Exercise Sheet 10

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1. Consider a steady flow in a pipe with circular and constant cross-section of radius R. The flow is induced by a pressure difference Δp across the length l of the pipe. Find:

   (a) The velocity profile $u(r)$

   (b) The mass of fluid per unit time passing through any cross-section.

2. A viscous fluid of viscosity μ occupies the gap between two circular cylinders, the inner having radius R_1 and angular velocity Ω_1, the other one having radius R_2 and angular velocity Ω_2.

   (a) Assume stationary and purely rotary motion, $\mathbf{u} = u(r)\hat{\mathbf{e}}_\theta$, and show that

   $$u(r) = Ar + \frac{B}{r},$$

   where

   $$A = \frac{\Omega_2 R_2^2 - \Omega_1 R_1^2}{R_2^2 - R_2^2}, \qquad B = \frac{(\Omega_1 - \Omega_2) R_1^2 R_2^2}{R_2^2 - R_1^2}.$$

   (b) Determine the moment M_1 of the frictional forces acting on the inner cylinder cylinder:

   $$M_1 = -\frac{4\pi\mu(\Omega_1 - \Omega_2) R_1^2 R_2^2}{R_2^2 - R_1^2}.$$

   How can this flow be used to determine the viscosity of the fluid?

3. A semi-infinite body of fluid with constant density ρ and viscosity μ is bounded by a flat plate. The plate is in the xy plane. The system is initially at rest. At time $t = 0$, the plate is suddenly set in motion in the x-direction with a constant velocity U.

   (a) Assume the flow has the form $\mathbf{u} = [u(z,t), 0, 0]$ with

   $$u = Uf(\eta), \qquad \eta = \frac{z}{\sqrt{4\nu t}},$$

   where $\nu = \mu/\rho$, and determine the velocity field.

(b) Evaluate the vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{u}$ and show that it is exponentially small beyond a distance of order $\sqrt{\nu t}$ from the boundary (flat plate).

4. Obtain the relevant dimensionless parameters in a problem involving shallow water waves described by the Navier-Stokes equations.