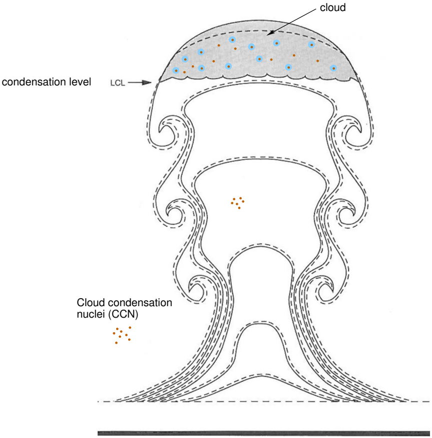
On models of stochastic condensation in clouds

Gustavo Abade

University of Warsaw · Institute of Geophysics · Faculty of Physics

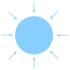
Atmospheric Physics Seminar

March 5, 2021

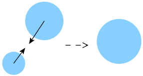


Microphysical processes

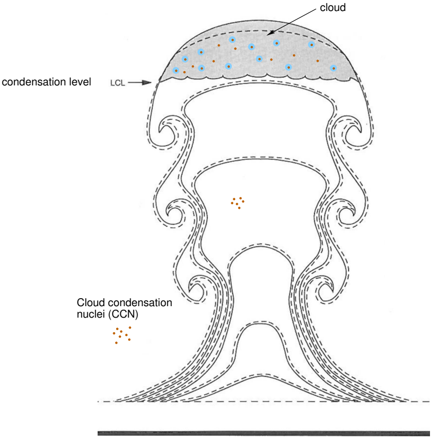
condensation

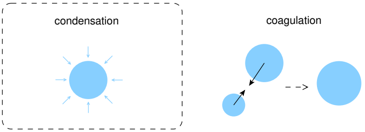


coagulation

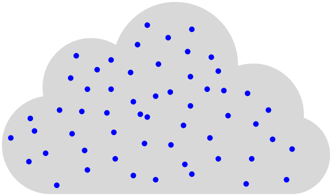


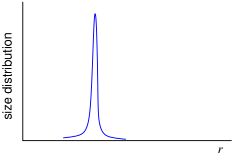
Microphysical processes





Narrow size distribution - **stable** cloud





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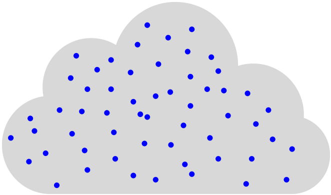
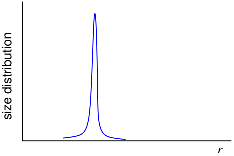
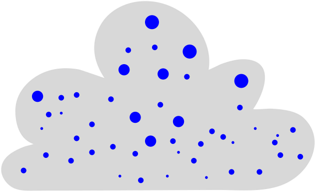
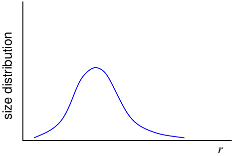




Broad size distribution - **unstable** cloud





Which is the **most likely** distribution?

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Maximum entropy principle

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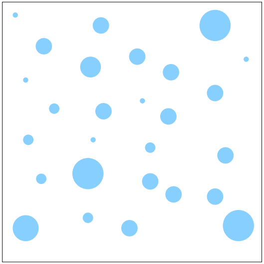
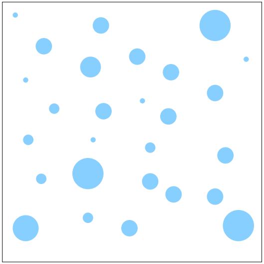
Maximum entropy principle

Liu, Atmos. Res., **12** (1995); Yano, J.-I., JAS, **76** (2019)

Closed cloud parcel



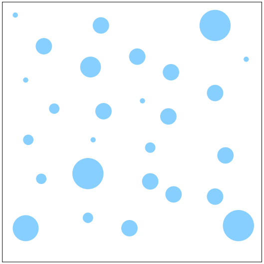
Closed cloud parcel



Given:

- N - number of droplets
- M - total mass of liquid water

Closed cloud parcel

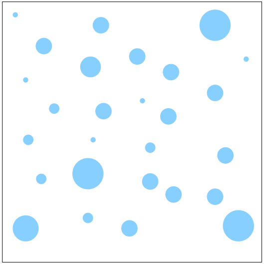


Given:

- N - number of droplets
- M - total mass of liquid water

Which is the **most likely** distribution $f(r)$?

Closed cloud parcel



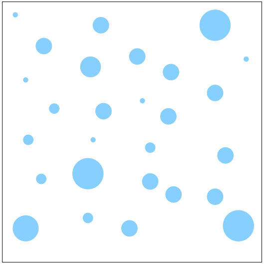
The most likely f **maximizes** the spectral **entropy**:

$$H = -\int f(x)[\ln f(x)]\,\mathrm{d}x$$

x - droplet mass

Liu, Atmos. Res., **12** (1995); Yano, J.-I., JAS, **76** (2019)

Closed cloud parcel



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x - droplet mass

$+$ constraints

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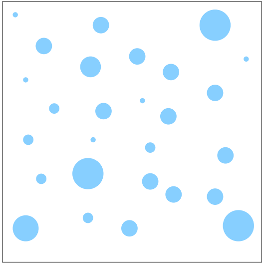
Closed cloud parcel



The most likely f **maximizes** the spectral **entropy**:

$$H = -\int f(x)[\ln f(x)]\, \mathrm{d}x$$

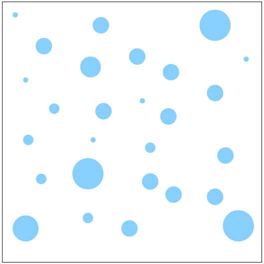
x - droplet mass

$$f(x)\mathrm{d}x = f(r)\mathrm{d}r$$

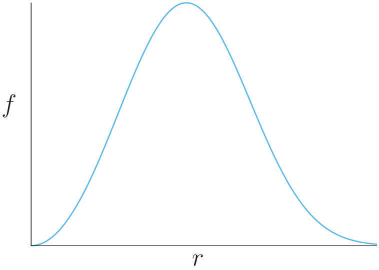
Liu, Atmos. Res., **12** (1995); Yano, J.-I., JAS, **76** (2019)

Weibull distribution

Closed cloud parcel



$$f(r) \sim r^2 \exp(-\beta r^3)$$



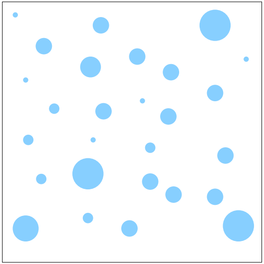
Liu, Atmos. Res., **12** (1995)

Which is the **least likely** distribution?

Which is the **least likely** distribution?

Maximum energy principle

Closed cloud parcel

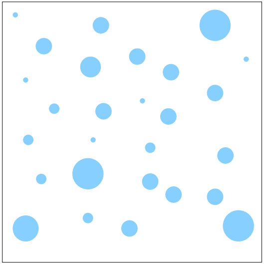


The least likely f **maximizes** the populational **energy**:

$$E = E_{\text{latent}} + E_{\text{surface}} + \dots$$

Liu, Atmos. Res., **12** (1995)

Closed cloud parcel



The least likely f **maximizes** the populational **energy**:

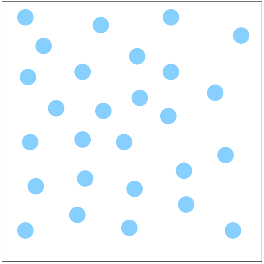
$$E = E_{\text{latent}} + E_{\text{surface}} + \ldots$$

- constraints

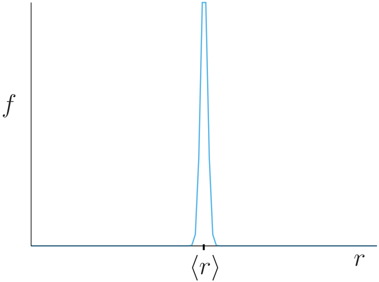
Liu, Atmos. Res., **12** (1995)

Monodisperse cloud

Closed cloud parcel

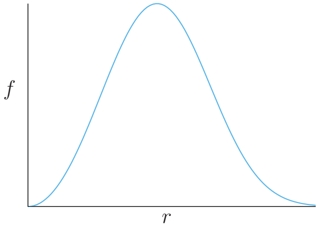


$$f(r) \sim \delta(r - \langle r \rangle)$$

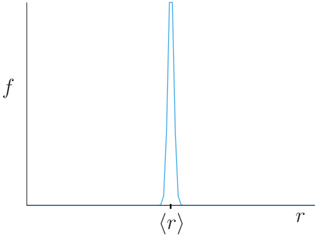


Liu, Atmos. Res., **12** (1995)

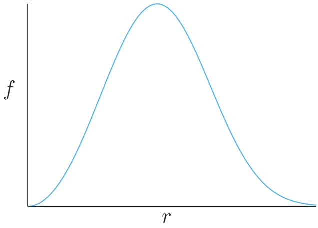
Most likely



Least likely



Most likely



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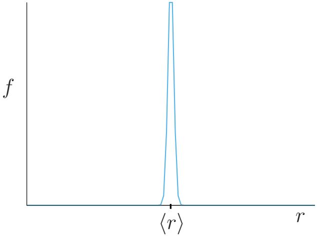
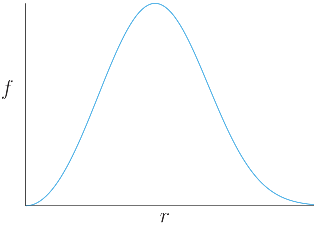


Many issues:

Most likely



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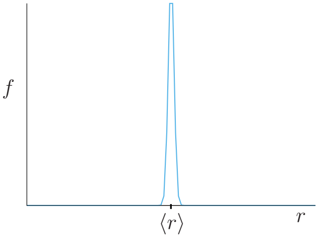
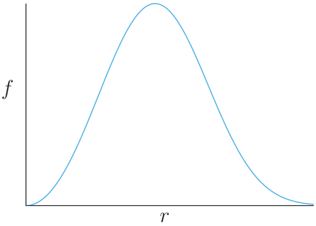


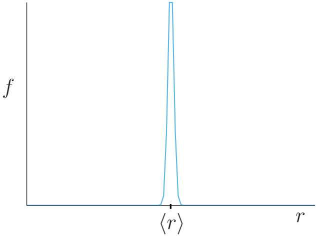
Many issues:

- “no dynamics”

Most likely



Least likely

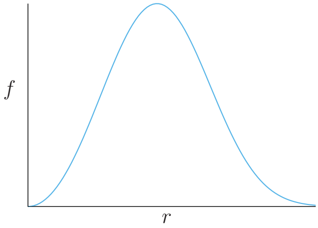


Many issues:

- “no dynamics”

- equilibrium × non-equilibrium

Most likely



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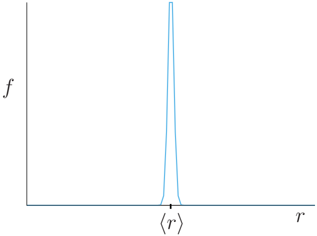


Many issues:

- "no dynamics"
- equilibrium $\times$ non-equilibrium
- closed system $\times$ open system

Let us describe the **condensation process**

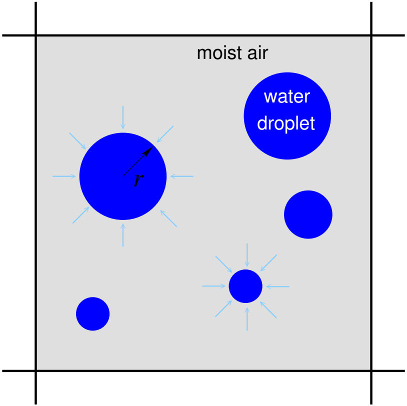
Diffusional growth

Droplet growth

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{r} D \langle S \rangle$$

$\langle S \rangle$ - mean-field supersaturation

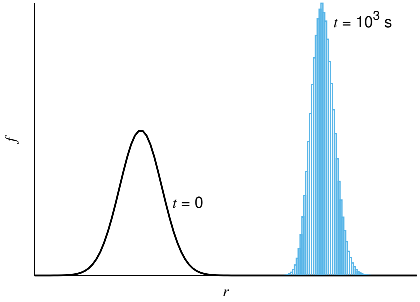
LES grid box



$\Delta \in$ inertial range

Equation for $f(r, t)$

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial r}(\dot{r} f) + \cdots$$

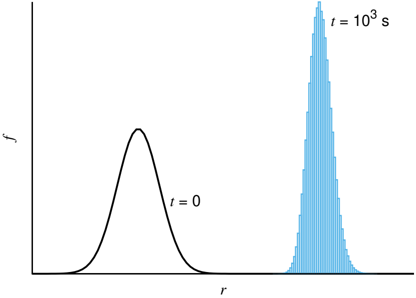


Equation for $f(r,t)$

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial r}(\dot{r} f) + \cdots$$

Advection in radius space with velocity

$$\dot{r} = \frac{D\langle S\rangle}{r}$$



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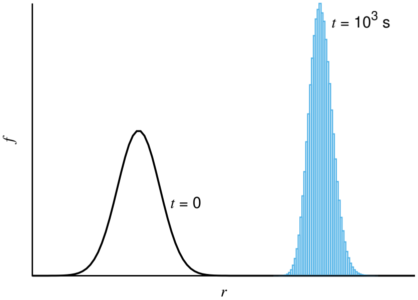
Narrow size distribution!



Eulerian stochastic model

JOURNAL OF THE ATMOSPHERIC SCIENCES

Toward the Theory of Stochastic Condensation in Clouds. Part I: A General Kinetic Equation

VITALY I. KHVOROSTYANOV

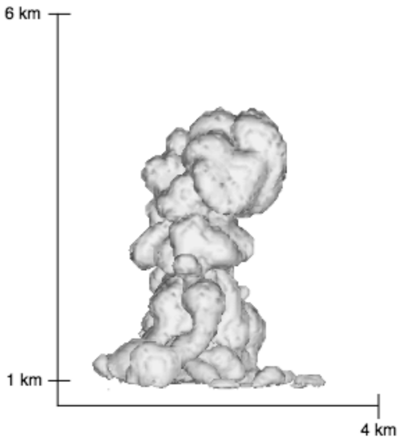
Department of Meteorology, University of Utah, Salt Lake City, Utah

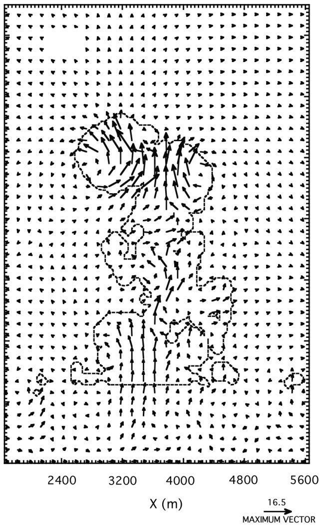
JUDITH A. CURRY

Department of Aerospace Engineering Sciences, Program in Atmospheric and Oceanic Sciences, University of Colorado, Boulder, Colorado

(Manuscript received 7 August 1997, in final form 11 February 1999)

Numerical simulation





Lasher-Trapp *et al.*, QJRMS, **131** (2005)

Kinetic equation for $f(r; \mathbf{x}, t)$

$$\frac{\partial f}{\partial t} + \nabla \cdot (\mathbf{u} f) = -\frac{\partial}{\partial r}(\dot{r} f) + \cdots \qquad \dot{r} = \frac{DS}{r}$$

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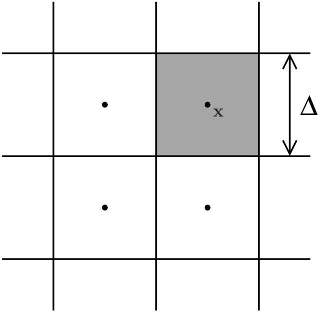

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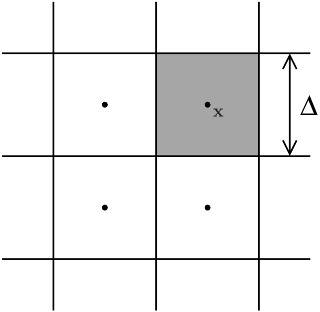

dependent variable = mean + fluctuation

Kinetic equation for $f(r; \mathbf{x}, t)$

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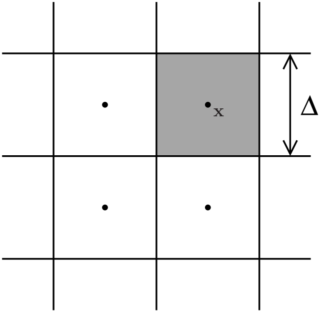

$$f = \langle f \rangle + f'$$

$$\mathbf{u} = \langle \mathbf{u} \rangle + \mathbf{u}'$$

$$\dot{r} = \langle \dot{r} \rangle + \dot{r}'$$

$$S = \langle S \rangle + S'$$

$$\frac{\partial \langle f \rangle}{\partial t} + \nabla \cdot [\langle \mathbf{u} \rangle \langle f \rangle] = -\frac{\partial}{\partial r}[\langle \dot{r} \rangle \langle f \rangle] + \nabla \cdot \langle \mathbf{u}' f' \rangle + \frac{\partial}{\partial r} \langle \dot{r}' f' \rangle + \cdots$$

$$
\begin{aligned}
\frac{\partial \langle f \rangle}{\partial t} + \nabla \cdot [\langle \mathbf{u} \rangle \langle f \rangle] &= -\frac{\partial}{\partial r}[\langle \dot{r} \rangle \langle f \rangle] \\
&+ \nabla \cdot \langle \mathbf{u}' f' \rangle + \frac{\partial}{\partial r} \langle \dot{r}' f' \rangle + \cdots
\end{aligned}
$$

Mean growth rate (narrowing):

$$
\langle \dot{r} \rangle = \frac{D \langle S \rangle}{r}
$$

$$\begin{aligned}\frac{\partial\langle f\rangle}{\partial t} + \nabla\cdot[\langle \mathbf{u}\rangle\langle f\rangle] &= -\frac{\partial}{\partial r}[\langle \dot{r}\rangle\langle f\rangle] \\ &+ \nabla\cdot\langle \mathbf{u}' f'\rangle + \frac{\partial}{\partial r}\langle \dot{r}' f'\rangle + \cdots\end{aligned}$$

Mean growth rate (narrowing):

$$\langle \dot{r}\rangle = \frac{D\langle S\rangle}{r}$$

Turbulent effect (broadening):

$$\langle \mathbf{u}' f'\rangle = ? \qquad \langle \dot{r}' f'\rangle = ?$$

Slow microphysics

$$\langle \mathbf{u}' f' \rangle = -K \nabla \langle f \rangle$$

K – turbulent diffusivity

Slow microphysics

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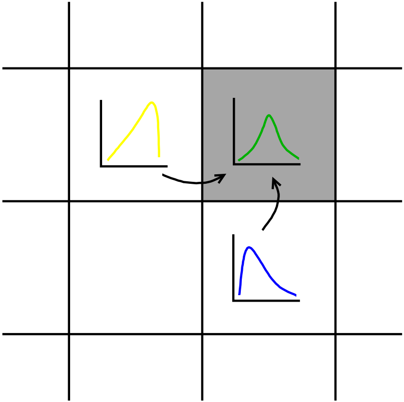
K − turbulent diffusivity



Slow microphysics

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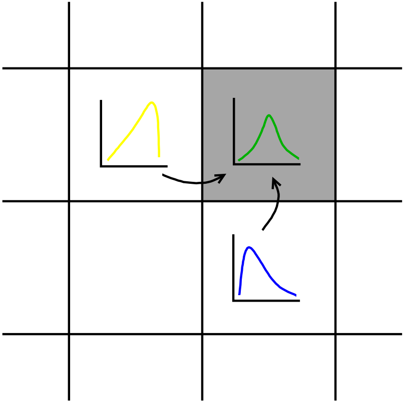


Cannot explain the observed spectrum broadening

Buikov, M. V. (1960's)

Fast microphysics

$$\langle \mathbf{u}' f' \rangle = -K \, \nabla \langle f \rangle \; + \; ?$$

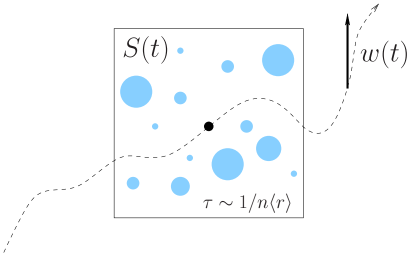
$$\langle \dot{r}' f' \rangle = \; ? \; + \; ?$$

Fast microphysics

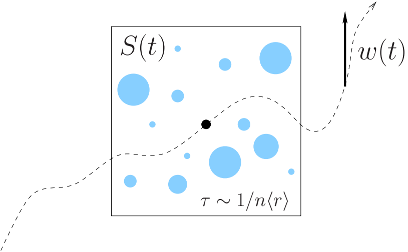
$$\langle \mathbf{u}' f' \rangle = -K \, \nabla \langle f \rangle \;\; + \;\; ?$$

$$\langle \dot{r}' f' \rangle = \; ? \; + \; ?$$

supersaturation fluctuations $\leftrightarrow$ vertical velocity fluctuations

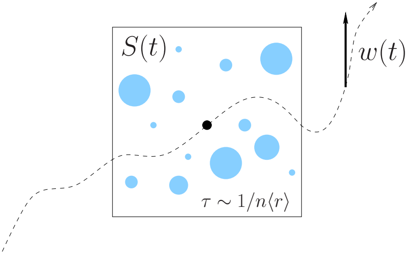


$S(t)$
$w(t)$
$\tau \sim 1/n\langle r \rangle$



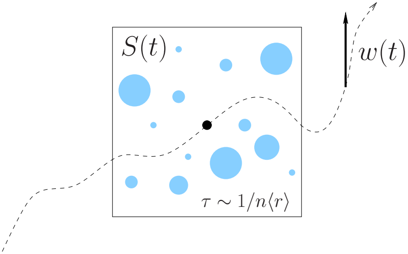
$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{S}{\tau} + a\, w(t)$$

τ – phase relaxation time



$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{1}{\tau}\left[S - S_{\mathrm{eq}}(t)\right]$$

$$S_{\mathrm{eq}}(t) = a\, w(t)\, \tau$$



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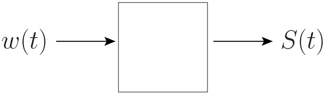

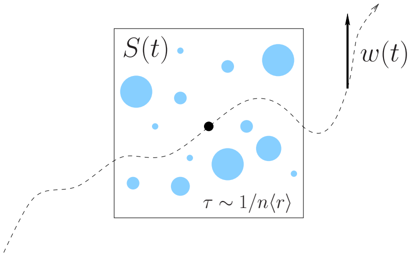

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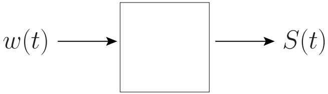
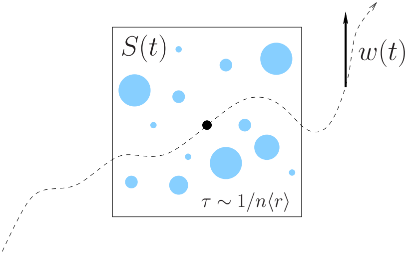
Slow microphysics (τ big)

$$\langle S' w' \rangle = 0$$





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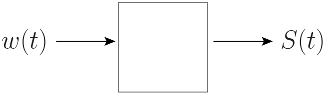
Slow microphysics (τ big)

$$\langle S' w' \rangle = 0$$

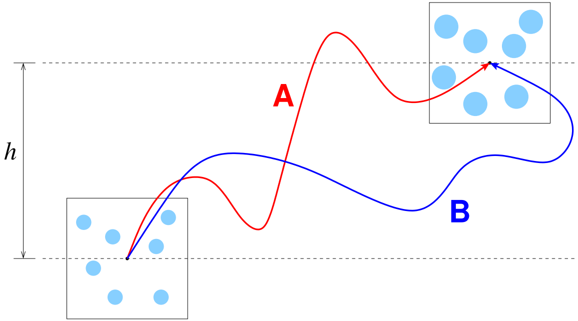


Fast microphysics (τ small)

$$S(t) \sim w(t)$$

Condensation reversibility

$$S(t) \sim w(t)$$



Condensation reversibility

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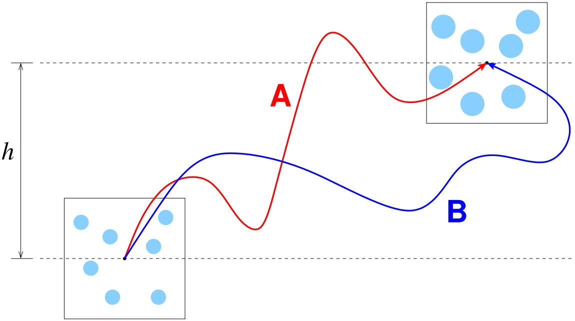
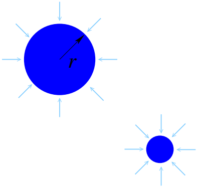


$$\dot{m} \sim \frac{\mathrm{d}r^3}{\mathrm{d}t} = C_1 w(t) \qquad m(z=h) - m(z=0) = C_2 h$$

A. Khain *et al.*, Atmos. Res., **55** (2000)



Growth rate

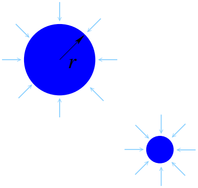
$$\dot{r} = \frac{D\langle S\rangle}{r} \sim \frac{1}{r}$$

Mass rate

$$\dot{m} \sim \frac{\mathrm{d}r^3}{\mathrm{d}t} \sim r^2\,\dot{r} \sim r$$

Supersaturation absorption is faster for larger droplets

Effective **microscale** supersaturation



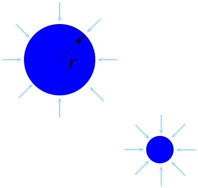
$$S_{\text{eff}} \approx \langle S \rangle \, \langle r \rangle^{-1} \, r$$

$$\dot{r}_{\text{eff}} = \frac{D S_{\text{eff}}}{r} \approx \frac{D \langle S \rangle}{\langle r \rangle}$$

Suppress the growth of smaller droplets

Effective **microscale** supersaturation

$$S_{\text{eff}} \approx \langle S \rangle \, \langle r \rangle^{-1} \, r$$



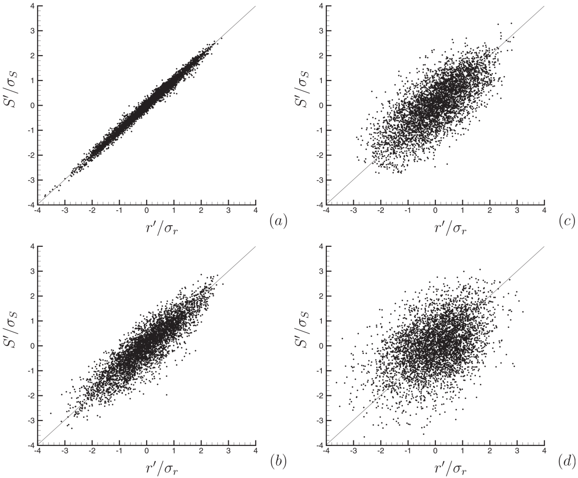
$$\dot{r}_{\text{eff}} = \frac{D S_{\text{eff}}}{r} \approx \frac{D \langle S \rangle}{\langle r \rangle}$$

Fluctuation in growth rate

$$\dot{r}' \approx \frac{D S'}{\langle r \rangle}$$

Suppress the growth of smaller droplets

Direct Numerical Simulations



$$S' = S - \langle S \rangle$$

$$r' = r - \langle r \rangle$$

Paoli and Sharif, JAS, **66** (2009)

$$\langle \mathbf{u}' f' \rangle = -K \nabla \langle f \rangle - K_1 \partial_r \langle f \rangle$$

$$\langle \mathbf{u}' f' \rangle = -K\nabla\langle f\rangle - K_1\partial_r\langle f\rangle$$

$$\langle \dot{r}' f' \rangle = -K_2\nabla\langle f\rangle - K_3\partial_r\langle f\rangle$$

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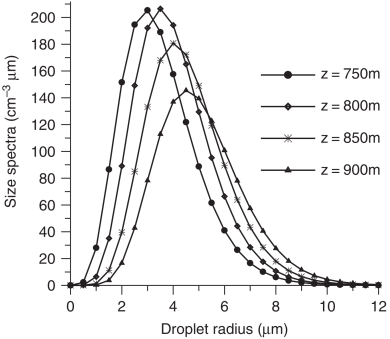
K_i – effective diffusion coefficients

Gamma distribution

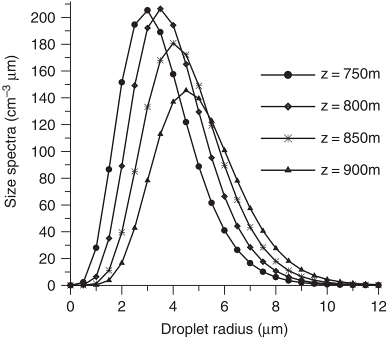
$$f(r) \sim r^p \exp(-\beta r)$$



Khvorostyanov and Curry, JAS, **66** (1999)

Gamma distribution

$$f(r) \sim r^p \exp(-\beta r)$$



Khvorostyanov and Curry, JAS, **66** (1999)

- power law $p \sim 5 - 10$

Gamma distribution

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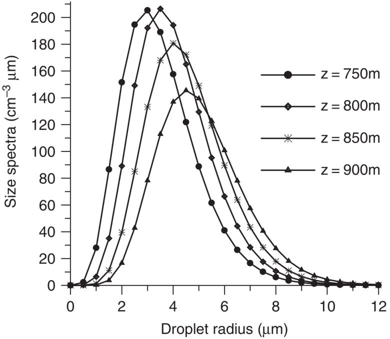

- power law $p \sim 5 - 10$
- exponential tail

Khvorostyanov and Curry, JAS, **66** (1999)

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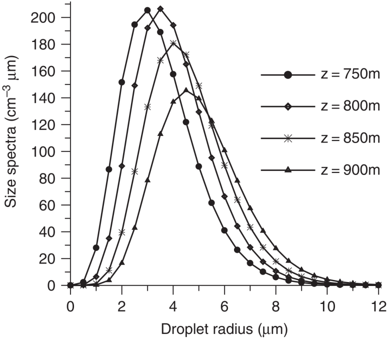
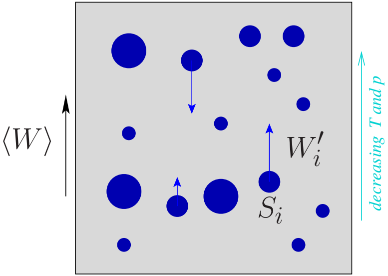


- power law $p \sim 5 - 10$
- exponential tail
- only one mode!

Khvorostyanov and Curry, JAS, **66** (1999)

Lagrangian stochastic model

Lagrangian model



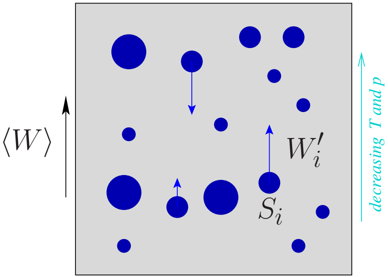
$$\frac{\mathrm{d}S_i'}{\mathrm{d}t} = -\frac{S_i'}{\tau_\mathrm{c}} - \frac{S_i'}{\tau_\mathrm{m}} + aW_i'(t)$$

$$\tau_\mathrm{c} \sim \frac{1}{N\langle r \rangle} \quad \text{(condensation)}$$

$$\tau_\mathrm{m} \sim \text{eddy turnover time} \quad \text{(mixing)}$$

Celani *et al.*, EPL, **70** (2005); Grabowski and Abade, JAS, **74** (2017)

Lagrangian model



$$\frac{\mathrm{d}S_i'}{\mathrm{d}t} = -\frac{S_i'}{\tau_c} - \frac{S_i'}{\tau_m} + aW_i'(t)$$

$$\tau_c \sim \frac{1}{N\langle r \rangle} \quad \text{(condensation)}$$

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- $W'(t)$: prescribed stochastic process.

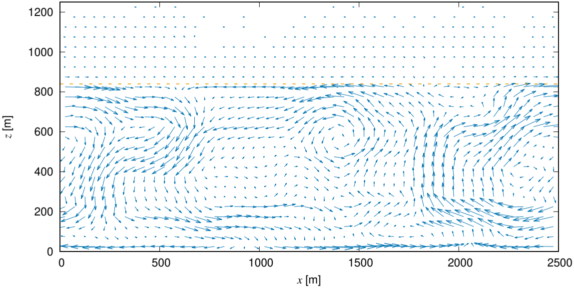
Celani *et al.*, EPL, **70** (2005); Grabowski and Abade, JAS, **74** (2017)

Kinematic framework

Synthetic turbulent-like flow

Turbulent-like flow

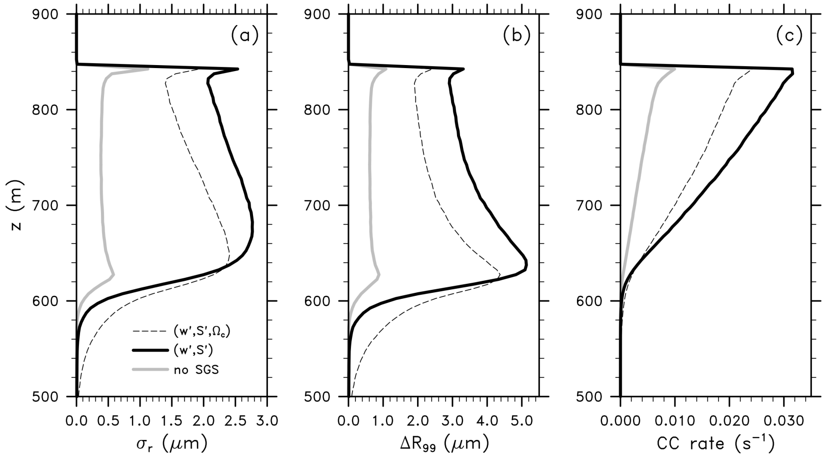
$$\mathbf{u} = (u, w) = \left(\frac{\partial\psi}{\partial z}, -\frac{\partial\psi}{\partial x}\right) \qquad \psi(\mathbf{r}, t) = \sum \text{random harmonics}$$



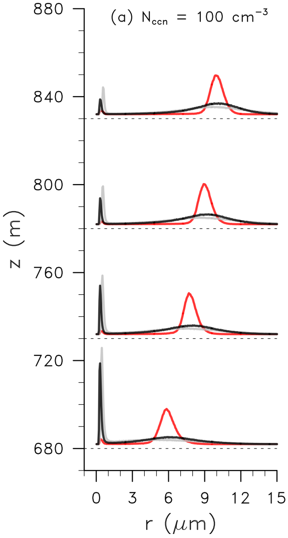
$$\langle w^2 \rangle = \sigma_w^2(z) \qquad \langle w(x', z) w(x' + x, z) \rangle = \hat{C}_w(x)\, \sigma_w^2(z)$$

Pinsky *et al.*, JAS, **65** (2008), ..., Magaritz-Ronen *et al.*, ACP, **16** (2016)

Vertical profiles

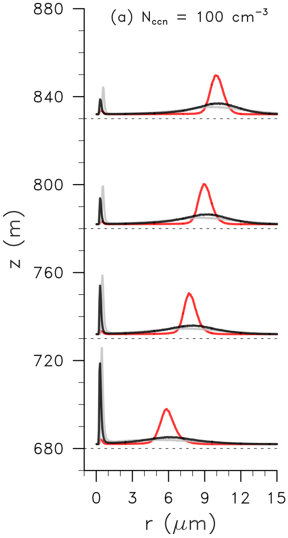


Size distribution at different heights



— no turbulence — turbulence (model 1) — turbulence (model 2)

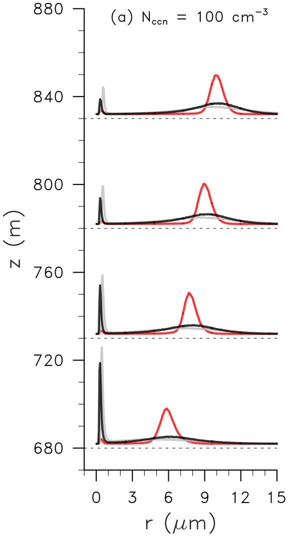
Size distribution at different heights



- broader distribution in the presence of turbulence

— no turbulence — turbulence (model 1) — turbulence (model 2)

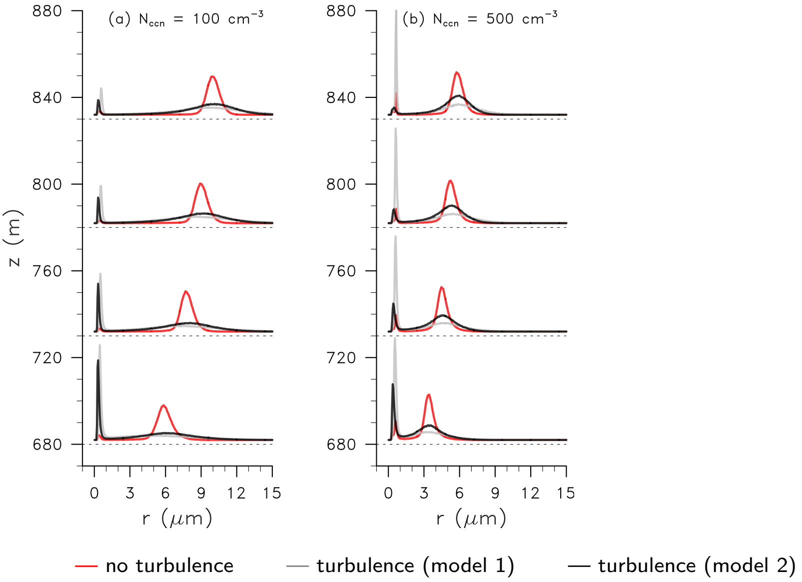
Size distribution at different heights



- broader distribution in the presence of turbulence
- bimodal distributions

— no turbulence — turbulence (model 1) — turbulence (model 2)

Size distribution at different heights



Summary

- Problem of narrow $\times$ broad size distributions in clouds

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- Subgrid fluctuations in Eulerian models are tricky

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THANK YOU!