Adiabatic Liquid Water Content

See Chapter 6.5 in Judith A. Curry & Peter J. Webster, Thermodynamics of Atmospheres & Oceans. Academic Press International Geophysics Series, Volume 65.

The amount of water vapor condensed ($q_l = m_l/m$, where m_l is the mass of liquid water, and m is the mass of the cloudy air) in adiabatic process is given by:

$$dq_l = \frac{c_p}{L_v}\left(\Gamma_d - \Gamma_s\right) dz \tag{1}$$

where q_l is the specific mass of liquid water, c_p is the specific heat at constant pressure, and L_v is the latent heat of vaporization. $\Gamma_d = g/c_p$ is the dry adiabatic lapse rate,

$$\Gamma_s = \Gamma_d \frac{1 + \frac{L_v r_s}{R_d T}}{1 + \frac{\epsilon L_v^2 r_s}{c_p R_d T^2}} \quad \text{is the saturated moist adiabatic lapse rate.}$$

$r_s = \epsilon(e_s/p$ is the saturation water vapor mixing ratio, e_s is the saturation water vapor pressure (can be expressed as: $e_s = e_{s0} exp\frac{17.27(T-273.15)}{(T-273.15)+237.7}$, where $e_{s0} = 611Pa$).

Eq. 1 can be expressed as $dq_l = c_q dz$, where $c_q = \frac{c_p}{L_v}\left(\Gamma_d - \Gamma_s\right)$ is called the condensation rate. The condensation rate is a function of temperature, T, and pressure, p.

For shallow clouds (up to 500 m thick) the condensation rate, c_q is approximately constant and takes the same value as at the cloud base, $c_q(T_0, p_0)$. Eq.1 can be integrated:

$$q_l(z) = c_q(T_0, p_0)(z - z_0), \text{ where } z_0 \text{ is the cloud base height.}$$

The liquid water content (LWC) is:

$$LWC = \frac{m_l}{V} = \frac{m_l}{m} \cdot \frac{m}{V} = \rho q_l \text{ where } \rho \text{ is the air density.}$$

Eq.1 can be written in a form:

$$d\left(\frac{LWC}{\rho}\right) = \frac{c_p}{L_v}\left(\Gamma_d - \Gamma_s\right) dz \tag{2}$$

As in the case of the specific mass of liquid water, q_l, the liquid water content can be approximated by a linear function:

$$LWC(z) = c_{LWC}\,(z - z_0)$$

where $c_{LWC} = \rho_0 c_q(T_0, p_0)$, and ρ_0 is the density of the air at the cloud base.