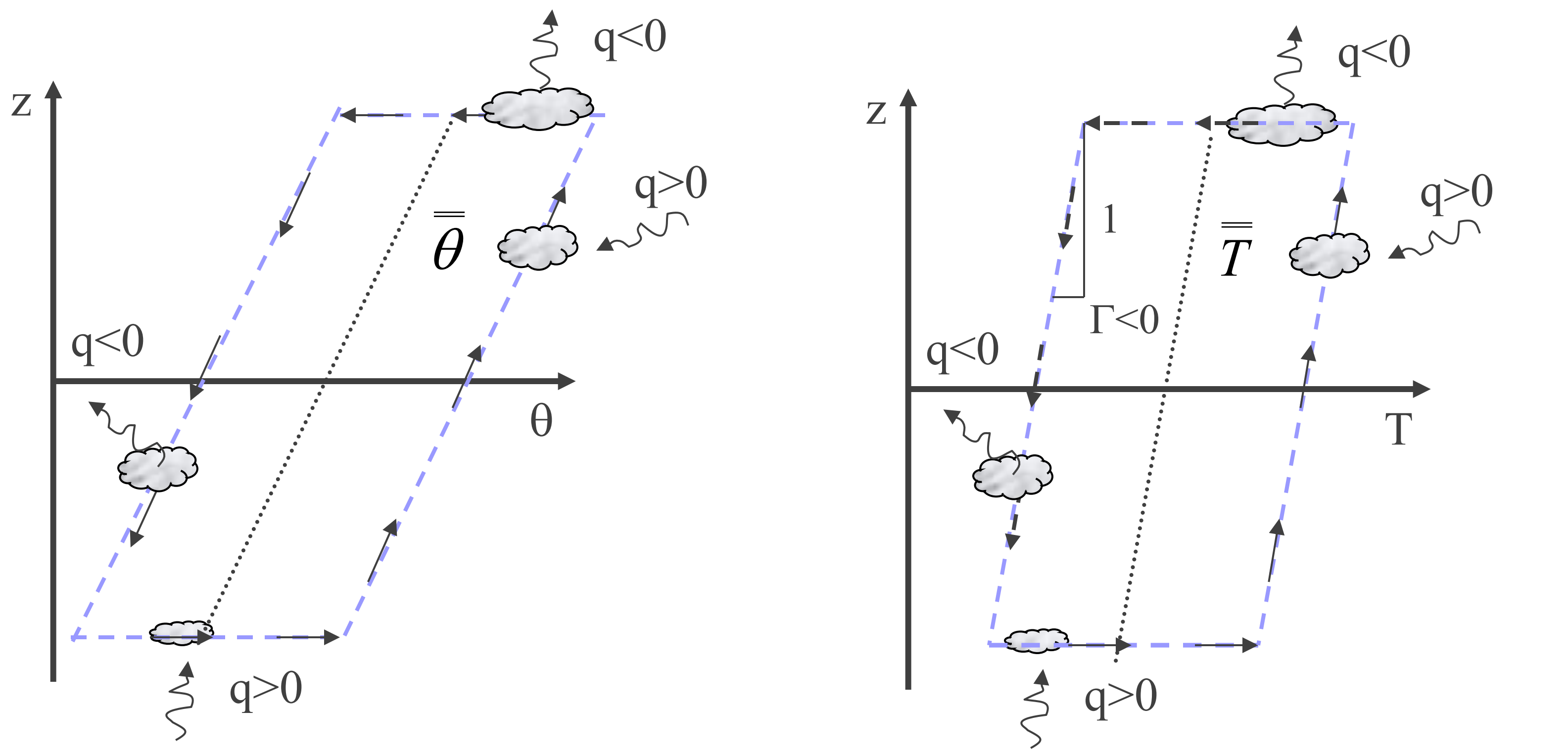
Idealized circuit followed by an air parcel during which it absorbs heat at the base of a layer and rejects heat at its top, with adiabatic vertical motion between. Fig. 6.6 Salby.

Adiabatic stratification (2)



Thermodynamic cycle followed by the air parcel in terms of (a) potential temperature and (b) temperature. Horizontally averaged behavior for a layer composed of many such parcels is indicated by dotted lines. Fig. 6.7 Salby

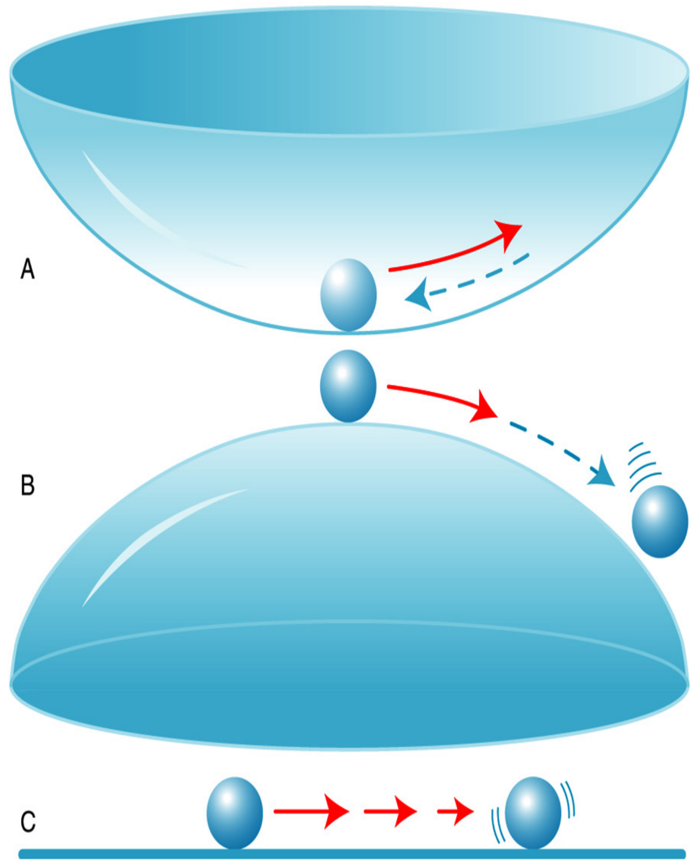
Diabatic stratification



Thermodynamic cycle followed by the air parcel whose vertical motion is diabatic and whose temperature increases with height. (a) potential temperature and (b) temperature. Horizontally averaged behavior for a layer composed of many such parcels is indicated by dotted lines. Fig. 6.8 Salby

Stability of vertical motion

- We will examine vertical displacements in a fluid that is in hydrostatic balance
- A parcel moving vertically within the fluid is subject to adiabatic expansion or compression, and hence the temperature will change
- As the parcel moves vertically, it may become warmer or cooler than the surrounding fluid at a particular level
- The parcel is subject to the Archimedes' force (buoyancy)
- If the buoyancy force acting on the displaced mass:
  - returns it to its initial position that the fluid is statically stable,
  - accelerates it away from its initial position, then the fluid is statically unstable
  - remains in balance with its surroundings, then the fluid is in a state of neutral equilibrium.



Parcel method

- We consider a small mass, or parcel, that is displaced vertically in a fluid at rest and in hydrostatic equilibrium.
- Symplifying assumption adopted in the parcel method:
  - the parcel retains its identity and does not mix with its environment
  - the parcel motion does not disturb its environment
  - the pressure of the parcel adjusts instantaneously to the ambient pressure of the fluid surrounding the parcel
  - the parcel moves isentropically, so that its potential temperature remains constant.

Why do we study stability in the atmosphere?

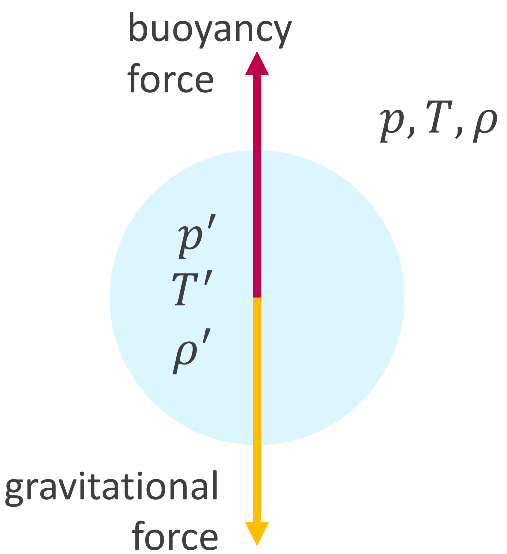
- The static stability of the atmosphere is important in the explanation and prediction of:
  - cumulus convection and severe storms
  - rainfall
  - boundary layer turbulence
  - large-scale atmospheric dynamics

Stability criteria

Primes parameters will describe properties of a parcel; non-primes parameters describe properties of parcel's environment.

The fluid environment is assumed to be in hydrostatic equilibrium; the gradient force is balanced by the gravitational force.

$$0 = -g - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

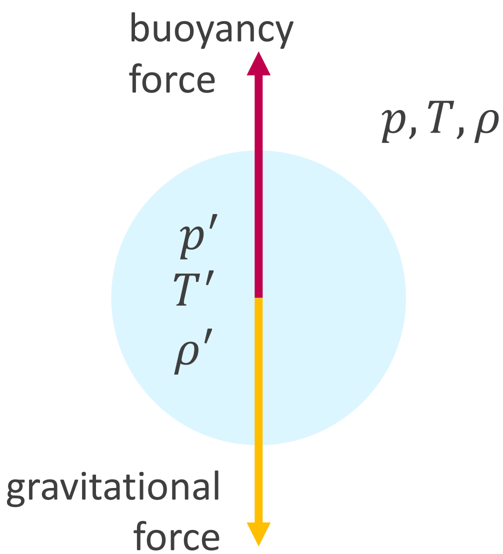


We will consider a small displacement of the parcel in the vertical direction.

From Newton's second law of motion, the acceleration of the parcel must be equal to sum of the gravitational and pressure gradient force.

$$\frac{d^2 z}{dt^2} = -g - \frac{1}{\rho'}\frac{\partial p'}{\partial z}$$

Hydrostatic balance - 1



$$\frac{d^2 z}{dt^2} = -g - \frac{1}{\rho'}\frac{\partial p'}{\partial z}$$

the environment is in hydrostatic balance

$$0 = -g - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

the pressure p' adjusts instantaneously to the ambient pressure p

$$p = p' \quad \longrightarrow \quad \frac{\partial p}{\partial z} = \frac{\partial p'}{\partial z}$$

$$\frac{d^2 z}{dt^2} = \frac{\rho - \rho'}{\rho'} g$$

$$f_b = \left(\frac{\rho - \rho'}{\rho'}\right) g$$

the net buoyancy force per unit mass

If the parcel is less dense than its surrounding, then it will accelerate upwards.

Hydrostatic balance - 2

$$\frac{d^2z}{dt^2} = \frac{\rho - \rho'}{\rho'} g$$

We will write this equation in terms of vertical density gradients by considering a small vertical displacement of the parcel from its initial location.

Let z=0 at the initial location, where the parcel density is equal to the density in the surrounding.

$$\rho_0' = \rho_0$$

We expand the density of the parcel and the density of the environment about the initial location by using Taylor's theorem. We will ignore higher-order terms involving powers of z if the vertical displacement is small.

$$\rho' = \rho_0' + \left(\frac{d\rho'}{dz}\right) z + \cdots \qquad \rho = \rho_0 + \left(\frac{d\rho}{dz}\right) z + \cdots$$

$$\frac{d^2z}{dt^2} = \frac{g}{\rho'}(\rho - \rho') = \frac{g}{\rho'}\left[\left(\frac{d\rho}{dz}\right) - \left(\frac{d\rho'}{dz}\right)\right] z \approx \frac{g}{\rho_0}\left[\left(\frac{d\rho}{dz}\right) - \left(\frac{d\rho'}{dz}\right)\right] z$$

Hydrostatic balance - 3

The Brunt-Väisälä frequency, N, is defined as:

$$N^2 = \frac{g}{\rho_0}\left[\left(\frac{d\rho'}{dz}\right) - \left(\frac{d\rho}{dz}\right)\right]$$

It is also referred to as the buoyancy frequency.

The equation of parcel's motion becomes:

$$\frac{d^2z}{dt^2} + N^2 z = 0$$

$$z = A_1 \exp(iNt) + B_2 \exp(-iNt)$$

$$z = A_1 \cos(Nt) + B_1 \sin(Nt)\ , \qquad N^2 \quad > 0$$

$$z = A_1 exp(|N|t) + B_1 exp(-|N|t)\ , \quad N^2 \quad < 0$$

Criteria of static stability - 1

$$\frac{d^2z}{dt^2} + N^2 z = 0$$

$N^2 > 0$: stable, period of oscilation: $$\tau_g = \frac{2\pi}{N}$$

$N^2 = 0$: neutral

$N^2 < 0$: unstable

Criteria of static stability - 2

$$N^2 = \frac{g}{\rho_0}\left[\left(\frac{d\rho'}{dz}\right) - \left(\frac{d\rho}{dz}\right)\right]$$

For the moist (but unsaturated) atmosphere using the ideal gas law and ignoring pressure fluctuations:

in the parcel

$$p' = R_d T_v' \rho'$$

$$\frac{d\rho'}{dz} = -\frac{p'}{R_d T_v'^2}\frac{dT_v'}{dz} = \frac{\rho'}{T_v'}\Gamma_d$$

in the environment

$$p = R_d T_v \rho$$

$$\frac{d\rho}{dz} = -\frac{p}{R_d T_v^2}\frac{dT_v}{dz} = -\frac{\rho}{T_v}\frac{dT_v}{dz}$$

$$N^2 = \frac{g}{\rho_0}\left[\frac{\rho'}{T_v'}\Gamma_d + \frac{\rho}{T_v}\frac{dT_v}{dz}\right]$$

$$T_v' = T_v = T_0$$

$$\rho' = \rho = \rho_0$$

$$N^2 = \frac{g}{T_0}\left[\Gamma_d + \frac{dT_v}{dz}\right]$$

Criteria of static stability - 3

From the definition of virtual potential temperature: $\theta_v = T_v \left(\frac{p_0}{p}\right)^{\kappa}$

$$\frac{1}{\theta_v}\frac{d\theta_v}{dz} = \frac{1}{T_v}\frac{dT_v}{dz} - \frac{R}{c_{pd}}\frac{1}{p}\frac{dp}{dz}$$

$$\frac{1}{\theta_v}\frac{d\theta_v}{dz} = \frac{1}{T_v}\left(\frac{dT_v}{dz} + \frac{g}{c_{pd}}\right)$$

$$N^2 = \frac{g}{\theta_o}\frac{d\theta_v}{dz}$$

stable	$N^2 > 0$	$\frac{d\theta_v}{dz} > 0$	or $-\frac{dT_v}{dz} < \Gamma_d$
neutral	$N^2 = 0$	$\frac{d\theta_v}{dz} = 0$	or $-\frac{dT_v}{dz} = \Gamma_d$
unstable	$N^2 < 0$	$\frac{d\theta_v}{dz} < 0$	or $-\frac{dT_v}{dz} > \Gamma_d$

$$\frac{d^2z}{dt^2} = \frac{\rho - \rho'}{\rho'} g$$

$$p = RT\rho\,, \qquad p = RT'\rho'$$

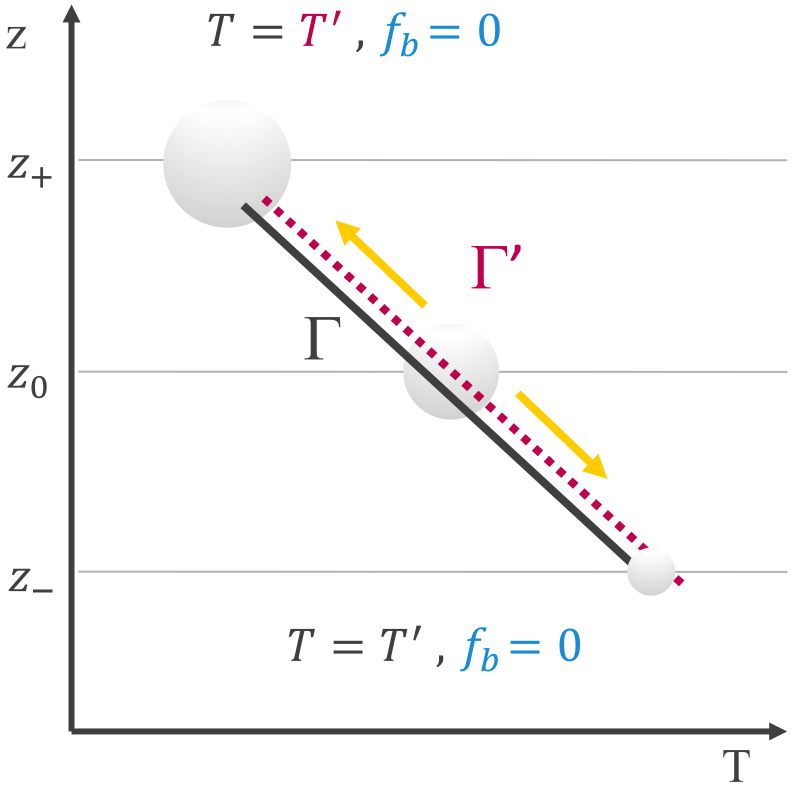
$$\frac{d^2z}{dt^2} = \frac{T' - T}{T} g$$

$$T' = T_0 - \Gamma' z \qquad \Gamma' = \Gamma_d \quad \text{or} \quad \Gamma' = \Gamma_s$$

$$T = T_0 - \Gamma z$$

$$\frac{d^2z}{dt^2} = \frac{g}{T}(\Gamma - \Gamma')z$$

Neutral stability, Γ=Γ'



$$\frac{d^2 z}{dt^2} = \frac{g}{T}(\Gamma - \Gamma')z$$

f_b – buoyancy force

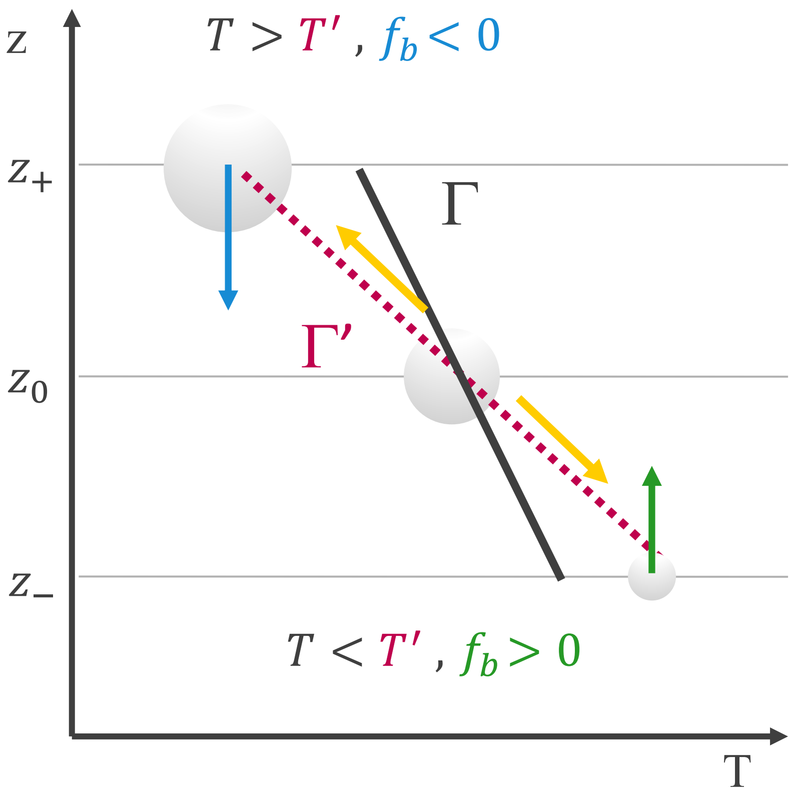
The parcel's temperature changes:

- at the dry adiabatic lapse rate (~1°C/100m) if the parcel is unsaturated; $\Gamma' = \Gamma_d$
- at the saturated adiabatic lapse rate if the parcel is saturated ; $\Gamma' = \Gamma_s$

Γ - the lapse rate in the environment (Γ=Γ')

If the temperature lapse rate in the environment is equal to the parcel's temperature lapse rate (either dry or wet adiabatic) then that parcel (dry or wet) does not experience a buoyancy force.

Stable/positive stability, Γ<Γ'



$$\frac{d^2z}{dt^2} = \frac{g}{T}(\Gamma - \Gamma')z$$

f_b – buoyancy force

The parcel's temperature changes:

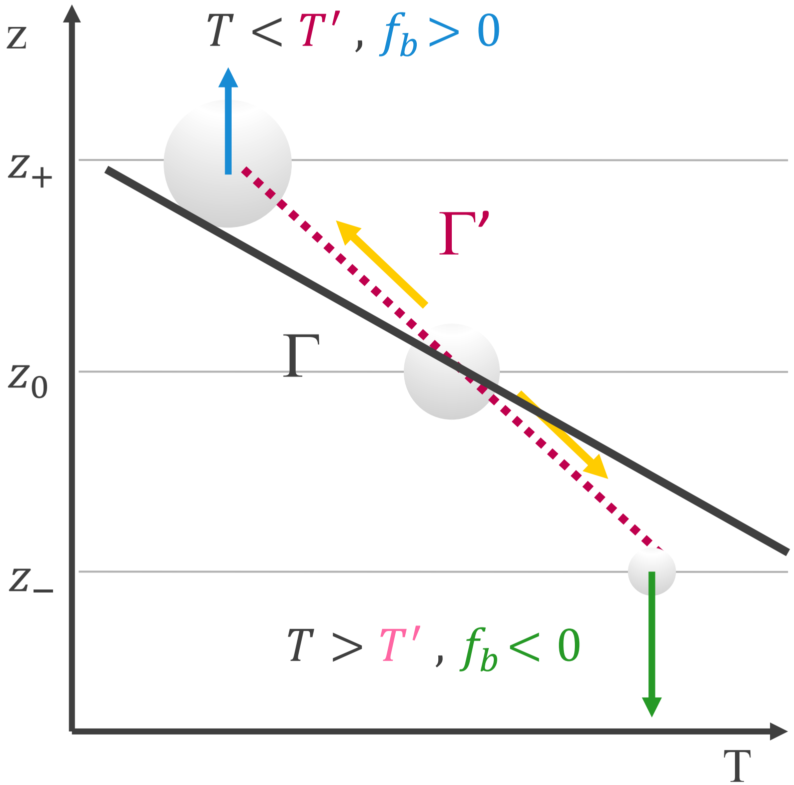
- at the dry adiabatic lapse rate (~1°C/100m) if the parcel is unsaturated; $\Gamma' = \Gamma_d$
- at the saturated adiabatic lapse rate if the parcel is saturated ; $\Gamma' = \Gamma_s$

Γ - the lapse rate in the environment (Γ=Γ')

If the temperature lapse rate in the environment is smaller to the parcel's temperature lapse rate (either dry or wet adiabatic) then that parcel (dry or wet) experience a buoyancy force that opposes the displacement.

If the parcel is displaced upward (downward), it becomes heavier (lighter) than its surroundings and thus negatively (positively) buoyant.

Unstable/negative stability, Γ>Γ'



$$\frac{d^2 z}{dt^2} = \frac{g}{T}(\Gamma - \Gamma')z$$

f_b – buoyancy force

The parcel's temperature changes:

- at the dry adiabatic lapse rate (~1°C/100m) if the parcel is unsaturated; $\Gamma' = \Gamma_d$
- at the saturated adiabatic lapse rate if the parcel is saturated ; $\Gamma' = \Gamma_s$

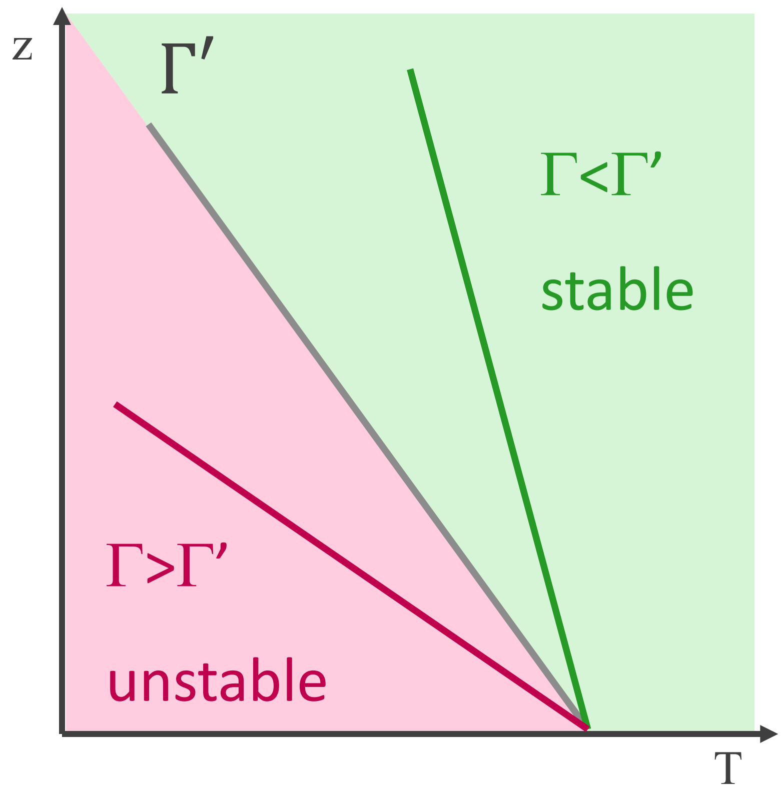
Γ - the lapse rate in the environment (Γ=Γ')

If the temperature lapse rate in the environment is bigger to the parcel's temperature lapse rate (either dry or wet adiabatic) then that parcel (dry or wet) experience a buoyancy force that reinforces the displacement.

If the parcel is displaced upward (downward), it becomes lighter (heavier) than its surroundings and thus positively (negatively) buoyant.

Stability of a dry parcel

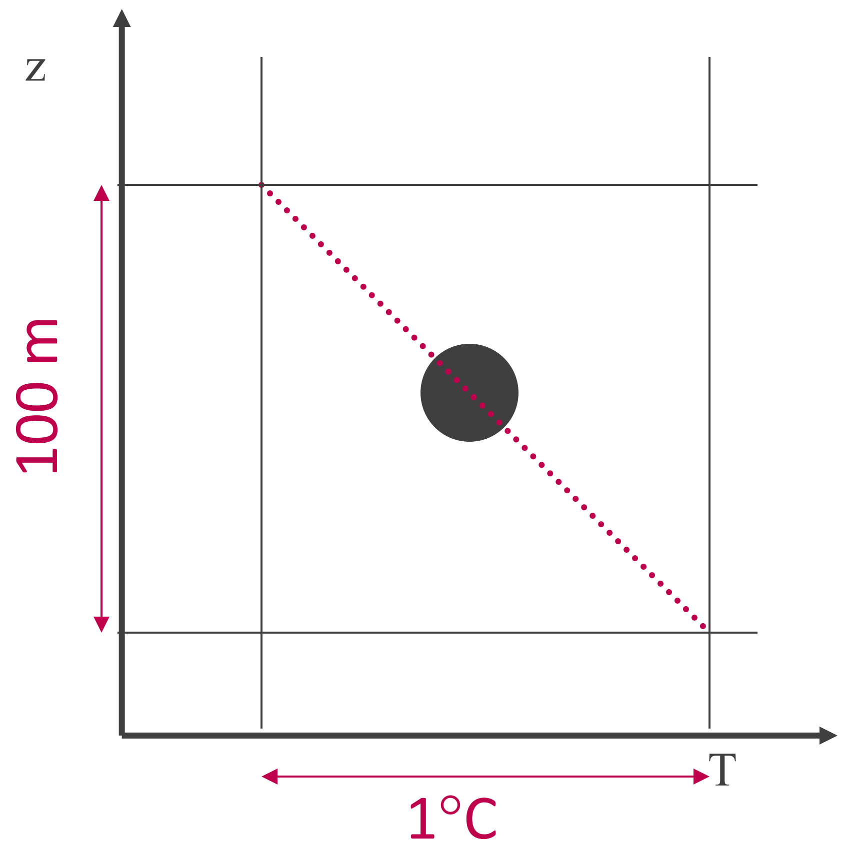
Γ' – the temperature lapse rate for a dry parcel (Γ_d)
Γ - the environmental lapse rate



Dry and wet (saturated) parcels

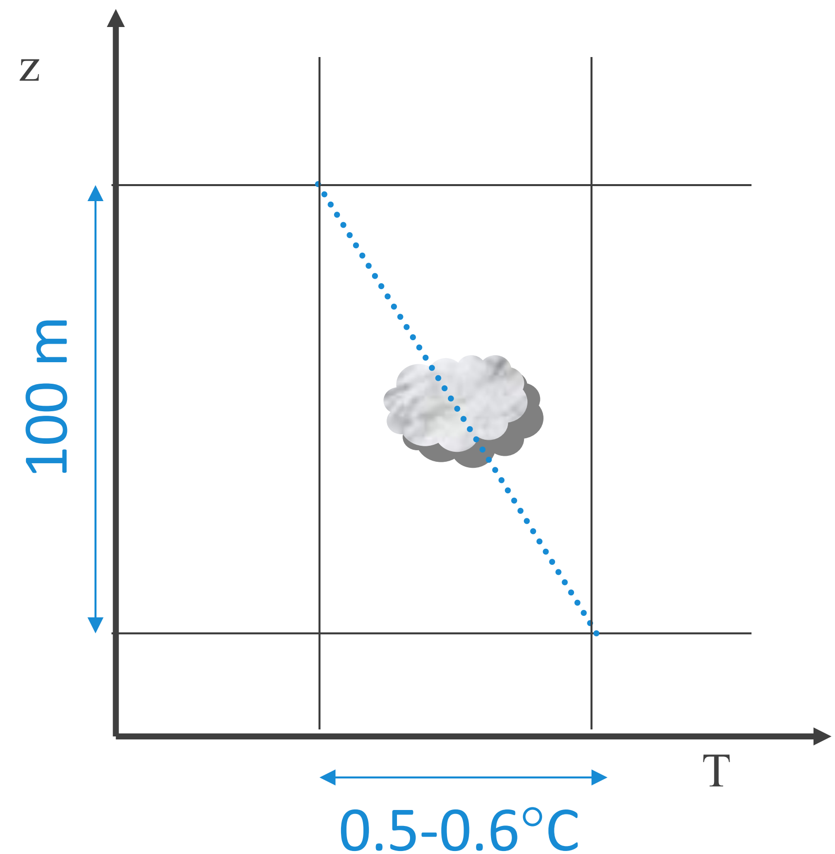
$$\Gamma' = \Gamma_d$$

dry adiabatic lapse rate



$$\Gamma' = \Gamma_s$$

wet adiabatic lapse rate



Stability of a wet (saturated) parcel

Vertical displacements of air parcels frequently result in phase change (release the latent heat), which affect the buoyancy of the air and thus the static stability criteria.

When a saturated parcel of air is displaced vertically, its temperature changes according to the saturated adiabatic lapse rate (Γ'_s).

The Brunt-Väisälä frequency is simillar as in the case of a dry parcel.

$$N^2 = \frac{g}{T_0}\left(\frac{dT_\rho}{dz} + \Gamma'_s\right)$$

Because of the weight of the condensed water, the density temperature is used instead of virtual temperature T_v.

$$T_\rho = T(1 + 0.608q_v - q_l)$$

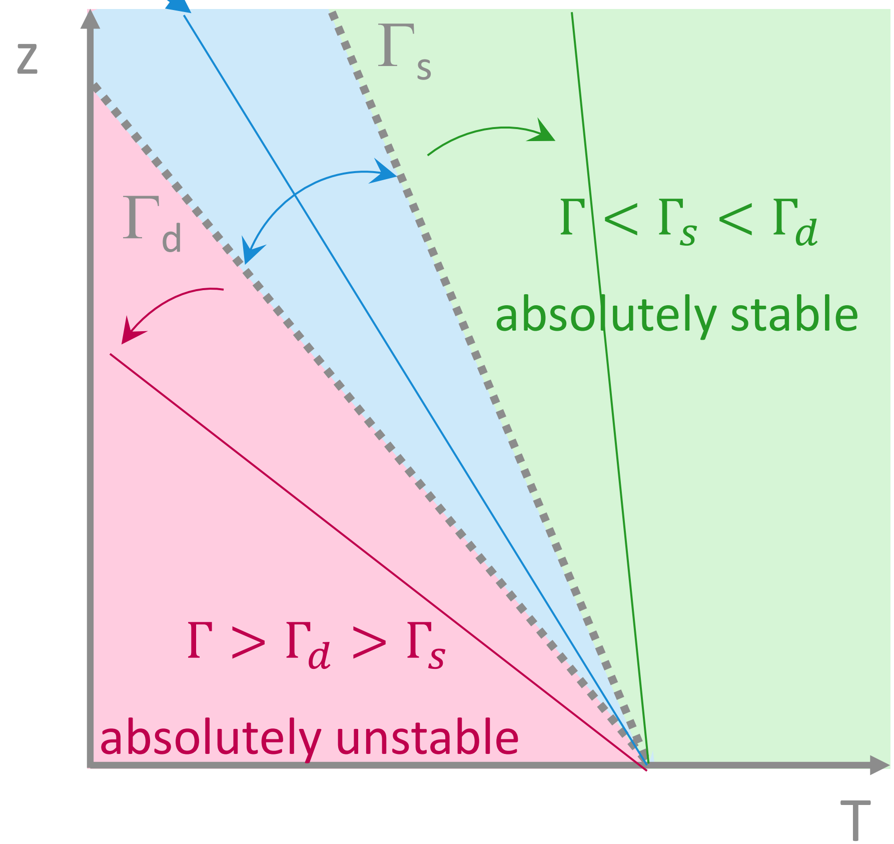
Note that $T_\rho = T_v$ when $q_l = 0$.

Stability criteria for dry and saturated parcels

$\Gamma_s < \Gamma < \Gamma_d$

Conditionally stable

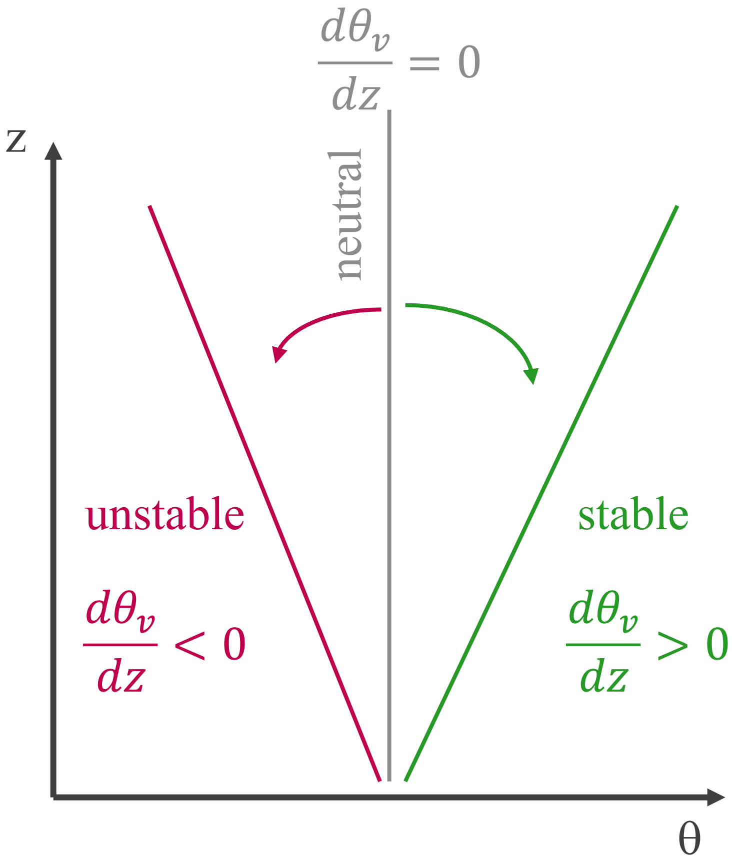
- Stable for dry particles
- Unstable for wet saturated particles



Stability criteria

absolutely stable	$-\frac{dT_\rho}{dz} < \Gamma_s$
neutral for saturated particles	$-\frac{dT_\rho}{dz} = \Gamma_s$
conditionally stable/unstable	$\Gamma_s < -\frac{dT_\rho}{dz} < \Gamma_d$
neutral for unsaturated particles	$-\frac{dT_\rho}{dz} = \Gamma_d$
absolutely unstable	$-\frac{dT_\rho}{dz} > \Gamma_d$

Stability criteria for dry particles in terms of potential temperature



$$\frac{d\theta_v}{dz} = \frac{\theta_v}{T}(\Gamma_d - \Gamma)$$

stable $\frac{d\theta_v}{dz} > 0$

neutral $\frac{d\theta_v}{dz} = 0$

unstable $\frac{d\theta_v}{dz} < 0$

Stability criteria for saturated particles in terms of equivalent potential temperature

If a parcel contains water vapor in saturated state the stability criteria are similar as for a dry parcel, but the virtual potential temperature has to be replaced by the equivalent potential temperature.

stable $\frac{d\theta_e}{dz} > 0$

neutral $\frac{d\theta_e}{dz} = 0$

unstable $\frac{d\theta_e}{dz} < 0$

The equivalent potential temperature can be written in an approximative form:

$$\theta_e = \theta \exp\left(\frac{L_{lv} q_s}{c_{pd} T}\right) \approx \theta\left(1 + \frac{L_{lv} q_s}{c_{pd} T}\right) = \theta + \frac{L_{lv} q_s}{c_{pd}} \left(\frac{p_0}{p}\right)^{\kappa}$$

For a layer to be unstable to wet-adiabatic displacements the potential temperature should decrease with altitude and/or the water vapor specific mass (specific humidity) should decrease with altitude.

Implication of stability for vertical motion - 1

- The stability of a layer determines its ability to support vertical motion and thus support transfers of heat, momentum, and constituents.
- Since vertical motion must be compensated by horizontal motion to conserve mass, hydrostatic stability also influences horizontal transport.
- Three-dimensional (3-D) turbulence that disperses atmospheric constituents involves both vertical and horizontal motion. Supressing vertical motion also supresses the horizontal component of 3-D eddy motion and thus turbulent dispersion.
- A layer that is stably stratified inhibits vertical motion. Small vertical displacements introduced mechanically by flow over elevated terrain or thermally through isolated heating are then opposed by the positive restoring force of buoyancy.
- A layer that is unstably stratified promotes vertical motion through the negative restoring force of buoyancy.
- Work performed by or against buoyancy reflects a conversion between potential and kinetic energy.

Implication of stability for vertical motion - 2

The vertical momentum balance for an unsaturated air parcel: $\frac{d^2z}{dt^2} + N^2 z = 0, \quad N^2 = g\frac{d\ln\theta}{dz}$

Equation was derived for small vertical displacements. Under positive or neutral stability, air displacements can remain small enough for the stratificition of a layer to be preserved.

A layer of negative stability (unstable) evolves differently. The solution of the equation takes the form:

$$z(t) = Ae^{\hat{N}t} + Be^{-\hat{N}t}, \quad N^2 = -\hat{N}^2 < 0$$

The parcel's displacement grows exponentially with time. The first term, which dominates the long-term behavior, violates the linear analysis applied in derivation of the equation. Except for small N, displacements amplify exponentially – even in the presence of friction.

Small initial disturbances then evolve into fully developed convection, in which nonlinear effects limit amplification by modifying the stratificationof the layer. By rearranging mass, convective cells alter N^2 and hence the buoyancy force experienced by individual air parcels. The simple linear description breaks down.

Implication of stability for vertical motion - 3

Amplifying motion is fueled by a conversion of potential energy, which is associated with the vertical distribution of mass, into kinetic energy, which is associated with convective motions.

Air motions modify stratification of the layer.

Fully developed convection, which results in efficient vertical mixing, rearranges the conserved property θ (θ_e) into a distribution that is statistically homogeneous. This limiting distribution corresponds to a state of neutral stability.

Thus, small disturbances to an unstable layer amplify and eventually evolve into fully developed convection, which neutralizes the instability by mixing θ (θ_e) into a uniform distribution.

In that limiting state, no more potential energy is available for conversion to kinetic energy, so convective motions decay through frictional dissipation.