Supplementary material 1

1. Two balls 4 cm in diameter are placed 100 m apart on a frictionless horizontal plane at 43 °N. If the balls are impulsively propelled directly at each other with equal speeds, at what speed must they travel so that they just miss each other?

So we have a situation where an object (ball) is going to move in a rotating frame. Because of that, there will be a Coriolis force acting on it. Assuming that we give the balls an initial velocity in y direction (v), then because of the Coriolis force, there will be a new velocity in x direction (u). Because of this velocity, the balls have a chance to miss each other. We want to know, what must be the velocity given to the balls in y direction, so the balls miss each other.

The balls miss each other, if after travelling 50 m in y direction, they also travel at least 2 cm in x direction. We assume that the only acceleration is due to the Coriolis effect, therefore:

$$\frac{\mathrm{D}\bar{u}}{\mathrm{D}t} = -\bar{f} \times \bar{u},$$

where $\bar{f} = [0, 0, f]$, $f = 2\Omega\text{sin}\phi$ and $\bar{u} = [u, v, 0]$. We only consider velocity in x direction. We assume that changes in y direction are very small. We will consider them later. The equation is now:

$$\frac{\mathrm{D}u}{\mathrm{D}t} = fv = 2\Omega\text{sin}\phi v,$$

assuming that v is constant (which we can plug in the equation later), we can integrate this equation to get:

$$x = \frac{1}{2} fvt^2 = \frac{1}{2} 2\Omega\text{sin}\phi vt^2 = \Omega\text{sin}\phi vt^2,$$

substituting $v = \frac{l/2}{t}$:

$$x = \Omega\text{sin}\phi\frac{l}{2}t.$$

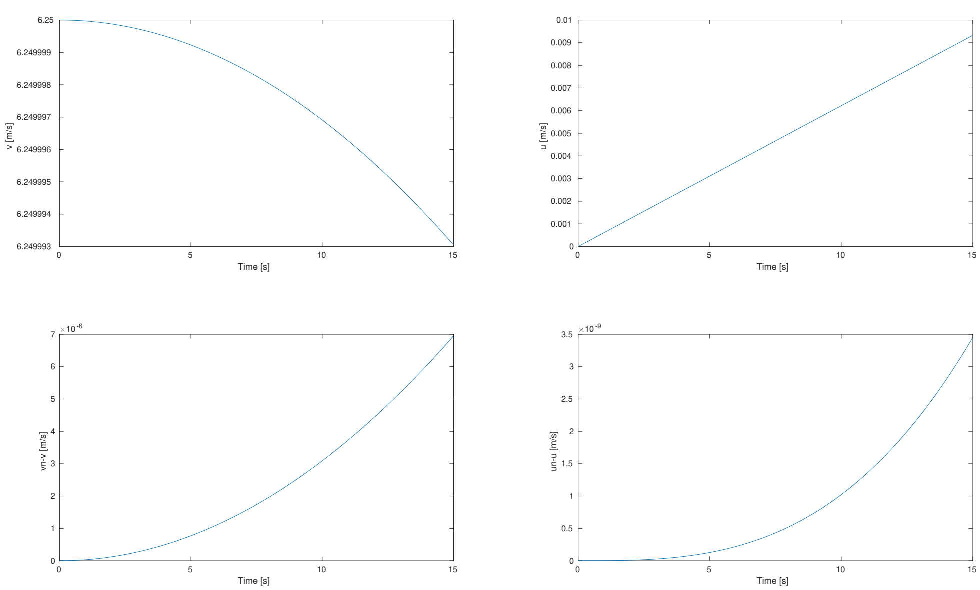
We can solve for t, and get $t = 8$s. Therefore $v = 6.25$m/s. The rest is in the second page.

We can also consider, that because of the new velocity in x, the balls will have some modification of the velocity in the y direction (just how because of the fact that they moved in the y direction, they gained velocity in x direction in the previous case). We end up with following equations:

$$\frac{\mathrm{D}u}{\mathrm{D}t} = fv = 2\Omega \mathrm{sin}\phi v(t),$$

$$\frac{\mathrm{D}v}{\mathrm{D}t} = -fu = -2\Omega \mathrm{sin}\phi u(t).$$

I don't know how to solve them analytically easily, but I solved them numerically, assuming initial velocity to be $\bar{u} = [0, 6.25]$ m/s. And below is a plot with some solutions:



Sorry for the size of the figures, but you can zoom them. The top figures are velocities with time calculated from the equations above, assuming that the velocity in y direction changes as well.

In the bottom figures, you can see difference between velocities from the top figures and velocities calculated using the assumption that the velocity in y direction is constant. You can see that this difference is very very small. Therefore we can assume that the effect on the velocity in y direction is negligible.