Turbulence and atmospheric boundary layer
Lecture 8

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Summary of lecture 7

1. 2D turbulence - flows in thin films
2. Vorticity formulation
3. Difference between 2D and 3D flows - vortex stretching term is absent in 2D turbulence !!!
4. Energy cascade in 2D turbulence
   - No dissipation anomaly = energy cannot be dissipated at small scales
   - No dissipation anomaly = inverse energy cascade
   - No dissipation anomaly = no anomalous scaling - Kolmogorov's predictions exactly satisfied
   - Enstrophy dissipation anomaly = direct enstrophy cascade
5. Kraichnan's picture of the double energy cascade
6. Scale break in the atmospheric turbulence

Phenomenology of the Atmospheric Boundary Layer.

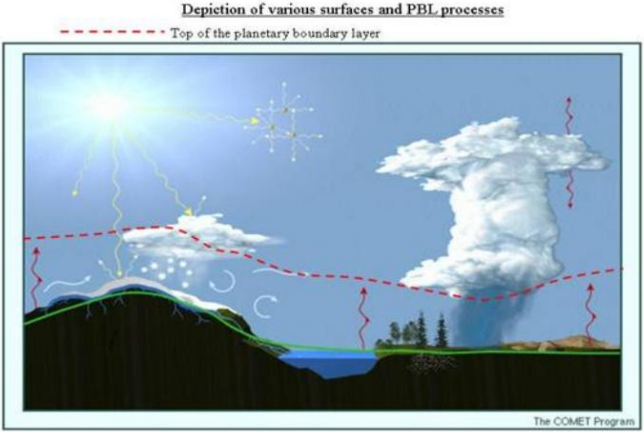


Figure: Earth Systems Research Laboratory, NOAA. Source: http://www.esrl.noaa.gov/research/themes/pbl/img/fig1.jpg

Boundary Layer

In classical fluid dynamics, a boundary layer is the layer in a nearly inviscid fluid next to a surface in which frictional drag associated with that surface is significant (term introduced by L. Prandtl, 1905).
Such boundary layers can be laminar or turbulent, and are often only mm thick.

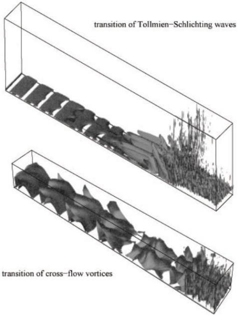


Figure: Author: Herbert.Oertel / www.prof.oertel.de, licence: CC BY 3.0

Planetary Boundary Layer - definitions

The Atmospheric Boundary Layer *(ABL, sometimes called P[lanetary] BL) is the layer of fluid directly above the Earths surface in which significant fluxes of momentum, heat and/or moisture are carried by turbulent motions whose horizontal and vertical scales are of the order of the boundary layer depth, and whose circulation timescale is a few hours or less.*
(J.R. Garratt, 1992.)
that part of the troposphere that is directly influenced by the presence of the earths surface, and responds to surface forcings with a timescale of about an hour or less
(Stull 1988, p. 2)
lowest kilometer or lowest portion of the atmosphere, which intensively exchanges heat as well as mass and momentum with the earths surface
(Sorbjan 1989,p.1)

Planetary Boundary Layer

The complexity of this definition is due to several complications compared to classical aerodynamics.

- Surface heat exchange can lead to thermal convection
- Moisture and effects on convection.
- Earths rotation
- Complex surface characteristics and topography

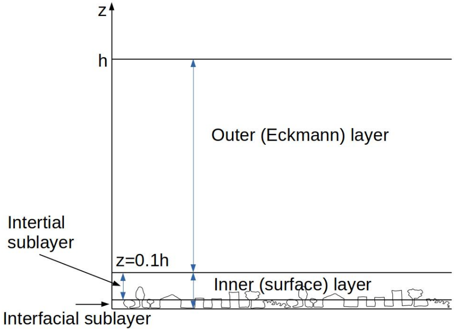
Boundary Layer Meteorology

Boundary Layer Meteorology is a field of science which studies physical, chemical and biological processes occurring in the lowest few kilometers of the Earth's atmosphere.

BLM traditionally includes the study of fluxes of heat, moisture and momentum between the atmosphere and the underlying surface. The investigations include the characterization of surfaces so as to predict these fluxes (this includes characterization of the surface roughness, radiative characteristics etc.). Different types of surfaces are e.g. plant canopies, water, ice, snow, bare ground, etc.

Most workers (but not all) include shallow cumulus in BL, but deep precipitating cumuli are usually excluded from scope of the Boundary Layer Meteorology (BLM) due to longer time for moist air to recirculate back from clouds into contact with surface.

Structure of the atmospheric boundary layer



The boundary layer itself exhibits dynamically distinct sublayers:

- Interfacial sublayer
- Inner (surface) layer
- Outer (Eckmann) layer

Structure of the atmospheric boundary layer

- Interfacial sublayer - molecular viscosity/diffusivity dominate vertical fluxes
- Inner (surface) layer - occupies the lowest 10% of the ABL, which is about 1 to 100m. It is the region at the bottom of the boundary layer where turbulent fluxes and stresses vary by less than 10% of their magnitude. the dominant scales of motion are still much less than the boundary layer depth.
- Outer (Eckmann) layer - turbulent fluid motions with scales of motion comparable to the boundary layer depth (large eddies). In this sublayer, force balance between pressure gradient force, Coriolis force and turbulent drag is observed. (Formation of the Eckmann spiral)

Capping inversion

At the top of the outer layer, the BL is often capped by an entrainment zone in which turbulent BL eddies are entraining non-turbulent free-atmospheric air. This entrainment zone is often associated with a stable layer or inversion.

This stable layer traps turbulence, pollutants, and moisture below it and prevents most of the surface friction from being felt by the free atmosphere.

Diurnal cycle

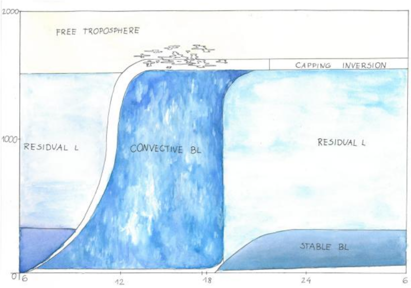


Figure: Diurinal cycle of PBL. From PhD thesis of E. O. Akinlabi, author: Z. Wacławczyk.

During fair weather we are accustomed to the diurnal (daily) cycle of changes in temperature, humidity, pollen, and winds that are governed by boundary-layer physics and dynamics. It is cool and calm at night; warm and gusty during daytime. (R. Stull, 2005)

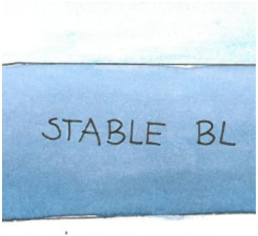
Diurnal cycle

During the day, bulk of heat from the sun reaches the Earths surface, and is then gradually transported from the surface upwards through the boundary layer.

The boundary layer is said to be unstable whenever the surface is warmer than the air, such as during a sunny day with light winds over land, or when cold air is advected over a warmer water surface. This boundary layer is in a state of free convection, with vigorous thermal updrafts and downdrafts. (R. Stull, 2005)

Diurnal cycle

The boundary layer is said to be stable when the surface is colder than the air, such as during a clear night over land, or when warm air is advected over colder water. (R. Stull, 2005)
As the night deepens, the surface cools downs, and the air immediately above also cools down, and becomes heavier, making the boundary layer stably stratified. (J.-I. Yano)



Diurnal cycle

Neutral boundary layers form during windy and overcast conditions, and are in a state of forced convection. That is, the heat transfer is possible through the turbulent motion of the air masses, without density variations.

Importance of the BLM

- Climate models and numerical weather predictions - ABL is so thin that a typical atmospheric model does not resolve its detailed structure well. A good parametrization of surface characteristics, air-surface exchange, BL thermodynamics fluxes and friction is crucial for the climate models.
- Air Pollution and Urban Meteorology - the study of pollutant dispersal, interaction of BL with mesoscale circulations. Urban heat island effects.
- Agricultural meteorology - Prediction of frost, dew, evapotranspiration
- Aviation - Prediction of fog formation and dissipation, dangerous wind-shear conditions
- Remote Sensing - Satellite-based measurements of surface winds, skin temperature, etc. Involve the interaction of BL and surface, and must often be interpreted in light of a BL model to be useful for NWP

Historical background

- around 1900 - Theoretical explanations of some observations from agriculture (creation of local frost hollows, etc.). Measuring of the solar radiation with the first radiometers, baloon measurements of changes in temperature and humidity with height.
- the first half of XXth century - development of turbulence theories: mixing length theory, eddy diffusivity - von Karman, Prandtl, Kolmogorov (1941) similarity theory of turbulence, Monin and Obuhkov (1954) theory - buoyancy effects on surface layer.
- 1960 -1970 The Golden Age of BLM. Accurate observations of a variety of boundary layertypes, including convective, stable and trade-cumulus. Verification/calibrationof surface similarity theory.
- 1970-1980 Introduction of resolved 3D computer modelling of BL turbulence (large-eddysimulation or LES). Application of higher-order turbulence closure theory.

Historical background

- 1980 - 1990 Major field efforts in stratocumulus-topped boundary layers and land-surface, vegetation parameterization. Mesoscale modeling.
- 1990 - The Age of Technology.
  New surface remote sensing tools (lidar, cloud radar) and extensive space-based coverage of surface characteristics.
  LES as a tool for improving parameterizations and bridging to observations.
  Improved BL parameterizations required by coupled ocean-atmosphere-ice-biosphere and medium-range forecast models.
  Accurate routine mesoscale modelling for urban air flow; coupling to air pollution.

Reynolds-averaged equations

$$u_i = \overline{u_i} + u_i', \quad b = \overline{b} + b', \quad p = \overline{p} + p'$$

Momentum balance

$$\underbrace{\frac{\partial \overline{u_i}}{\partial t} + \overline{u_j}\frac{\partial \overline{u_i}}{\partial x_j}}_{(1)} + \underbrace{\frac{\partial \overline{u_i' u_j'}}{\partial x_j}}_{(2)} = \underbrace{-\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i}}_{(3)} + \underbrace{\nu \frac{\partial^2 \overline{u_i}}{\partial x_j \partial x_j}}_{(4)} + \underbrace{\delta_{i3}\overline{b}}_{(5)} + \underbrace{\epsilon_{ij3} f \overline{u_j}}_{(6)}$$

1. rate of change following a point moving with mean velocity
2. Reynolds-stress term
3. mean pressure term
4. mean viscous term (important only in the interfacial layer)
5. mean buoyancy terms
6. mean Coriolis force term

Reynolds-averaged equations

Momentum balance

$$\underbrace{\frac{\partial \overline{u_i}}{\partial t} + \overline{u_j}\frac{\partial \overline{u_i}}{\partial x_j}}_{(1)} + \underbrace{\frac{\partial \overline{u_i' u_j'}}{\partial x_j}}_{(2)} = \underbrace{-\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial x_i}}_{(3)} + \underbrace{\nu \frac{\partial^2 \overline{u_i}}{\partial x_j \partial x_j}}_{(4)} + \underbrace{\delta_{i3}\overline{b}}_{(5)} + \underbrace{\epsilon_{ij3} f \overline{u_j}}_{(6)}$$

$$b = g\frac{\theta_v}{\theta_0}, \quad \theta_V = \theta\left(1 + 0.61 q_V - q_l\right)$$

where θ_V is the virtual potential temperature, ρ_0 and θ_0 are characteristic ABL density and potential temperature, q_V - water vapor mixing ratio, q_l - liquid water mixing ratio

Reynolds-averaged equations

Energy balance

$$\underbrace{\frac{\partial \overline{\theta}}{\partial t} + \overline{u_j}\frac{\partial \overline{\theta}}{\partial x_j}}_{(1)} + \underbrace{\frac{\partial \overline{\theta' u_j'}}{\partial x_j}}_{(2)} = \underbrace{\kappa \frac{\partial^2 \overline{\theta}}{\partial x_j \partial x_j}}_{(3)} + \underbrace{S_\theta}_{(4)}$$

Continuity

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0$$

1. rate of change following a point moving with mean velocity
2. buoyancy-flux term
3. mean molecular transport term
4. Source or sink term due to radiation - proportional to the divergence of the net radiative heat flux

Reynolds-averaged equations

Balance equation for the mixing ratio $q = q_V + q_l$

$$\underbrace{\frac{\partial \overline{q}}{\partial t} + \overline{u_j}\frac{\partial \overline{q}}{\partial x_j}}_{(1)} + \underbrace{\frac{\partial \overline{q' u_j'}}{\partial x_j}}_{(2)} = \underbrace{\kappa \frac{\partial^2 \overline{q}}{\partial x_j \partial x_j}}_{(3)} + \underbrace{S_q}_{(4)}$$

1. rate of change following a point moving with mean velocity
2. mixing ratio turbulent flux term
3. mean molecular transport term
4. Source due to precipitation

For cloud-topped boundary layers condensation, precipitation and evaporation can also be important

BL approximation

H - vertical length scale,
$L >> H$ horizontal length scale
$\mathcal{U}$ - longitudinal scale of mean velocity
$\mathcal{V}$ - characteristic scale of vertical component of the mean velocity
u' - velocity scale of turbulent perturbations,

Continuity

$$\underbrace{\frac{\partial \overline{u}}{\partial x}}_{\mathcal{U}/L} + \underbrace{\frac{\partial \overline{v}}{\partial y}}_{\mathcal{U}/L} + \underbrace{\frac{\partial \overline{w}}{\partial z}}_{\mathcal{V}/H} = 0$$

The terms are of the same order, hence

$$\frac{\mathcal{U}}{L} \sim \frac{\mathcal{V}}{H}$$

BL approximation

We can non-dimensionalize the equation

Continuity

$$\frac{\mathcal{U}}{L}\frac{\partial \overline{u^*}}{\partial x^*} + \frac{\mathcal{U}}{L}\frac{\partial \overline{v^*}}{\partial y^*} + \frac{\mathcal{V}}{H}\frac{\partial \overline{w^*}}{\partial z^*} = 0$$

We assume

$$\frac{\mathcal{U}}{L} = \frac{\mathcal{V}}{H}$$

Continuity equation in the non-dimensional form

$$\frac{\partial \overline{u^*}}{\partial x^*} + \frac{\partial \overline{v^*}}{\partial y^*} + \frac{\partial \overline{w^*}}{\partial z^*} = 0$$

where u^*, v^*, w^*, x^*, y^*, z^* are non-dimensional variables

BL approximation

Momentum balance - longitudinal component

$$\underbrace{\frac{\partial \overline{u}}{\partial t}}_{\mathcal{U}/T} + \underbrace{\overline{u}\frac{\partial \overline{u}}{\partial x}}_{\mathcal{U}^2/L} + \underbrace{\overline{v}\frac{\partial \overline{u}}{\partial y}}_{\mathcal{U}^2/L} + \underbrace{\overline{w}\frac{\partial \overline{u}}{\partial z}}_{\mathcal{U}\mathcal{V}/H} + \underbrace{\frac{\partial \overline{u'^2}}{\partial x}}_{u'^2/L} + \underbrace{\frac{\partial \overline{u'v'}}{\partial y}}_{u'^2/L} + \underbrace{\frac{\partial \overline{u'w'}}{\partial z}}_{u'^2/H} =$$

$$\underbrace{-\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial x}}_{\mathcal{U}^2/L} + \underbrace{\nu\frac{\partial^2 \overline{u}}{\partial x^2}}_{\nu\mathcal{U}/L^2} + \underbrace{\nu\frac{\partial^2 \overline{u}}{\partial y^2}}_{\nu\mathcal{U}/L^2} + \underbrace{\nu\frac{\partial^2 \overline{u}}{\partial z^2}}_{\nu\mathcal{U}/H^2} + \underbrace{f\overline{v}}_{f\mathcal{U}}$$

We note

$$\frac{u'^2}{H} >> \frac{u'^2}{L}, \quad \nu\frac{\mathcal{U}}{H^2} >> \nu\frac{\mathcal{U}}{L^2}$$

because $L >> H$

BL approximation

So, some terms cancel because of $\frac{u'^2}{H} >> \frac{u'^2}{L}, \quad \nu\frac{\mathcal{U}}{H^2} >> \nu\frac{\mathcal{U}}{L^2}$

Momentum balance - longitudinal component

$$\underbrace{\frac{\partial \overline{u}}{\partial t}}_{\mathcal{U}/T} + \underbrace{\overline{u}\frac{\partial \overline{u}}{\partial x}}_{\mathcal{U}^2/L} + \underbrace{\overline{v}\frac{\partial \overline{u}}{\partial y}}_{\mathcal{U}^2/L} + \underbrace{\overline{w}\frac{\partial \overline{u}}{\partial z}}_{\mathcal{U}\mathcal{V}/H} + \underbrace{\cancel{\frac{\partial \overline{u'^2}}{\partial x}}}_{\cancel{u'^2/L}} + \underbrace{\cancel{\frac{\partial \overline{u'v'}}{\partial y}}}_{\cancel{u'^2/L}} + \underbrace{\frac{\partial \overline{u'w'}}{\partial z}}_{u'^2/H} =$$

$$\underbrace{-\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial x}}_{\mathcal{U}^2/L} + \underbrace{\cancel{\nu\frac{\partial^2 \overline{u}}{\partial x^2}}}_{\cancel{\nu\mathcal{U}/L^2}} + \underbrace{\cancel{\nu\frac{\partial^2 \overline{u}}{\partial y^2}}}_{\cancel{\nu\mathcal{U}/L^2}} + \underbrace{\nu\frac{\partial^2 \overline{u}}{\partial z^2}}_{\nu\mathcal{U}/H^2} + \underbrace{f\overline{v}}_{f\mathcal{U}}$$

BL approximation

We obtain

Momentum balance - longitudinal component

$$\frac{\mathcal{U}}{T}\frac{\partial \overline{u^*}}{\partial t^*} + \frac{\mathcal{U}^2}{L}\overline{u^*}\frac{\partial \overline{u^*}}{\partial x^*} + \frac{\mathcal{U}^2}{L}\overline{v^*}\frac{\partial \overline{u^*}}{\partial y^*} + \frac{\mathcal{U}\mathcal{V}}{H}\overline{w^*}\frac{\partial \overline{u^*}}{\partial z^*} + \frac{u'^2}{H}\frac{\partial \overline{u'^* w'^*}}{\partial z^*} =$$
$$-\frac{\mathcal{U}^2}{\rho_0 L}\frac{\partial \overline{p^*}}{\partial x^*} + \nu\frac{\mathcal{U}}{H^2}\frac{\partial^2 \overline{u^*}}{\partial z^{*2}} + f\mathcal{U}\overline{v^*}$$

We assume $T = L/\mathcal{U}$ and from the continuity we have $\mathcal{U}/L = \mathcal{V}/H$. In the vicinity of the surface all terms are of the same order, in particular:

$$\frac{\mathcal{U}^2}{L} \sim \nu\frac{\mathcal{U}}{H^2},$$

hence

$$H^2 \sim \frac{\nu L}{\mathcal{U}}, \quad \text{and} \quad \frac{H}{L} \sim \sqrt{\frac{\nu}{\mathcal{U}L}} = \frac{1}{\sqrt{Re}}$$

Further from the surface the viscous terms are negligible.

BL approximation

Dividing both sides of the above equation by $\mathcal{U}^2/L$ we obtain

Momentum balance - non-dimensional form

$$\frac{\partial \overline{u^*}}{\partial t^*} + \overline{u^*}\frac{\partial \overline{u^*}}{\partial x^*} + \overline{v^*}\frac{\partial \overline{u^*}}{\partial y^*} + \overline{w^*}\frac{\partial \overline{u^*}}{\partial z^*} + \frac{u'^2}{\mathcal{U}^2}\frac{L}{H}\frac{\partial \overline{u'^* w'^*}}{\partial z^*} =$$
$$-\frac{\partial \overline{\mathcal{P}^*}}{\partial x^*} + \frac{L}{H}\frac{\nu}{H\mathcal{U}}\frac{\partial^2 \overline{u^*}}{\partial z^{*2}} + \frac{fL}{\mathcal{U}}\overline{v^*}$$

The terms with non-dimensional prefactors may become negligible, if the order of the term is smaller than the order of remaining terms in the equation e.g. the viscous term is negligible further from the surface, the Coriolis term can be neglected in the surface layer.

BL approximation

Momentum balance - vertical component

$$\underbrace{\frac{\partial \overline{w}}{\partial t}}_{\mathcal{V}/T} + \underbrace{\overline{u}\frac{\partial \overline{w}}{\partial x}}_{\mathcal{UV}/L} + \underbrace{\overline{v}\frac{\partial \overline{w}}{\partial y}}_{\mathcal{UV}/L} + \underbrace{\overline{w}\frac{\partial \overline{w}}{\partial z}}_{\mathcal{V}^2/H} + \underbrace{\frac{\partial \overline{w'^2}}{\partial x}}_{u'^2/L} + \underbrace{\frac{\partial \overline{w'v'}}{\partial y}}_{u'^2/L} + \underbrace{\frac{\partial \overline{w'^2}}{\partial z}}_{u'^2/H} =$$

$$\underbrace{-\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial z}}_{\mathcal{U}^2/H} + \underbrace{\nu\frac{\partial^2 \overline{w}}{\partial x^2}}_{\nu\mathcal{V}/L^2} + \underbrace{\nu\frac{\partial^2 \overline{w}}{\partial y^2}}_{\nu\mathcal{V}/L^2} + \underbrace{\nu\frac{\partial^2 \overline{w}}{\partial z^2}}_{\nu\mathcal{V}/H^2} + \overline{b}$$

Momentum balance - vertical component

$$\frac{\partial \overline{w'^2}}{\partial z} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial z} + \overline{b}$$

BL approximationn

Energy balance

$$\frac{\partial\overline{\theta}}{\partial t} + \overline{u}\frac{\partial\overline{\theta}}{\partial x} + \overline{v}\frac{\partial\overline{\theta}}{\partial y} + \overline{w}\frac{\partial\overline{\theta}}{\partial z} + \frac{\partial\overline{\theta' w'}}{\partial z} = \kappa\frac{\partial^2\overline{\theta}}{\partial z^2} + S_\theta$$

Balance of mean mixing ratio

$$\frac{\partial\overline{q}}{\partial t} + \overline{u}\frac{\partial\overline{q}}{\partial x} + \overline{v}\frac{\partial\overline{q}}{\partial y} + \overline{w}\frac{\partial\overline{q}}{\partial z} + \frac{\partial\overline{q' w'}}{\partial z} = \kappa\frac{\partial^2\overline{q}}{\partial z^2} + S_q$$

References

J.R. Garratt (1992)
The atmospheric boundary layer
Cambridge University Press

The End