Wavelet-based Analysis: Intermittency, Coherency, Pulses:

by

Jun-Ichi Yano

Météo France

Toulouse

Background:

Standard Turbulence Theories

(cf., Kolmogorov):

Homogeneity, Isotropy

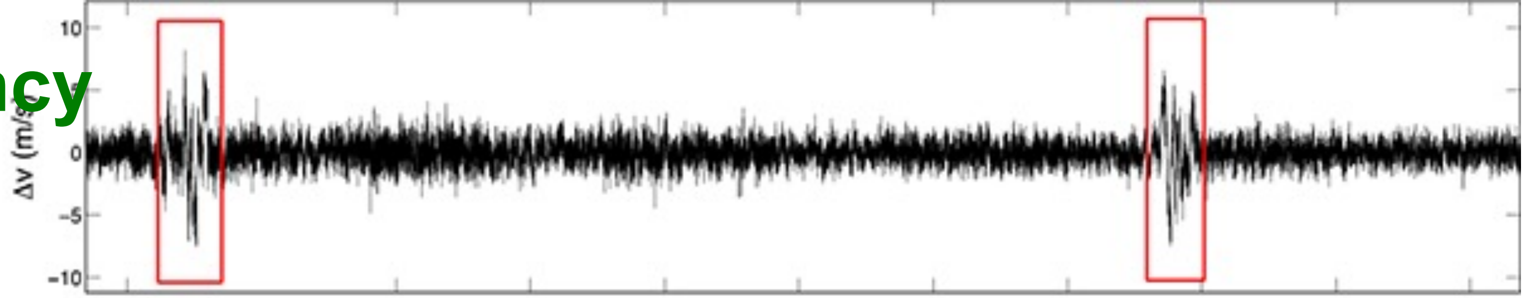
(i.e., totally disorganized)

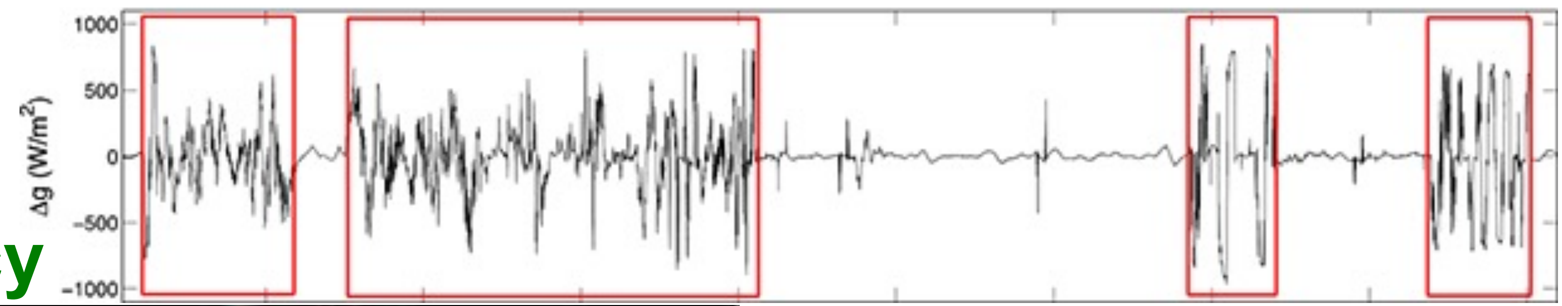
Observed Turbulence:

Inhomogeneous, Nonisotropic:

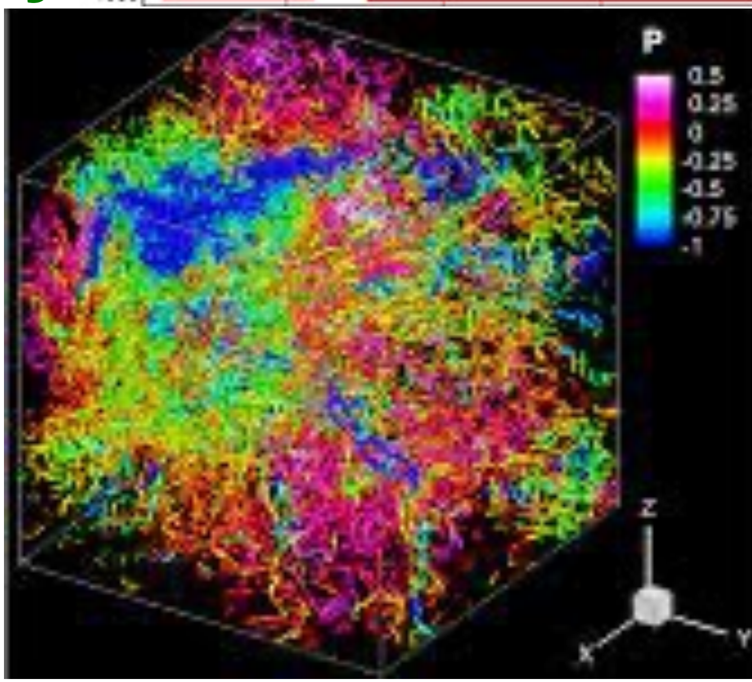
Intermittency, Coherency, Pulses

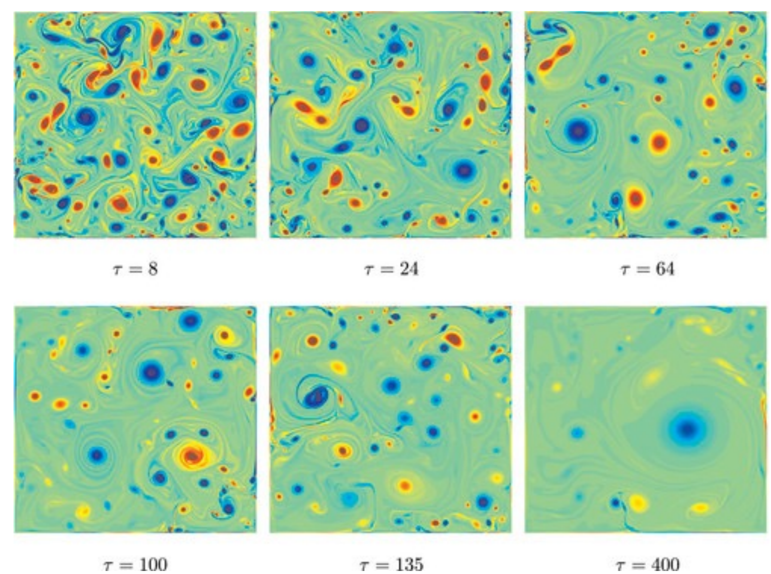
Intermittency





Coherency





Intermittency, Coherency:

Isolated Structures in Time and Space :

Pulses

How to Characterize the Intermittency, Coherency, Pulses?:

i.e.,

Isolated Structures in Time and Space?:

Wavelet

Wavelet(Discrete Orthogonal:Meyer)

Two indices: i, j:

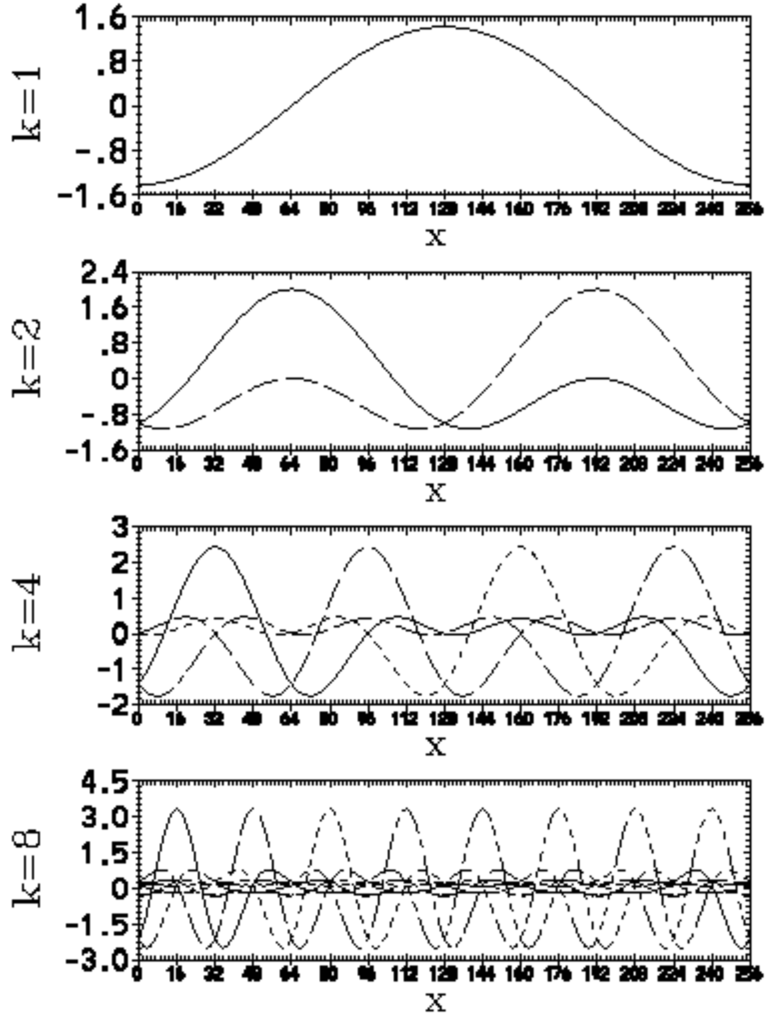
localizations:

$k=2^i$, i=0 — j=1

i=1 — j=1, 2

i=2 — j=1, ..., 4

i=3 — j=1, ..., 8



i=-1: domain average

Orthogonal&Complete

Fourier Expansion(Discrete Complete Set)
for a finite periodic domain, $[0, L]$:

$$\frac{1}{L}\int_0^L \phi_k(x)\phi_l(x) = \delta_{kl}$$

For any *well–behaved* function, $f(x)$:

$$f(x) = \sum_{k=0}^{\infty} \hat{f}_k \phi_k(x)$$

$$\hat{f}_k = \frac{1}{L}\int_0^L f(x)\phi_k(x)dx$$

wavlet method

$$\frac{1}{L}\int_0^L \phi_{i,j}(x)\phi_{i',j'}(x) = \delta_{ii'}\delta_{jj'}$$

For any *well–behaved* function, $f(x)$:

$$f(x) = \sum_{i=-1}^{i_{max}} \sum_{j=1}^{\mathrm{Max}(1,2^i)} \hat{f}_{i,j}\phi_{i,j}(x)$$

$$\hat{f}_{i,j} = \frac{1}{L}\int_0^L f(x)\phi_{i,j}(x)dx$$

Wavelet(Discrete Orthogonal:Meyer)

Wavelet-based Decomposition:

$$\varphi(x) = \sum_{i=-1}^{\ln_2 N-1} \sum_{j=1}^{j_{max}} \tilde{\varphi}_{i,j} \psi_{i,j}(x)$$

$$\varphi(x,y) = \sum_{i_x=-1}^{\ln_2 N_x-1} \sum_{i_y=-1}^{\ln_2 N_y-1} \sum_{j_x=1}^{j_{x,max}} \sum_{j_y=1}^{j_{y,max}} \tilde{\varphi}_{i_x,i_y,j_x,j_y} \psi_{i_x,j_x}(x) \psi_{i_y,j_y}(y)$$

Demonstrations:

Efficiency of Representing Intermittency, Coherency, and Pulses by Wavelet:

Compression

Decomposition

Extraction

Compression:

Let $l = l(i, j)$ with $l = 0, \ldots, N-1$:

$$\hat{f}_l^c = \begin{cases} \hat{f}_l, & |\hat{f}_l| > \alpha f_c \\ 0, & |\hat{f}_l| \leq \alpha f_c \end{cases}$$

where

$$f_c = (\sum_{l=1}^{N-1} \hat{f}_l^2)^{1/2}$$

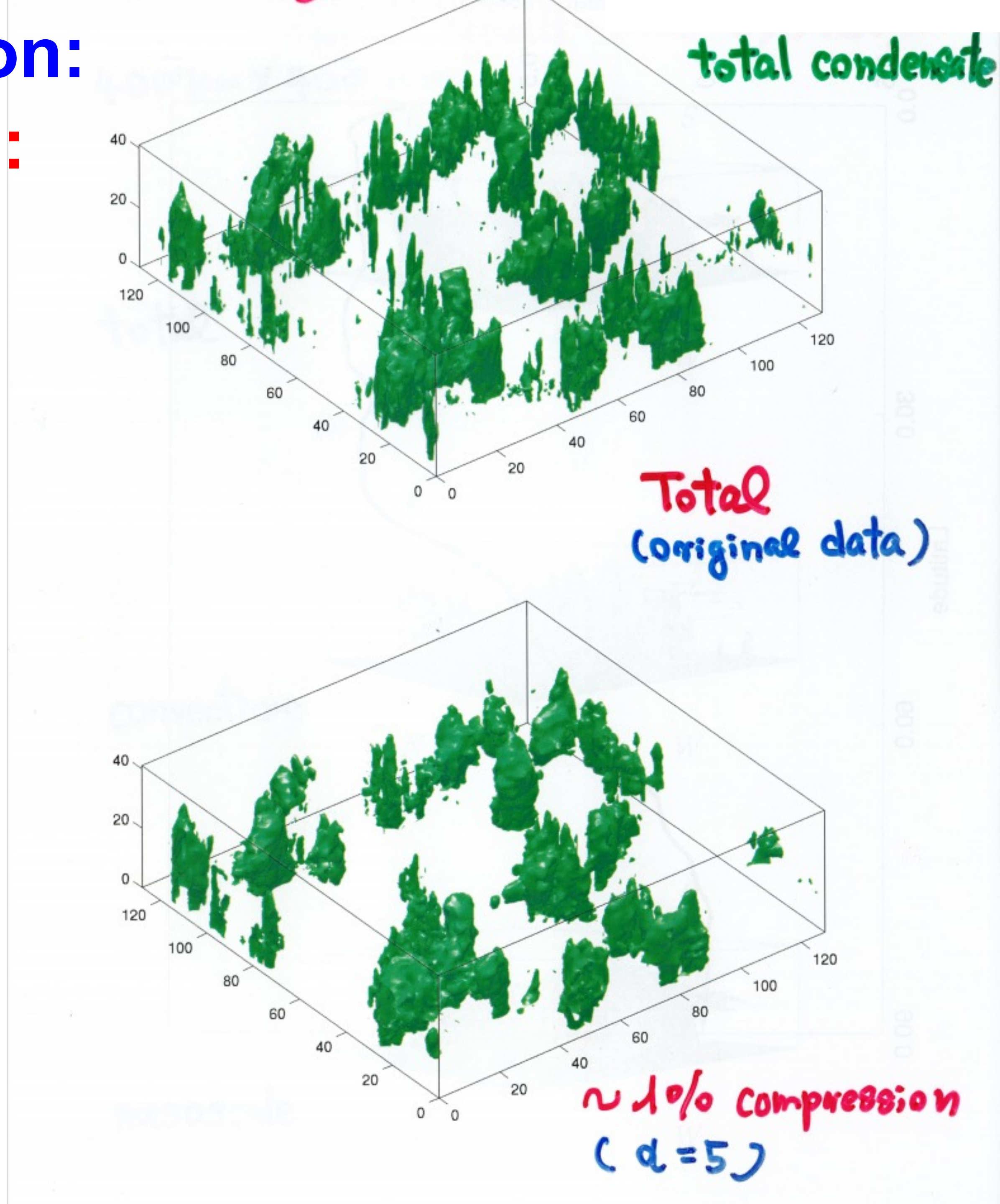
Compressed Representation:

$$f^c(x) = \sum_{l=0}^{N-1} \hat{f}_k^c \phi_k(x)$$

CRM Simulation:

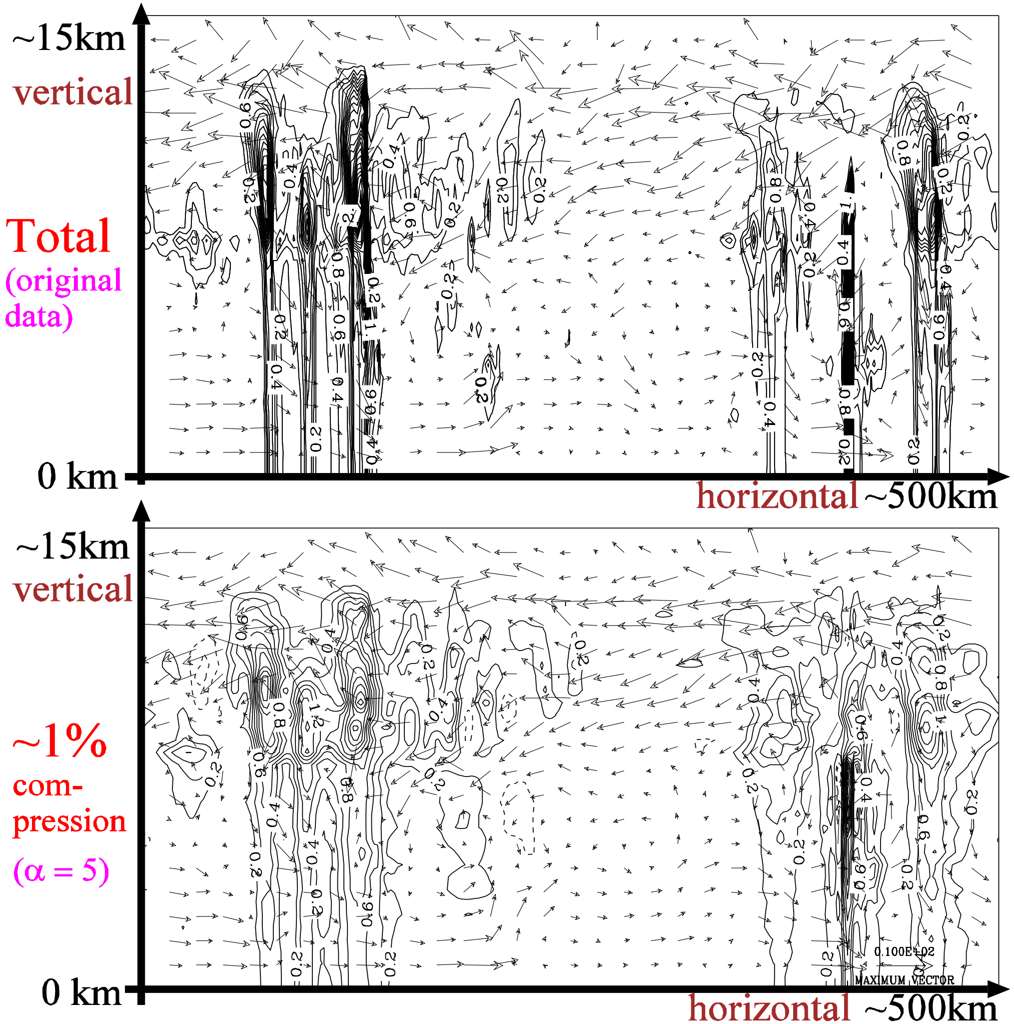
TOGA-COARE:

512 x 512 km^2 x 18km



Wavelet Compression of 3D CRM Data (TOGA period)

Example of a vertical-section total condensate + winds



Decomposition:

Choose a Threshold Variable: φ.

High Component:

$$\hat{\mathbf{f}}_l^H = \begin{cases} \hat{\mathbf{f}}_l, & |\hat{\varphi}_l| > \varphi_c \\ 0, & |\hat{\varphi}_l| \leq \varphi_c \end{cases}$$

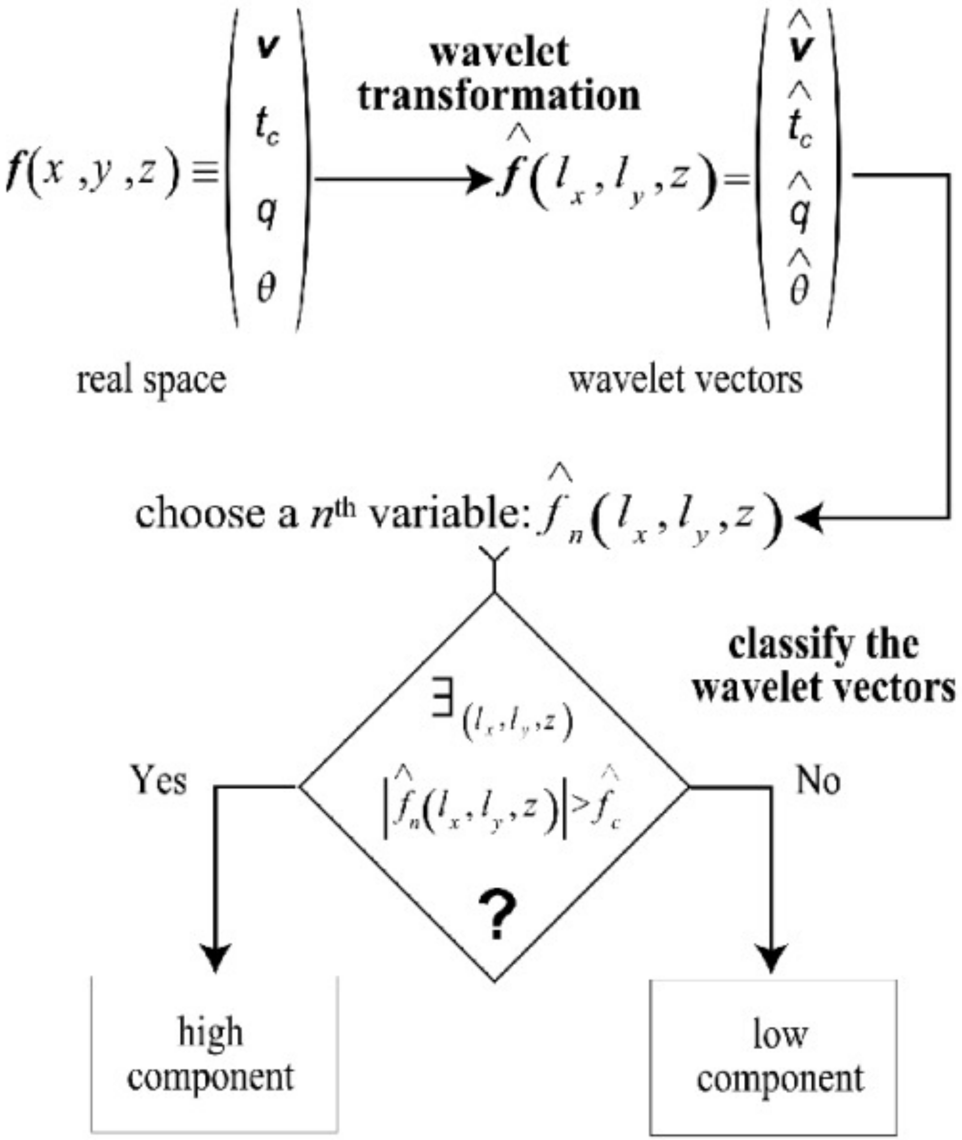
Low Component:

$$\hat{\mathbf{f}}_l^L = \begin{cases} 0, & |\hat{\varphi}_l| > \varphi_c \\ \hat{\mathbf{f}}_l, & |\hat{\varphi}_l| \leq \varphi_c \end{cases}$$

Here,

$$\varphi_c = (\sum_{l=1}^{N-1} \hat{\varphi}_l^2)^{1/2}$$

Decomposition:



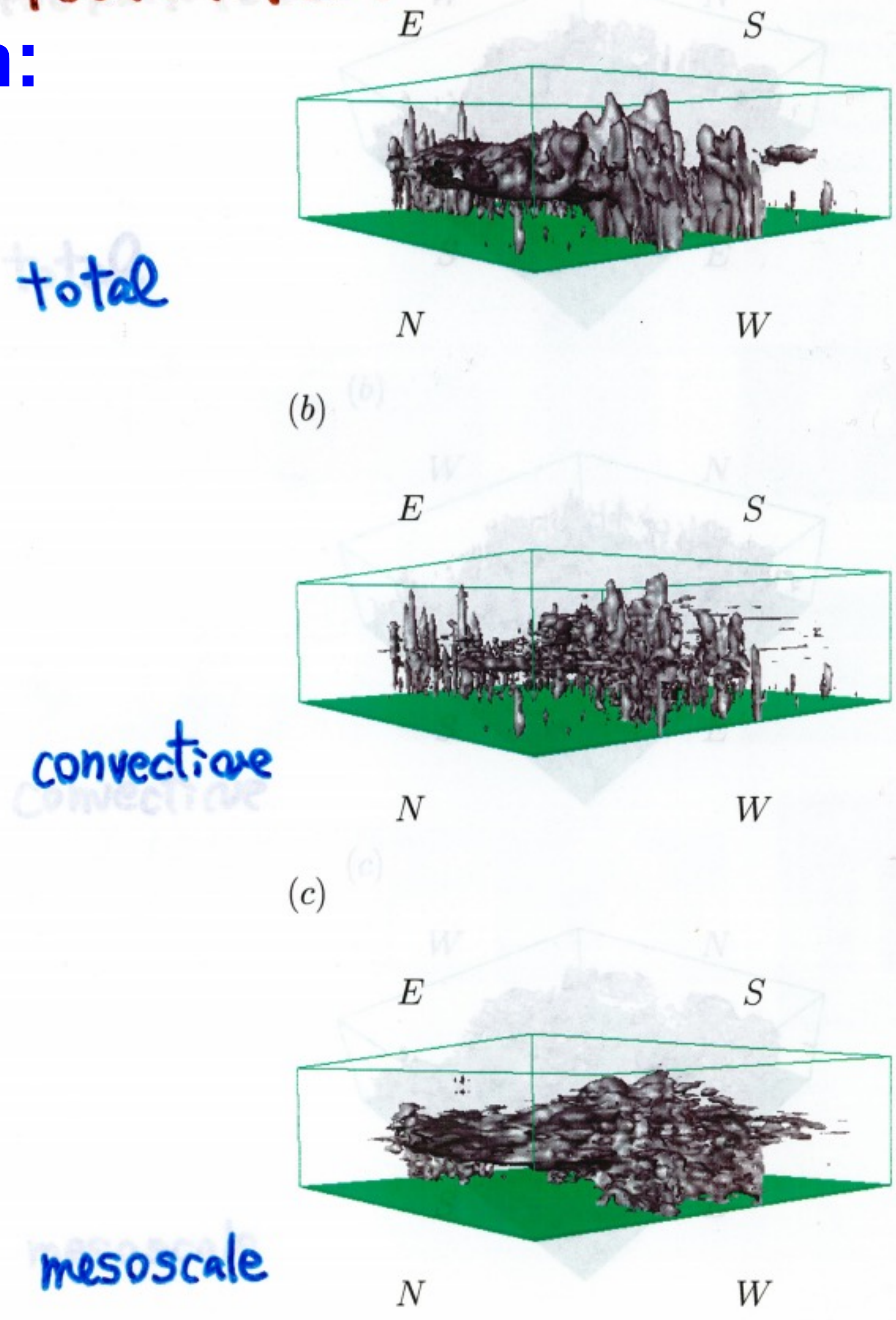
Decomposition:

Convective and Mesoscale Components:

Threshold Variable:

$$\phi = d|\mathbf{v}_H|/dz$$

CRM Simulation:
GATE:
Squall-Line System
400 x 400 km^2

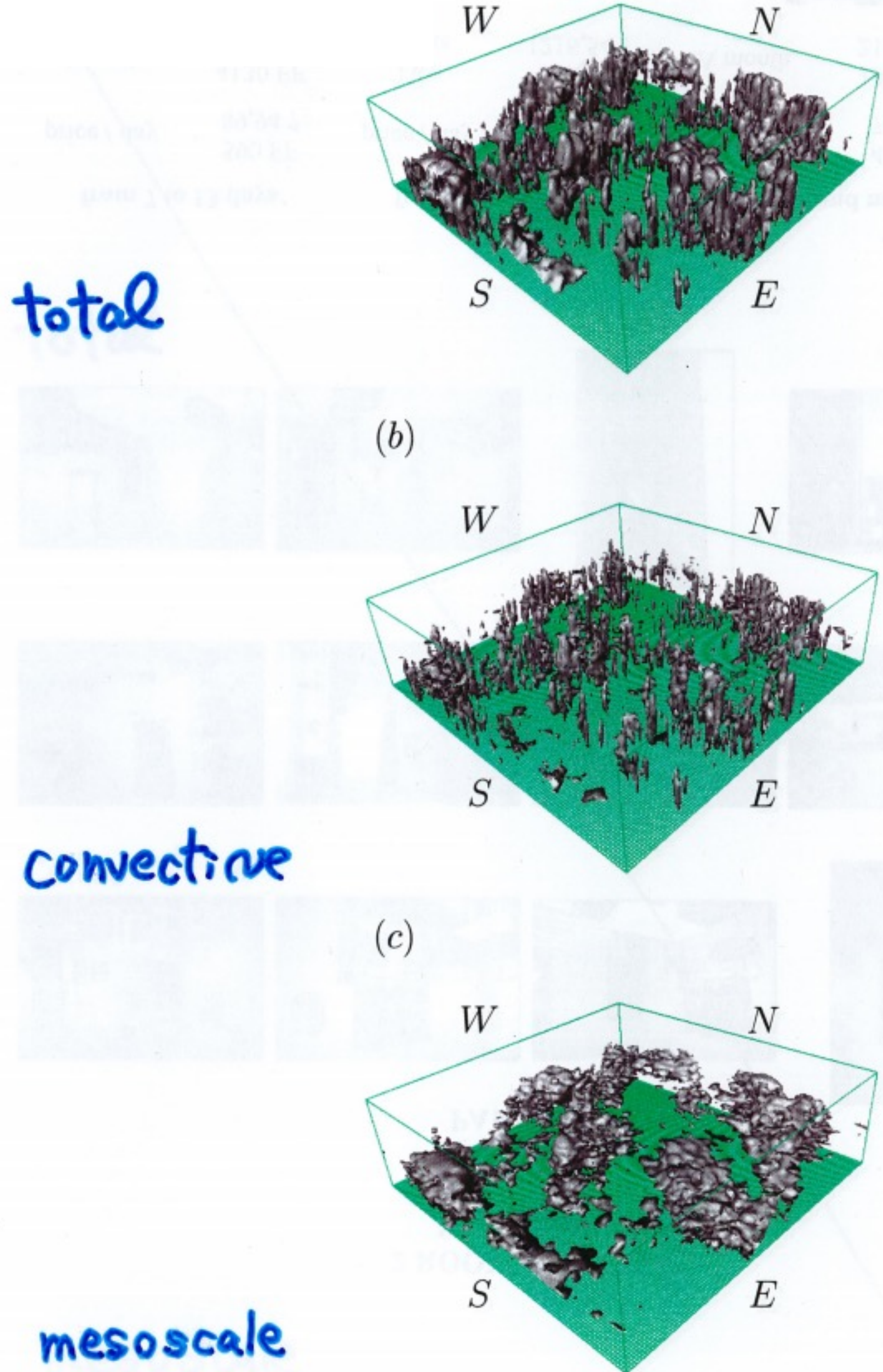


CRM Simulation:

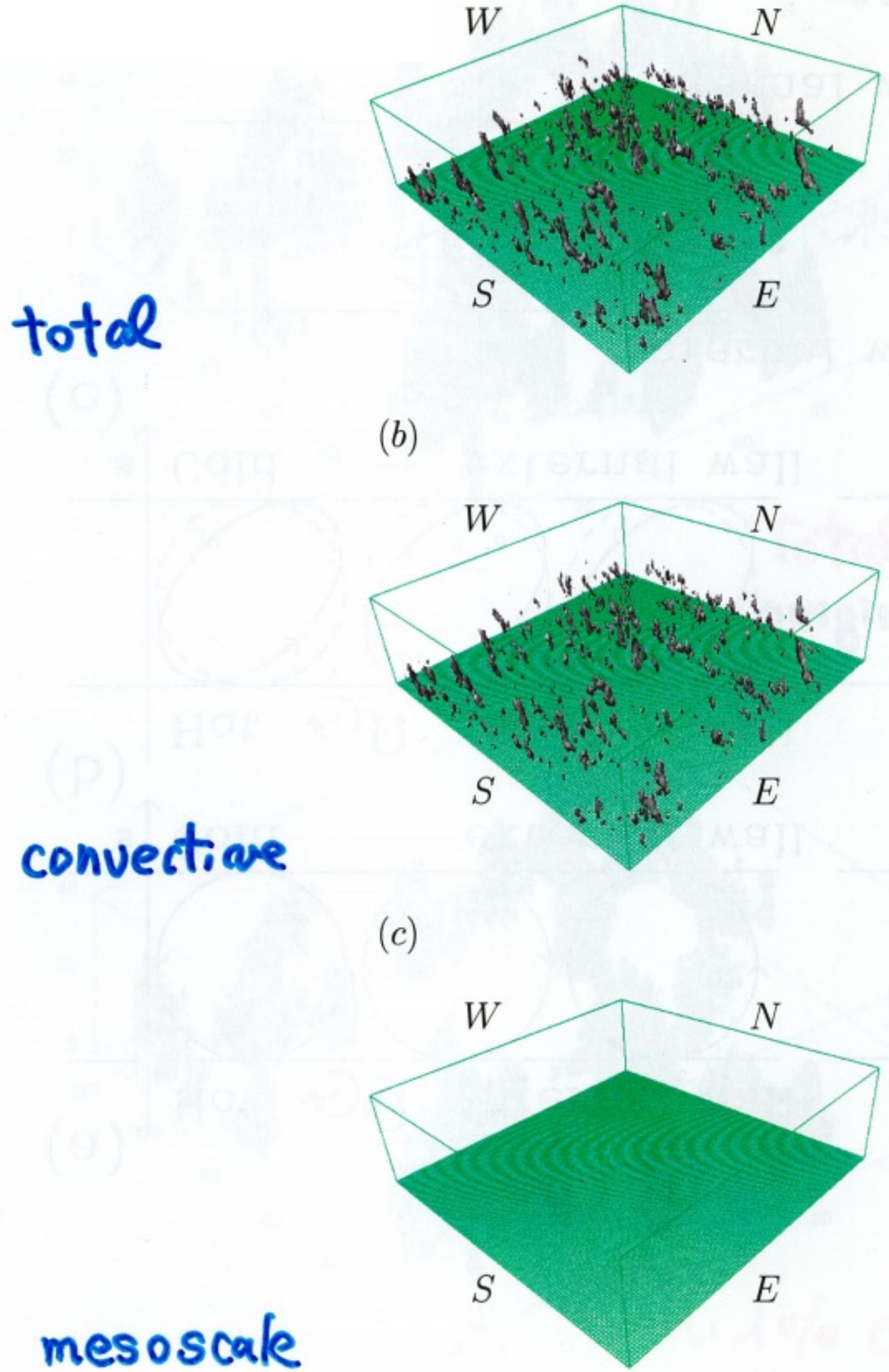
GATE:

Nonsquall-Line System

400 x 400 km^2



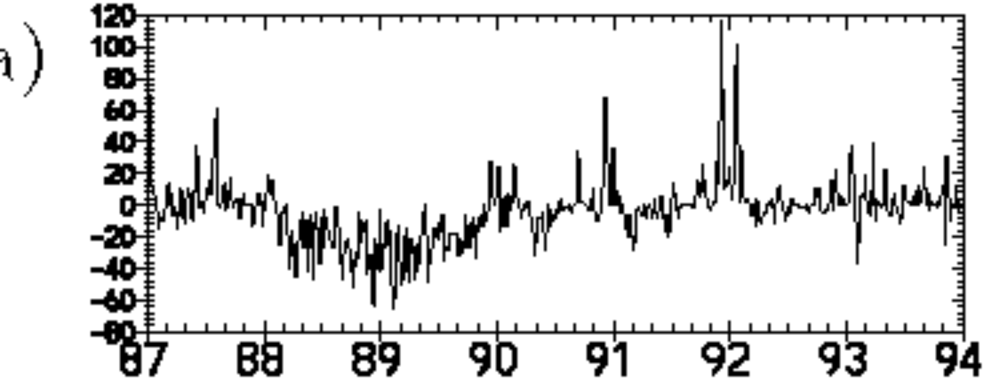
CRM Simulation:
GATE:
Scattered Convection
400 x 400 km^2

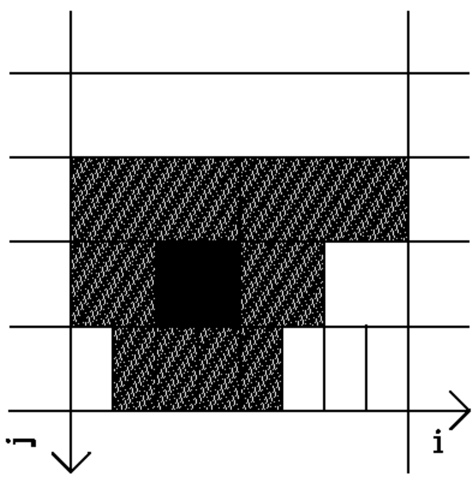


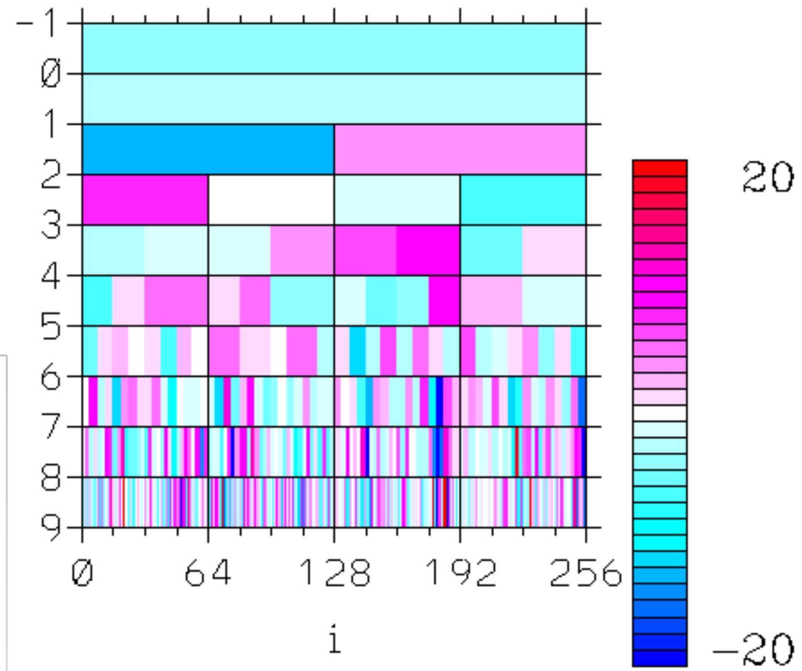
Extraction:

One-Dimensional Demonstration:

Zonal Wind over Western Pacific:



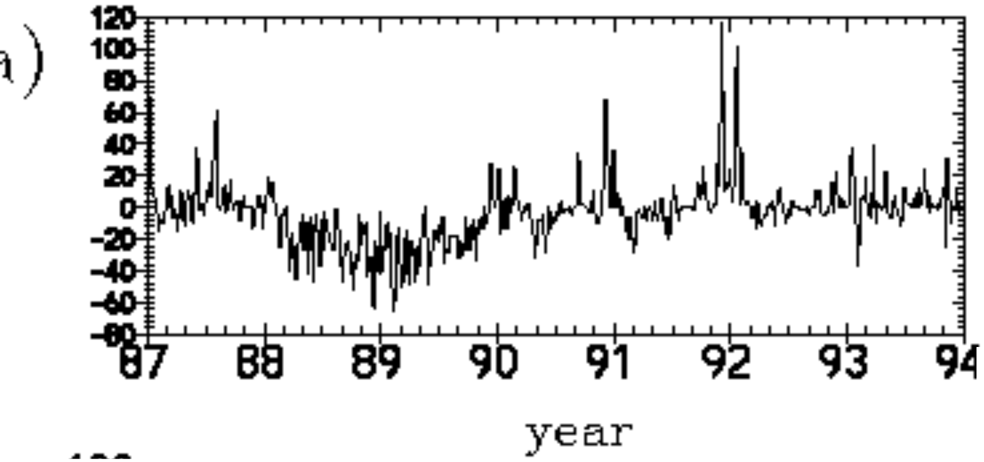


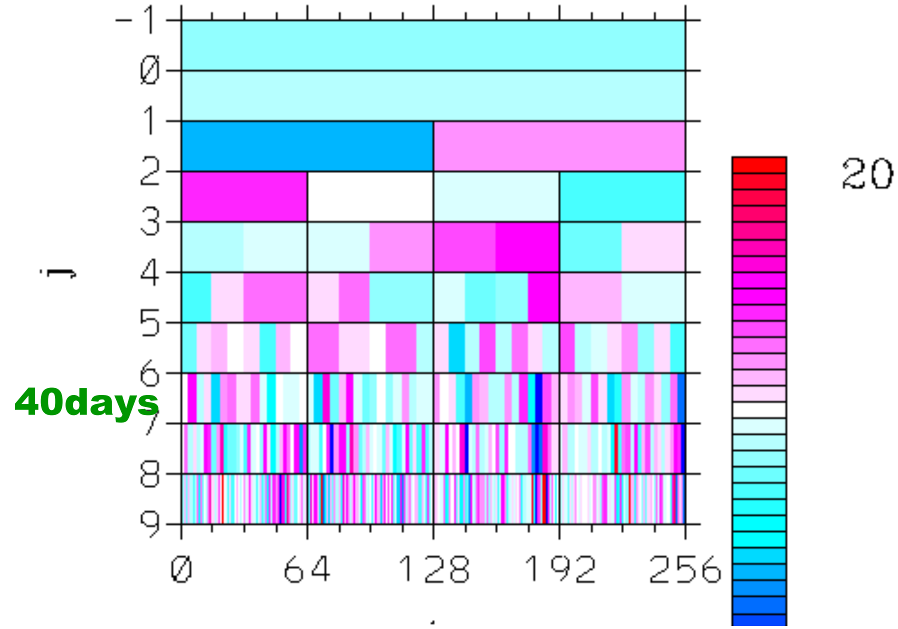


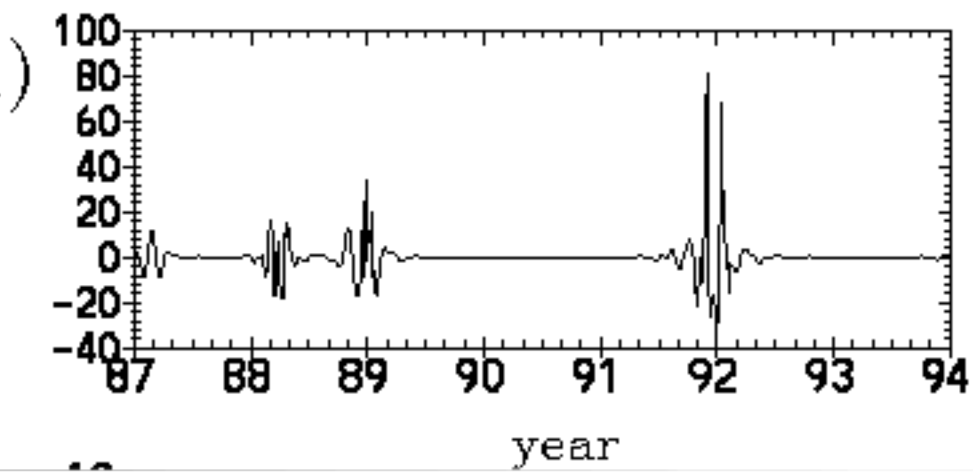
Extraction of Isolated Fe

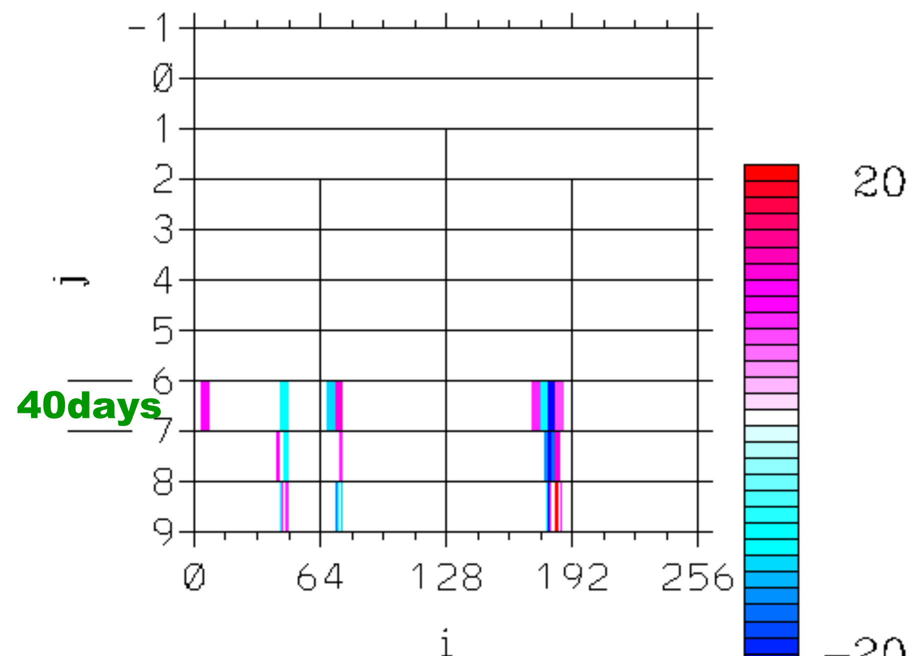
One-Dimensional Demon

Zonal Wind over Wester





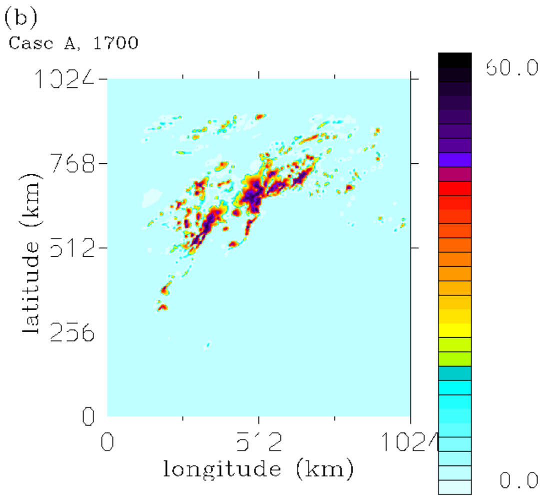




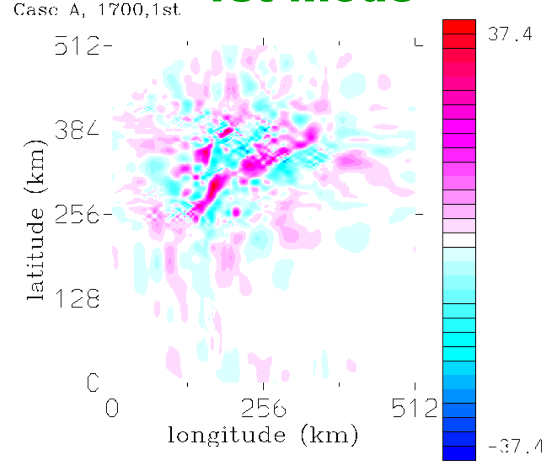
Extraction of Isolated Features:

Two-Dimensional Generalization:

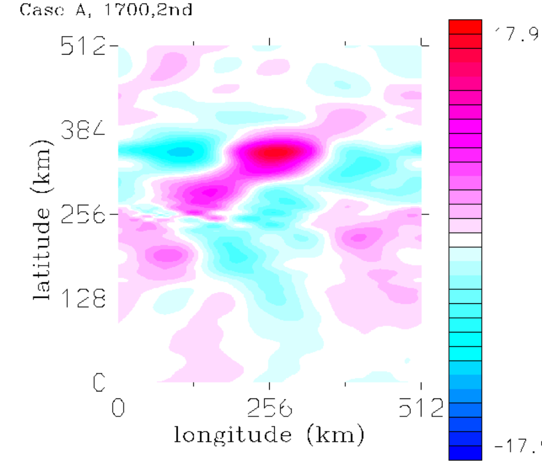
Conceptually Straightforward:



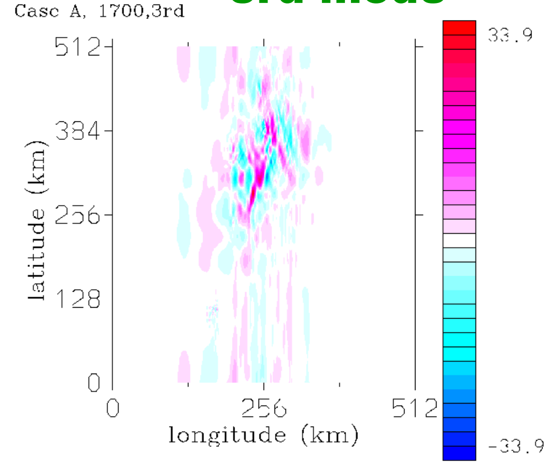
1st mode



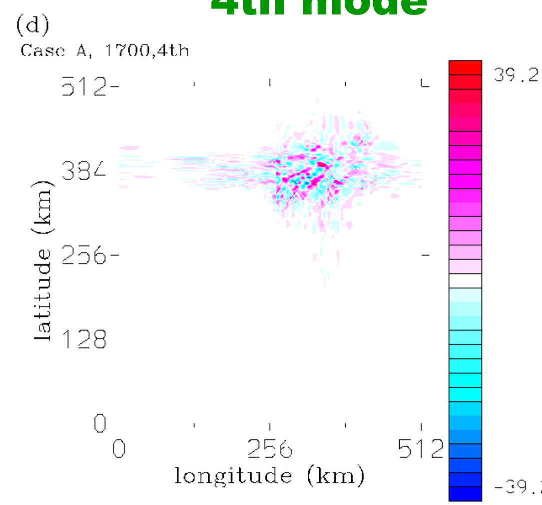
2nd mode



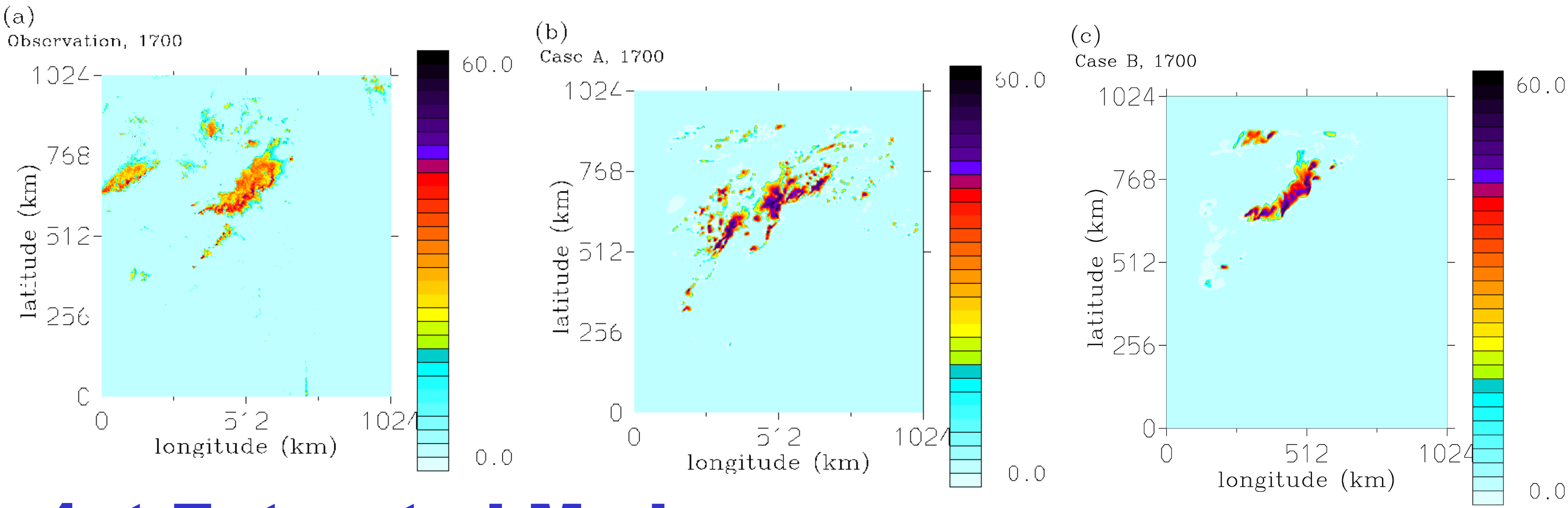
3rd mode



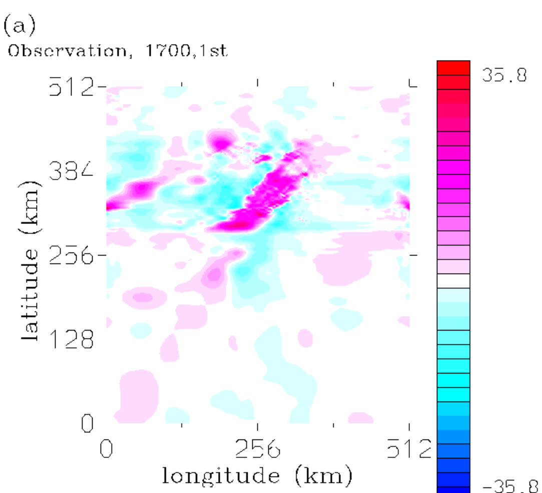
4th mode



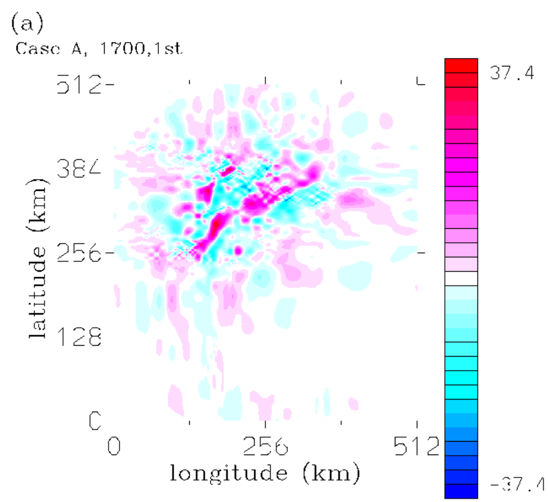
Extraction of Isolated Features:



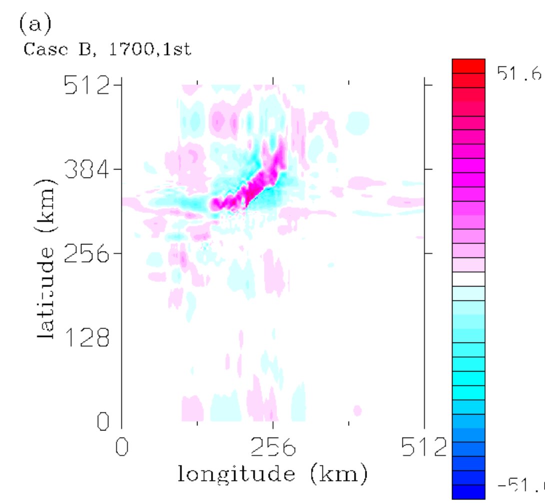
1st Extracted Modes:



Radar Observation



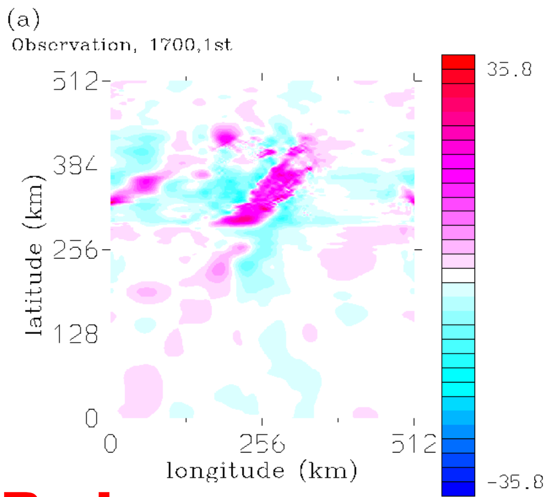
Forecast A



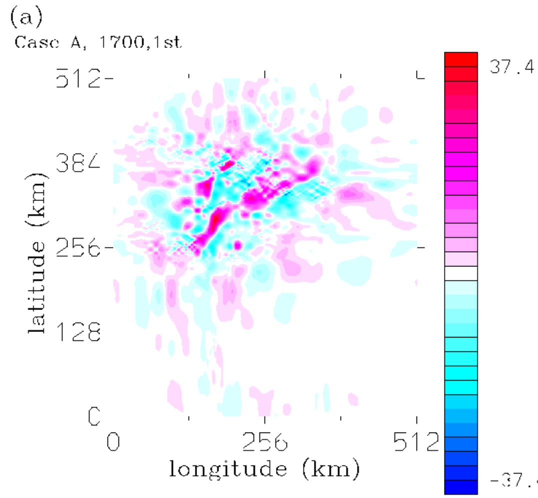
Forecast B

Extraction of Isolated Features:

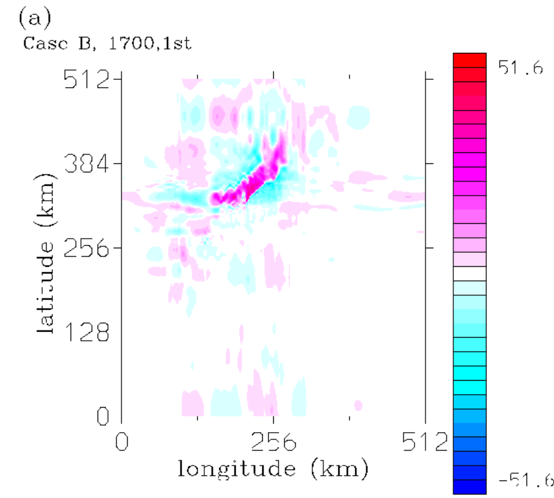
1st Extracted Modes:



Radar Observation

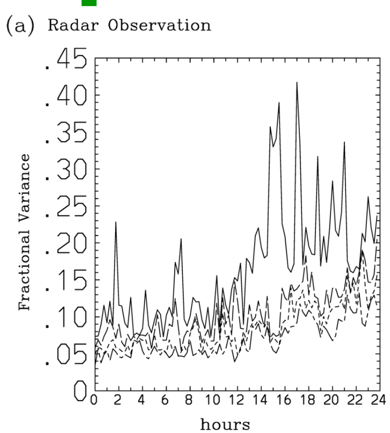


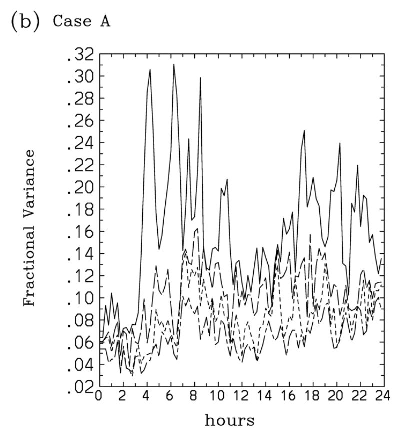
Forecast A

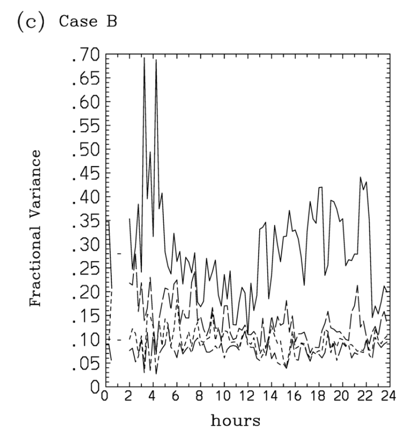


Forecast B

Explained Variance of First 4 Modes:





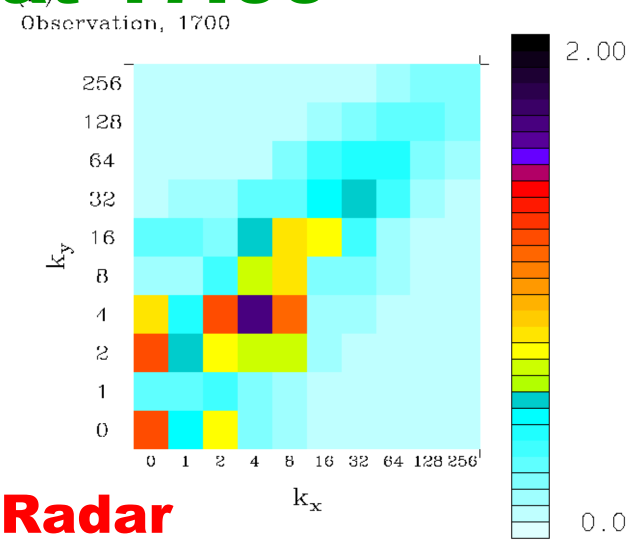


Fourier-Type Analysis:

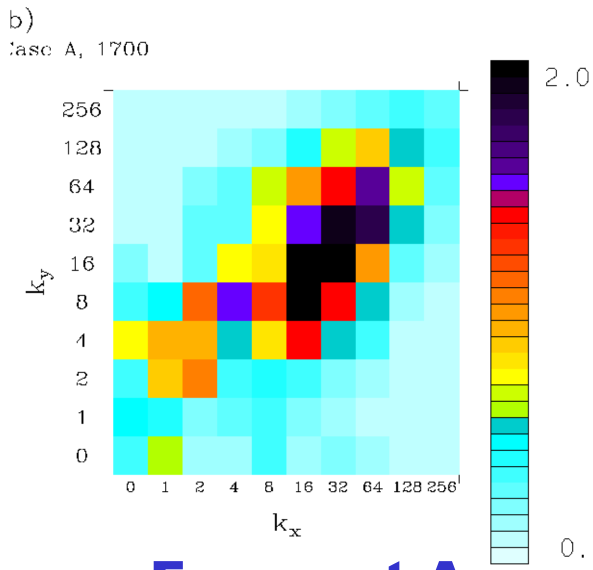
Wavenumer-Spectrum Power:

$$P(k_x, k_y) = \sum_{j_x=1}^{max(1,k_x)} \sum_{j_y=1}^{max(1,k_y)} \tilde{\varphi}^2_{i_x,i_y,j_x,j_y}$$

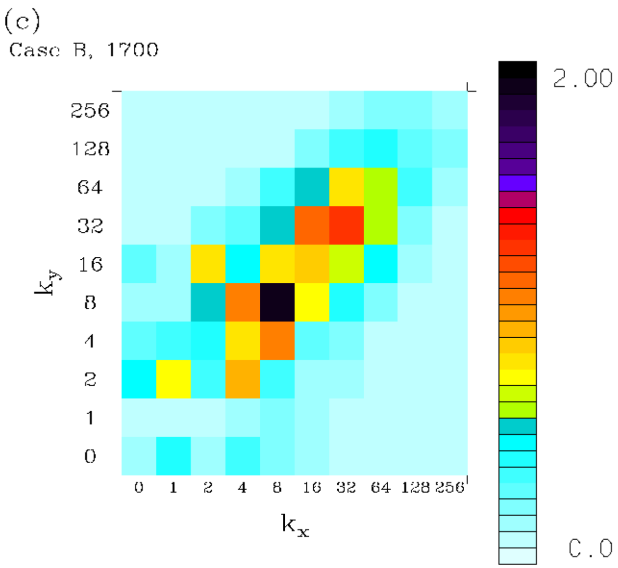
at 17:00



Radar Observation



Forecast A



Forecast B

Further Perspectives:

Applications to the Standard Fourier-Based Method for the Intermittency Analyses:

Just Replace Fourier by Wavelet

Advantage:

No Windowing Issue Any More

Summary

Wavelet:

- Capacity of Quantifying the Intermittency effectively:

- Orthogonality and Completeness
- Automatic Windowing
- Many Options with Flexibility

Q: What We Want to Quantify?

Further Possibility:

Careful Process Study in Wavelet Space

Bottom Line

Don't Use Continuous Wavelet

Thanks to Contributions of:
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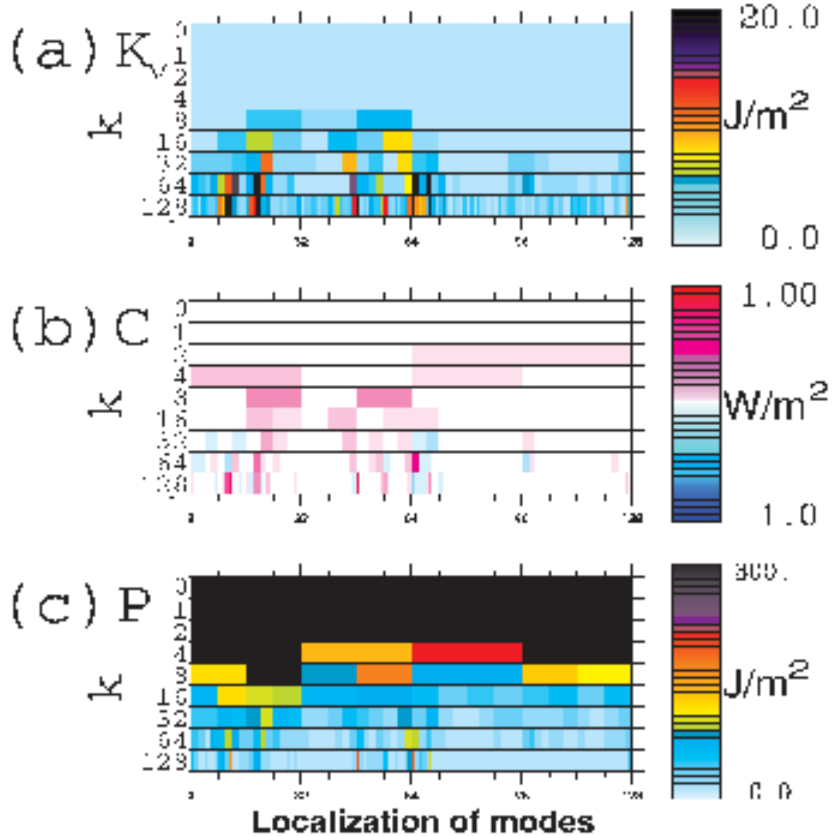
F. Guichard, B. Jakubiak

The Meyer Code is provided by:
Michio Yamada

Exmaple of Process Study in Wavelet Space:

Energy-Conversion Cycle

(Yano et al. 2005, QJ):



Konvective Kinetic Energy

Conversion Rate

Available Potential Energy