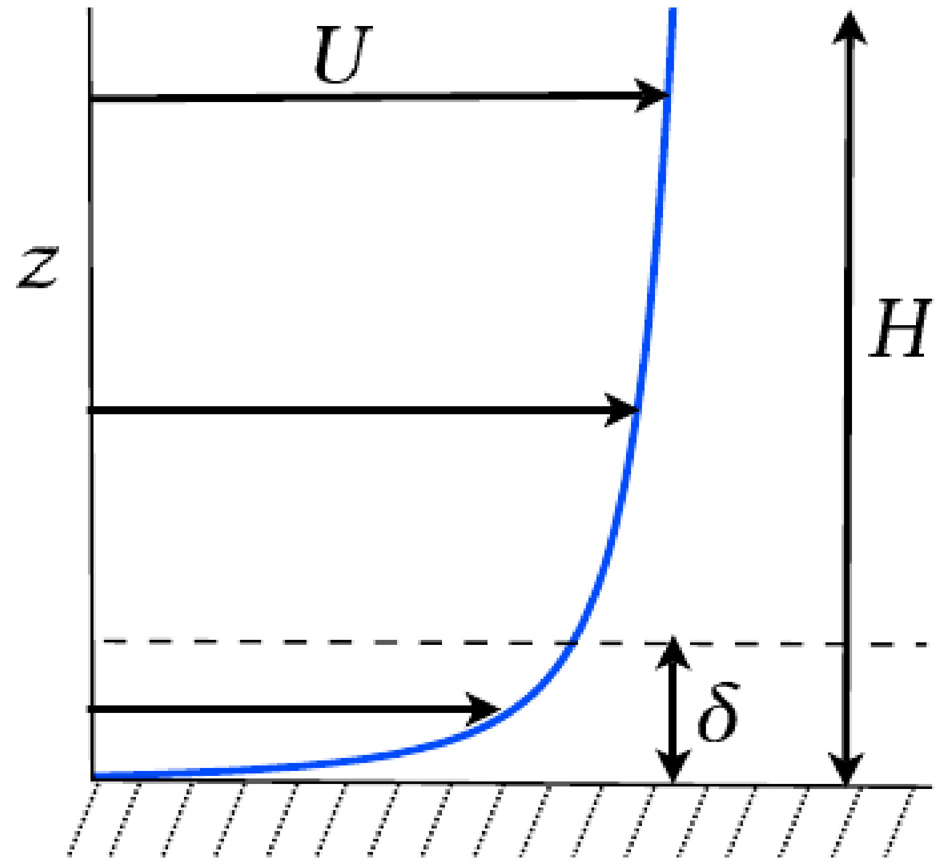
Dynamics of the Atmosphere and the Ocean

Lecture 7

Szymon Malinowski

2021 Fall



A schematic of a **boundary layer**. The values of a field, such as velocity, U , may vary rapidly with height in a boundary in order to satisfy the boundary conditions at a rigid surface. The parameter δ is a measure of the boundary layer thickness, and H is a typical scale of variation away from the boundary.

Boundary layer.

In classical fluid dynamics, a **boundary layer is the layer in a nearly inviscid fluid next to a surface in which frictional drag associated with that surface is significant** (term introduced by Prandtl, 1905).

Such boundary layers can be laminar or turbulent, and are often only mm thick.

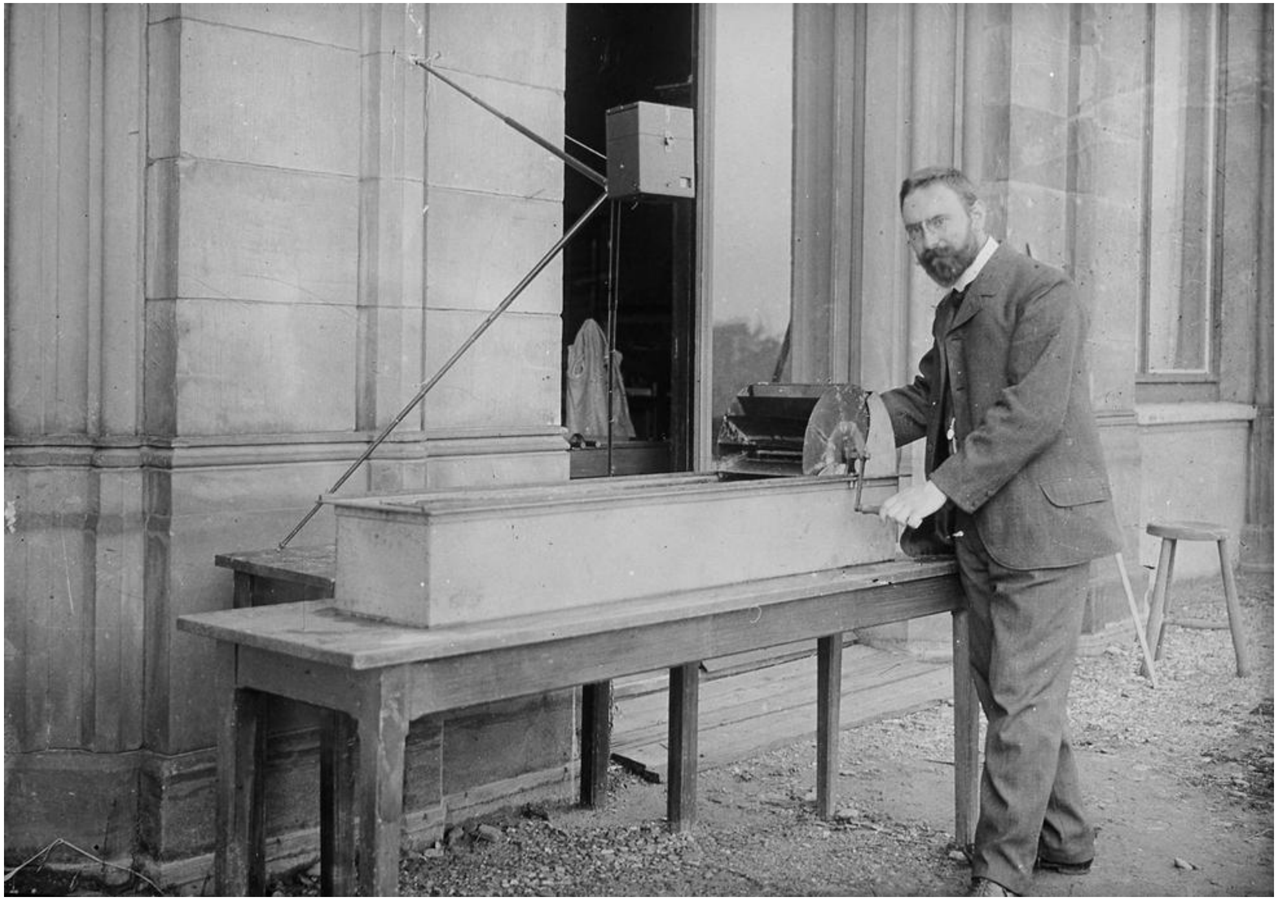
In atmospheric science, a similar definition is useful. The **atmospheric boundary layer (ABL, sometimes called P[lanetary] BL) is the layer of fluid directly above the Earth's surface in which significant fluxes of momentum, heat and/or moisture are carried by turbulent motions whose horizontal and vertical scales are on the order of the boundary layer depth, and whose circulation timescale is a few hours or less** (Garratt, p. 1).
A similar definition works for the ocean, but for a layer just below the ocean surface!

The complexity of this definition is due to several complications compared to classical aerodynamics.

i) Surface heat exchange can lead to thermal convection
ii) Moisture and effects on convection
iii) Earth's rotation
iv) Complex surface characteristics and topography.

BL is assumed to encompass surface-driven dry convection. Most workers (but not all) include shallow cumulus in BL, but deep precipitating cumuli are usually excluded from scope of BLM due to longer time for most air to recirculate back from clouds into contact with surface.

BLM also traditionally includes the study of fluxes of heat, moisture and momentum between the atmosphere and the underlying surface, and how to characterize surfaces so as to predict these fluxes (roughness, thermal and moisture fluxes, radiative characteristics). Includes plant canopies as well as water, ice, snow, bare ground, etc.



Ludwig Prandtl with his fluid test channel, 1904

Atmospheric boundary layer

sublayers:

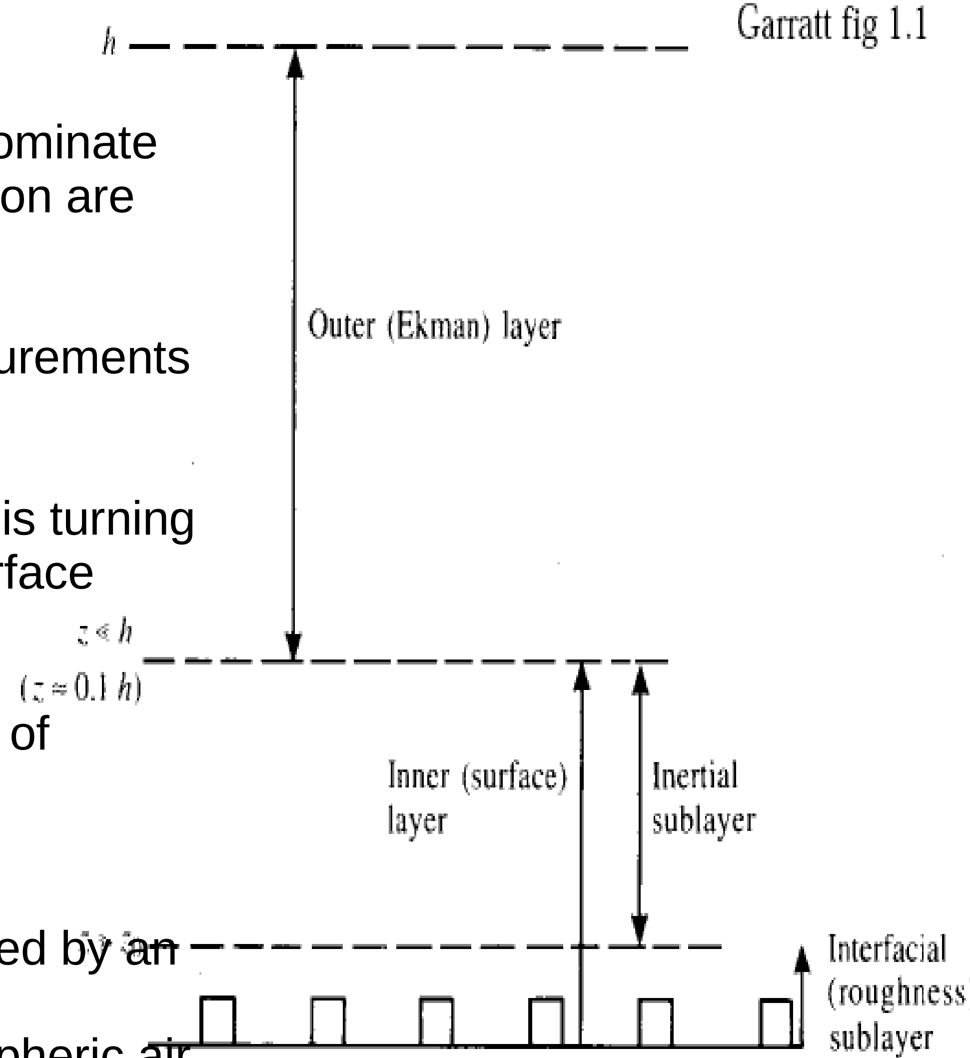
i) Interfacial sublayer - in which molecular viscosity/diffusivity dominate vertical fluxes
ii) Inertial layer - in which turbulent fluid motions dominate the vertical fluxes, but the dominant scales of motion are still much less than the boundary layer depth.

This is the layer in which most surface wind measurements are made.

• Layers (i) + (ii) comprise the surface layer. Coriolis turning of the wind with height is not evident within the surface layer.

iii) Outer layer - turbulent fluid motions with scales of motion comparable to the boundary layer depth (‘large eddies’).

• At the top of the outer layer, the BL is often capped by an entrainment zone in which turbulent BL eddies are entraining non-turbulent free-atmospheric air. This entrainment zone is often associated with a stable layer or inversion.
• For boundary layers topped by shallow cumulus, the outer layer is subdivided further into subcloud, transition, cumulus and inversion layer.



Garratt fig 1.1

Ekman layer

Vagn Walfrid Ekman
(3 May 1874 – 9 March 1954)
a Swedish oceanographer.

In many boundary layers in non-rotating flow the dominant balance in the momentum equation is between the advective and viscous terms.

In large scale atmospheric and oceanic flow the effects of rotation are large, and this results in a boundary layer, known as the Ekman layer, in which the dominant balance is between Coriolis and frictional terms.
Thus consider the effects of friction on geostrophic flow.
In practice a balance occurs between the Coriolis terms and the stress due to small-scale turbulent motion, and this gives rise to a boundary layer that has a typical depth of tens to hundreds of meters.

The atmospheric Ekman layer occurs near the ground, and the stress at the ground itself is due to the surface wind (and its vertical variation). In the ocean the main Ekman layer is near the surface, and the stress at ocean surface is largely due to the presence of the overlying wind.

ASSUMPTIONS:

1) The Ekman layer is Boussinesq. This is a very good assumption for the ocean, and a reasonable one for the atmosphere if the boundary layer is not too deep.

2) The Ekman layer has a finite depth that is less than the total depth of the fluid, this depth being given by the level at which the frictional stresses essentially vanish. Within the Ekman layer, frictional terms are important, whereas geostrophic balance holds beyond it.

3) The nonlinear and time dependent terms in the equations of motion are negligible, hydrostatic balance holds in the vertical, and buoyancy is constant, not varying in the horizontal.

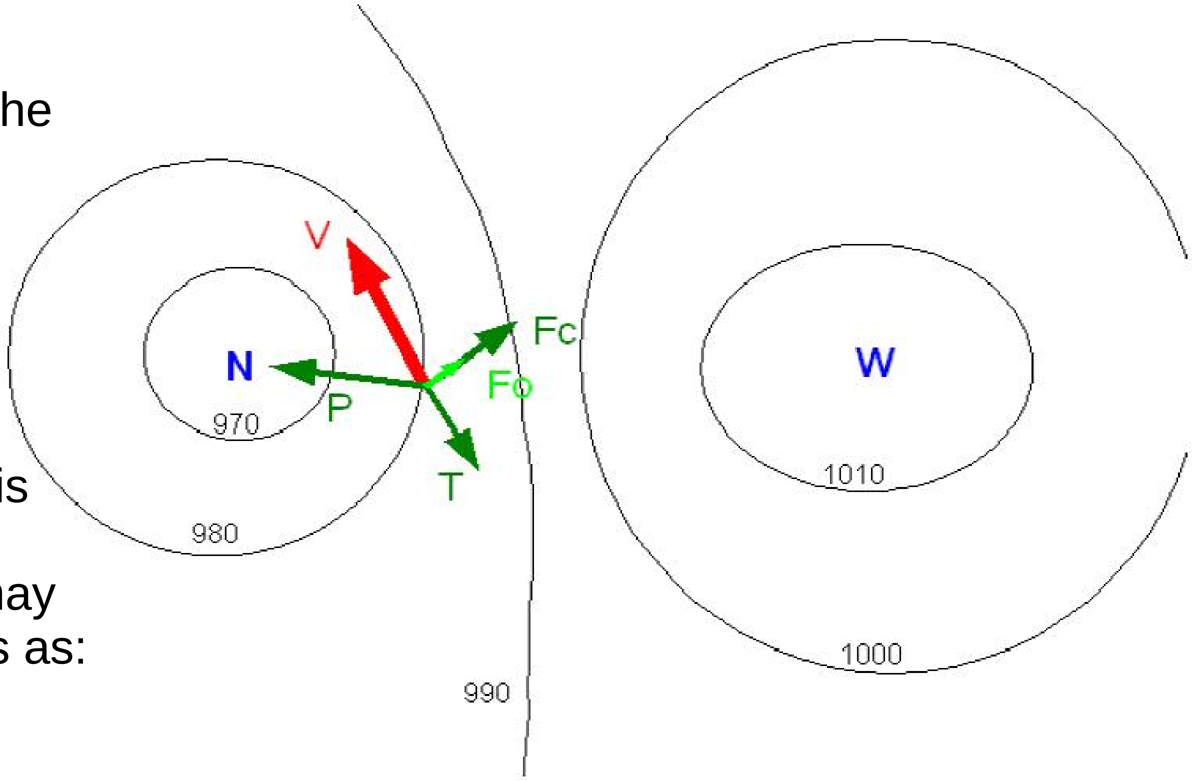
4) Friction can be parameterized by a viscous term of the form $\rho_0^{-1}\partial\boldsymbol{\tau}/\partial z = A\partial^2\boldsymbol{u}/\partial z^2$ where A is constant and τ is the stress.

In laboratory A may be the molecular viscosity, whereas in the atmosphere and ocean it is a so-called eddy viscosity.

Equations of motion and scaling

Frictional-geostrophic balance in the horizontal momentum equation is:

$$f \times u = -\nabla_z \phi + A \frac{\partial^2 u}{\partial z^2}.$$



The vertical momentum equation is hydrostatic balance.
Since buoyancy is constant, we may without loss of generality write this as:

$$\frac{\partial \phi}{\partial z} = 0.$$

The mass continuity equation is in form:

$$\nabla \cdot v = 0.$$

$$\boldsymbol{f} \times \boldsymbol{u} = -\nabla_z \phi + A \frac{\partial^2 \boldsymbol{u}}{\partial z^2}.$$

Let's non-dimensionalize the equations:

$$(u, v) = U(\hat{u}, \hat{v}), \quad (x, y) = L(\hat{x}, \hat{y}), \quad f = f_0 \hat{f}, \quad z = H\hat{z}, \quad \phi = \Phi \hat{\phi},$$

where hatted variables are non-dimensional and H is certain scaling for the height.

Geostrophic balance gives:

$$\Phi = f_0 U L.$$

And the non-dimensional equation of motion takes the form:

$$\hat{\boldsymbol{f}} \times \hat{\boldsymbol{u}} = -\hat{\nabla} \hat{\phi} + Ek \frac{\partial^2 \hat{\boldsymbol{u}}}{\partial \hat{z}^2},$$

Where *Ek* is the Ekman number:

$$Ek \equiv \left(\frac{A}{f_0 H^2} \right)$$

The Ekman number

It determines the importance of frictional terms in the horizontal momentum equation.
If *Ek*<<1 then the friction is small in the flow interior where $\hat{z} = \mathcal{O}(1)$.

But..... the friction term cannot necessarily be neglected in the boundary layer because it is of the highest differential order in the equation, and so determines the boundary conditions.

Case when *Ek* is small but the second term on the right-hand side of the momentum equation remains finite is a singular limit, meaning that it differs from the case with *Ek*=0.

If *Ek is* close or above 1 friction is important everywhere.

Momentum balance in the Ekman layer.

The fluid lies above a rigid surface at z= 0.
Far away from the boundary the velocity field is in geostrophic balance.
We write the velocity field and the pressure field as the sum of the interior geostrophic part, plus a boundary layer correction (subscript E):

$$\hat{\boldsymbol{u}} = \hat{\boldsymbol{u}}_g + \hat{\boldsymbol{u}}_E, \qquad \hat{\phi} = \hat{\phi}_g + \hat{\phi}_E,$$

negligible above the boundary layer.

Since $\partial \hat{\phi}_g / \partial \hat{z} = 0$, and $\partial \hat{\phi}_E / \partial \hat{z} = 0$, remembering that $\hat{\tilde{\phi}}_E = 0$ away from the boundary we conclude that there is no boundary layer in the pressure field.

Then the dominant force balance in the Ekman layer is thus between the Coriolis force and friction:

$$\boldsymbol{f} \times \boldsymbol{u}_E = \frac{\partial \tilde{\boldsymbol{\tau}}}{\partial z}.$$

$$\boldsymbol{f} \times \boldsymbol{u}_E = A \frac{\partial^2 \boldsymbol{u}_E}{\partial z^2}$$

$$Ek \equiv \left(\frac{A}{f_0 H^2} \right)$$

$$\hat{\boldsymbol{f}} \times \hat{\boldsymbol{u}}_E = Ek \frac{\partial^2 \hat{\boldsymbol{u}}_E}{\partial \hat{z}^2}$$

It is evident this equation can only be satisfied if $\hat{z} \neq \mathcal{O}(1)$, implying that H is not a proper scaling for z in the boundary layer. Rather, if the vertical scale in the Ekman layer is $\hat{\delta}$ (meaning $\hat{z} \sim \hat{\delta}$) we must have $\hat{\delta} \sim Ek^{1/2}$. In dimensional terms this means the thickness of the Ekman layer is

$$\delta = H\hat{\delta} = HEk^{1/2} \qquad \delta = \left(\frac{A}{f_0} \right)^{1/2}$$

$$Ek = \left(\frac{\delta}{H}\right)^2$$

That is, the Ekman number is equal to the square of the ratio of the depth of the Ekman layer to an interior depth scale of the fluid motion. In laboratory flows where A is the molecular viscosity we can thus estimate the Ekman layer thickness, and if we know the eddy viscosity of the ocean or atmosphere we can estimate the thickness of their respective Ekman layers. We can invert this argument and obtain an estimate of A if we know the Ekman layer depth. In the atmosphere, deviations from geostrophic balance are very small in the atmosphere above 1 km, and using this gives $A \approx 10^2\ \mathrm{m^2\,s^{-1}}$. In the ocean Ekman depths are about 50 m or less, and eddy viscosities about $0.1\ \mathrm{m^2\,s^{-1}}$.

Transport in the Ekman layer

In the Ekman layer itself we have

$$\boldsymbol{f} \times \boldsymbol{u}_E = \frac{\partial \widetilde{\boldsymbol{\tau}}}{\partial z}$$

with friction zero at the edge of the Ekman layer.

Consider either a top or bottom Ekman layer, and integrate over its thickness:

$$\boldsymbol{f} \times \boldsymbol{M}_E = \widetilde{\boldsymbol{\tau}}_t - \widetilde{\boldsymbol{\tau}}_b$$

$$\boldsymbol{M}_E = \int_{\mathrm{Ek}} \boldsymbol{u}_E \, \mathrm{d}z$$

M_E is the ageostrophic transport in the Ekman layer, and its, in a given latitude, depends on a difference between stress on the top and bottom of the layer.

The stress is zero on the top of the atmospheric Ekman layer and on the bottom of the oceanic Ekman layer.

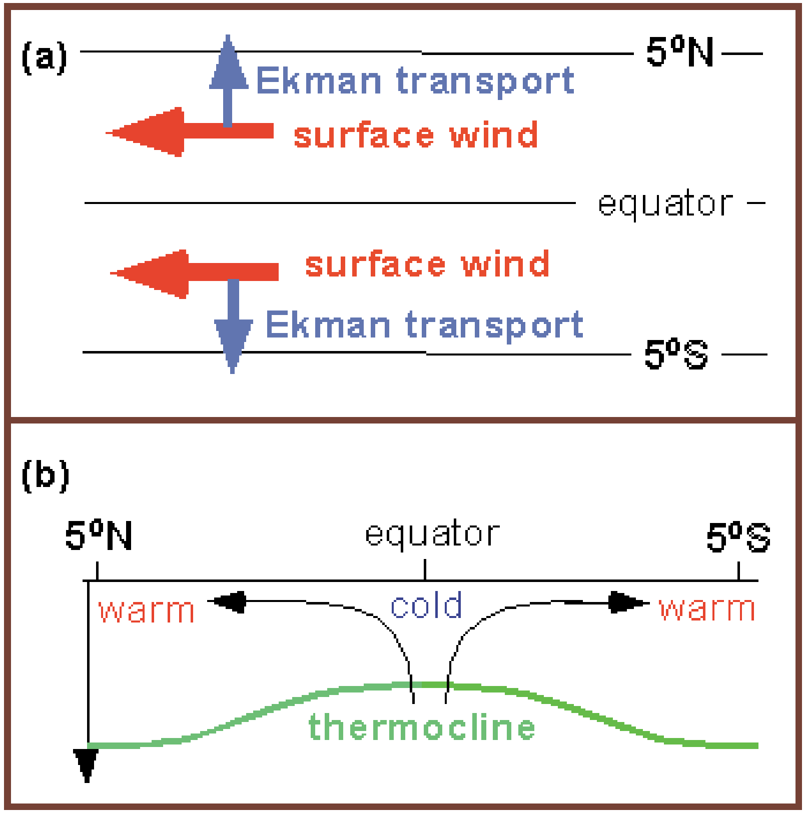
Top Ekman Layer:

$$\boxed{\boldsymbol{M}_E = -\frac{1}{f}\mathbf{k} \times \widetilde{\boldsymbol{\tau}}_t}$$

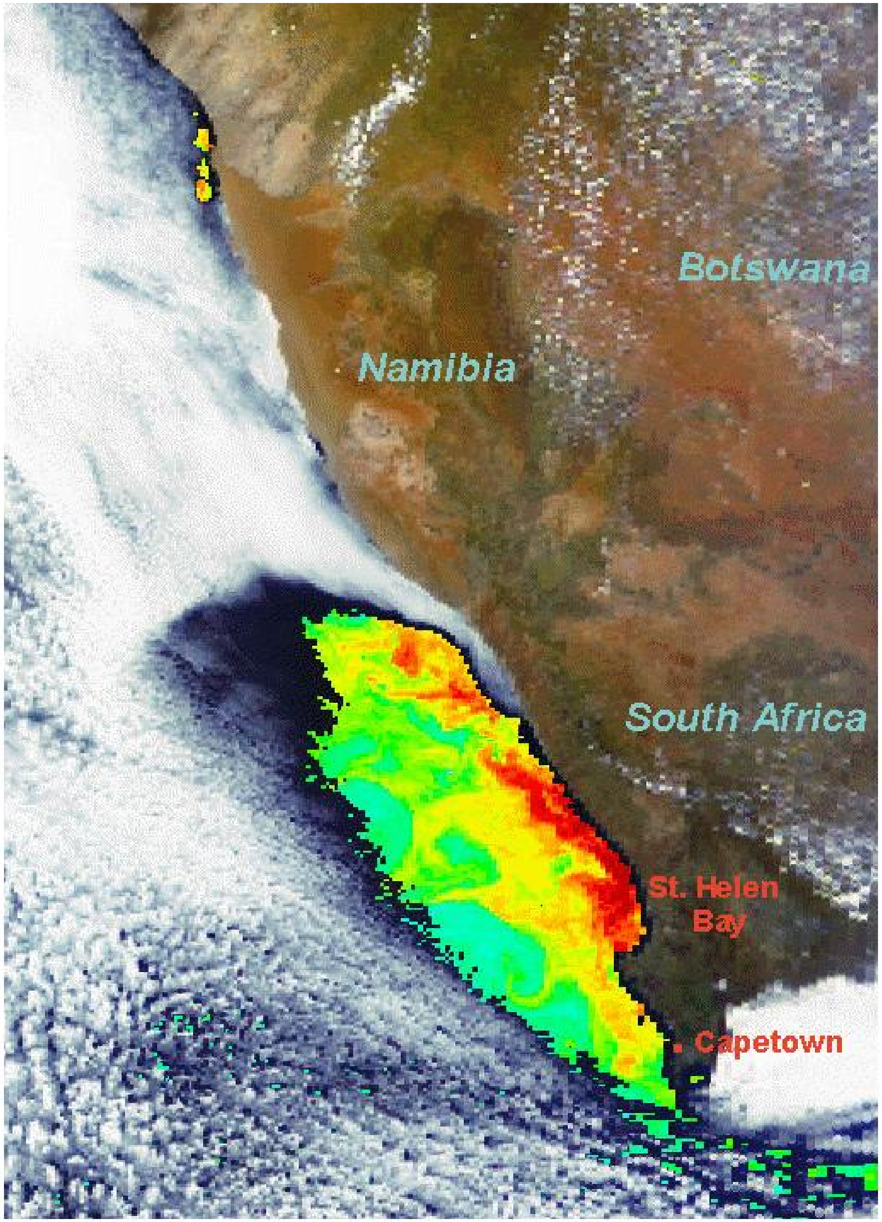
Bottom Ekman Layer:

$$\boxed{\boldsymbol{M}_E = \frac{1}{f}\mathbf{k} \times \widetilde{\boldsymbol{\tau}}_b}$$

The transport in the Ekman layer is thus at right-angles to the stress at the surface. This has a simple physical explanation: integrated over the depth of the Ekman layer the surface stress must be balanced by the Coriolis force, which in turn acts at right angles to the mass transport. This result is particularly useful in the ocean, where the stress at the upper surface is primarily due to the wind, and can be regarded as *independent* of the interior flow. If f is positive, as in the Northern hemisphere, then an Ekman transport is induced 90° to the right of the direction of the wind stress. This has innumerable important consequences — for example, in inducing coastal upwelling when, as is not uncommon, the wind blows parallel to the coast. Upwelling off the coast of California is one example. In the atmosphere, however, the stress arises as a consequence of the interior flow, and we need to parameterize the stress in terms of the flow in order to calculate the surface stress.



Ekman pumping along the equator. (a) shows a plan view of the prevailing surface wind and resulting water transport in the ocean's Ekman layer. (b) is a corresponding cross section, showing the upwelling and resulting SST anomalies.



Multispectral SEAWIFS satellite image at 11:20 UTC on 16 Sept 1997. The greytones reveal cloudiness (visible spectrum) while the colours display sea surface temperature in cloud-free areas. Note: red is colder and blue warmer.

Ekman pumping is also important along coasts, e.g. along the west coast of South Africa and Namibia

Vertical velocity induced by Ekman layer (Ekman pumping)

$$f \times \boldsymbol{M}_E = \tilde{\boldsymbol{\tau}}_t - \tilde{\boldsymbol{\tau}}_b$$

$$\boldsymbol{M}_E = \int_{\mathrm{Ek}} \boldsymbol{u}_E \, \mathrm{d}z$$

The mass conservation equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

It can be integrated across the Ekman layer depth resulting in:

$$\nabla \cdot \boldsymbol{M}_t = -(w_t - w_b)$$

$$\boldsymbol{M}_t = \int_{\mathrm{Ek}} \boldsymbol{u} \, \mathrm{d}z = \int_{\mathrm{Ek}} (\boldsymbol{u}_g + \boldsymbol{u}_E) \, \mathrm{d}z \equiv \boldsymbol{M}_g + \boldsymbol{M}_E,$$

Taking momentum equation we get:

$$\mathbf{k} \times (\boldsymbol{M}_t - \boldsymbol{M}_g) = \frac{1}{f}(\tilde{\boldsymbol{\tau}}_t - \tilde{\boldsymbol{\tau}}_b).$$

Taking the curl (the length of rotation operation) one obtains:

$$\nabla \cdot (\boldsymbol{M}_t - \boldsymbol{M}_g) = \mathrm{curl}_z[(\tilde{\boldsymbol{\tau}}_t - \tilde{\boldsymbol{\tau}}_b)/f]$$

curl_z operator on a vector $\boldsymbol{A}$ is defined by $\mathrm{curl}_z \boldsymbol{A} \equiv \partial_x A_y - \partial_y A_x$.

Finally, remembering that $\nabla \cdot \boldsymbol{M}_t = -(w_t - w_b)$ we get:

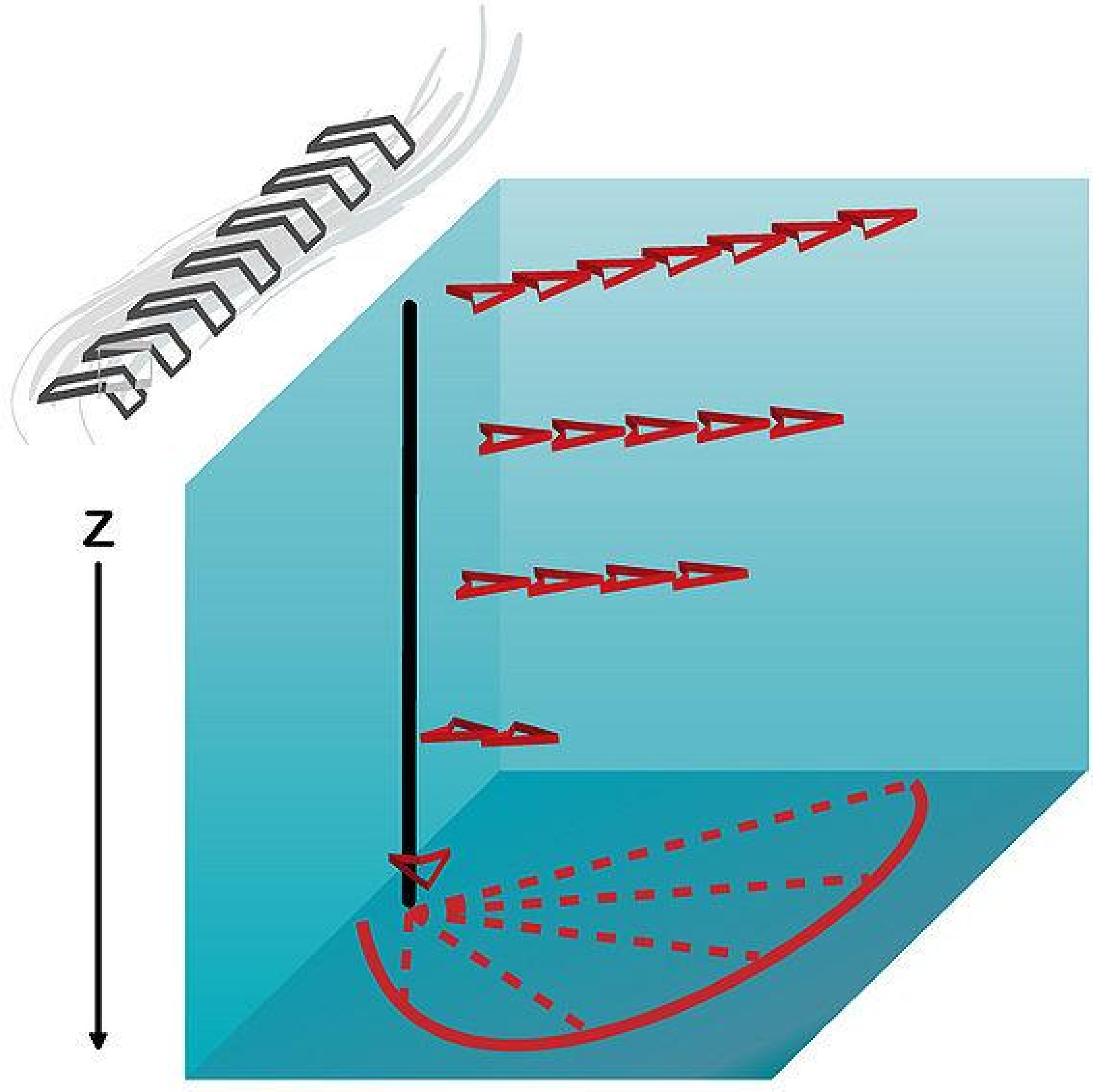
$$w_b = \mathrm{curl}_z \frac{\widetilde{\boldsymbol{\tau}}_t}{f} + \nabla \cdot \boldsymbol{M}_g, \qquad w_t = \mathrm{curl}_z \frac{\widetilde{\boldsymbol{\tau}}_b}{f} - \nabla \cdot \boldsymbol{M}_g$$

In the above

$$\nabla \cdot \boldsymbol{M}_g = -\beta \boldsymbol{M}_g / f$$

is the divergence of the geostrophic transport in the Ekman layer, often small compared to the other terms.
Thus, friction induces a vertical velocity at the edge of the Ekman layer, proportional to the curl of the stress at the surface,

Numerical models sometimes do not have the vertical resolution to explicitly resolve an Ekman layer, and the above provides a means of parameterizing the Ekman layer in terms of resolved or known fields.
It is particularly useful for the top Ekman layer in the ocean, where the stress can be regarded as a given function of the overlying wind.



Ekman Transport is the net motion of fluid as the result of a balance between Coriolis and turbulent drag forces.
In the picture above, valid for the ocean the wind blowing from South to North creates a surface stress and a resulting Ekman spiral is found below it in the water column.

Specific solution: a bottom (atmospheric) Ekman layer

The frictional-geostrophic balance may be written as

$$\boldsymbol{f} \times (\boldsymbol{u} - \boldsymbol{u}_g) = A \frac{\partial^2 \boldsymbol{u}}{\partial z^2}$$

$$f(u_g, v_g) = \left(-\frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial x} \right).$$

With no thermal wind $\partial u_g / \partial z = \partial v_g / \partial z = 0.$

Boundary conditions and solution

Appropriate boundary conditions for a bottom Ekman layer are:

At $z = 0:$	$u = 0, \quad v = 0$	(the no slip condition)	(2.296a)
As $z \to \infty:$	$u = u_g, \quad v = v_g$	(a geostrophic interior).	(2.296b)

Consider solution for geostrophic balance in the form:

$$u = u_g + A_0 e^{\alpha z}, \qquad v = v_g + B_0 e^{\alpha z}$$

Then, after substitution

$$A_0 f - B_0 A \alpha^2 = 0$$

$$-A_0 A \alpha^2 - B_0 f = 0$$

which can be solved for

$$\alpha^4 = -f^2 / A^2,$$

and gives solutions for wind components:

$$u = u_g - e^{-z/d} \left[u_g \cos(z/d) + v_g \sin(z/d) \right]$$

$$v = v_g + e^{-z/d} \left[u_g \sin(z/d) - v_g \cos(z/d) \right]$$

$$d = \sqrt{2A/f}$$

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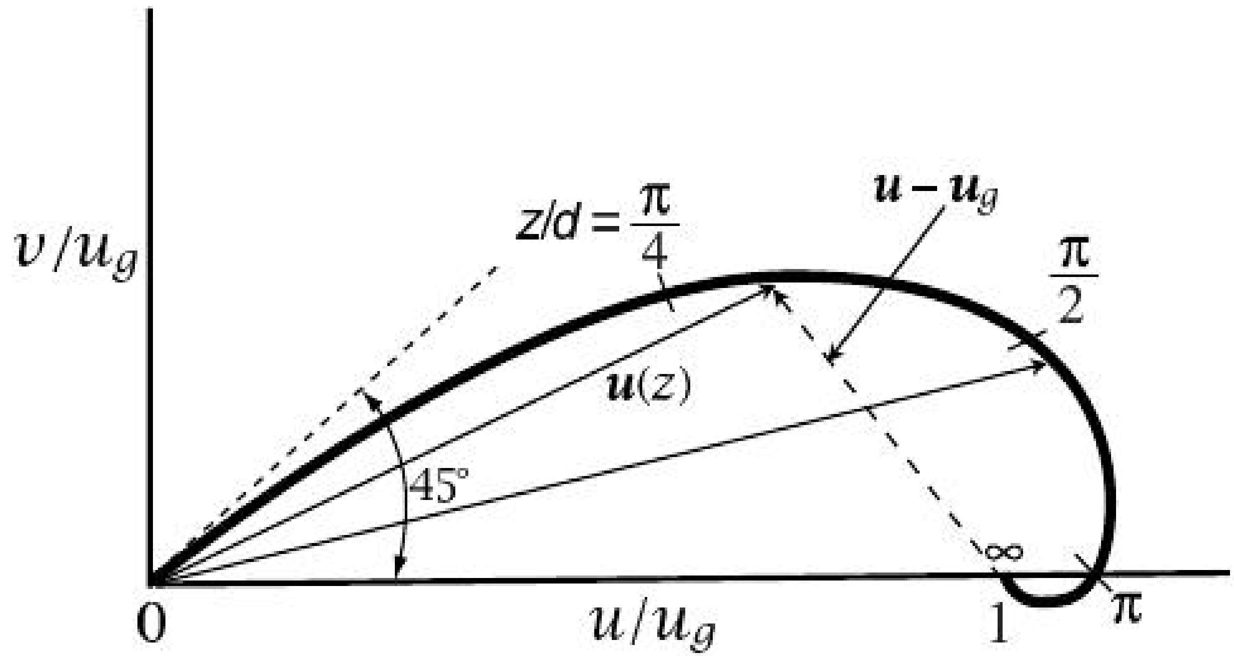
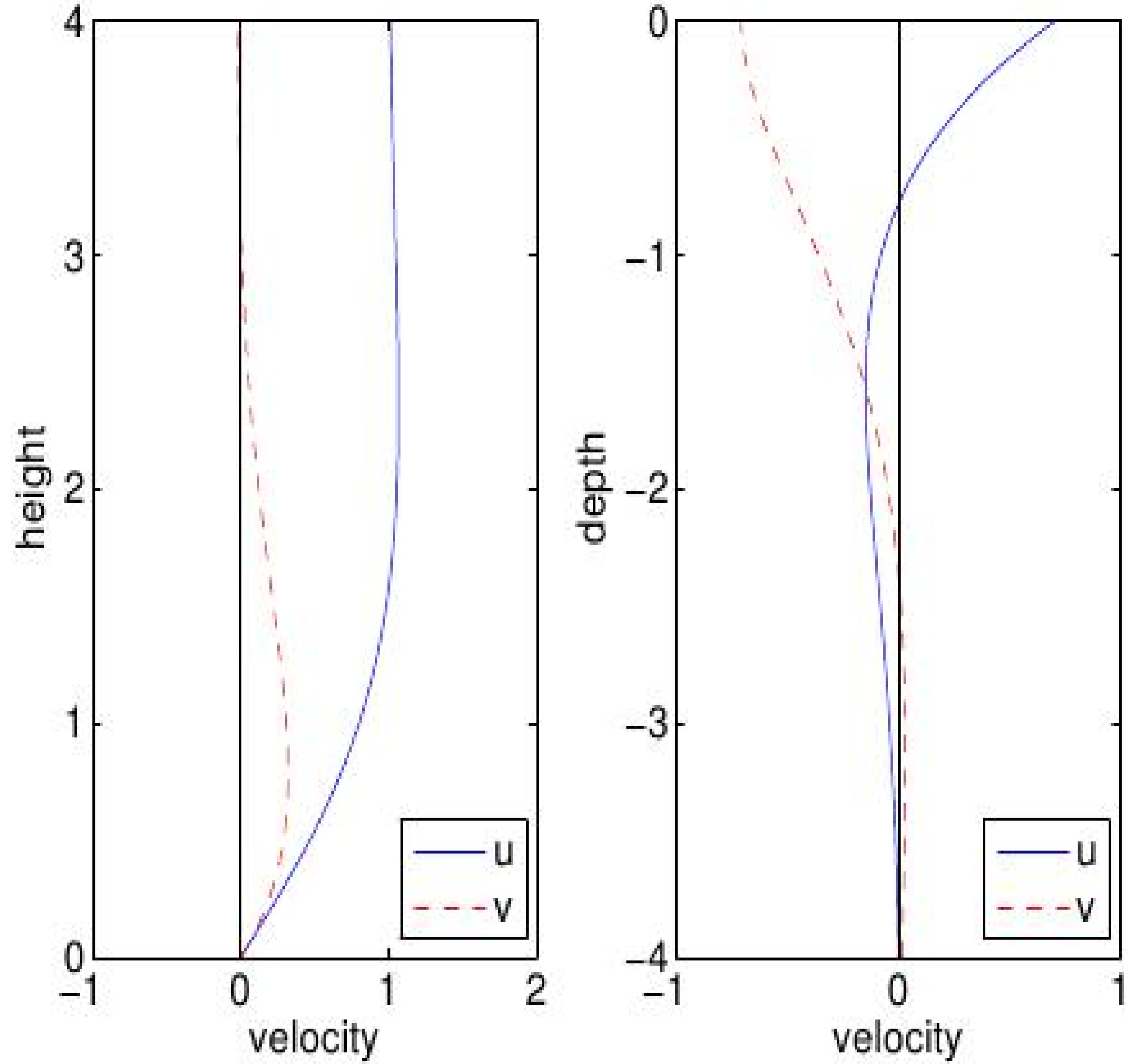


Figure 2.10 The idealised Ekman layer solution in the lower atmosphere, plotted as a hodograph of the wind components: the arrows show the velocity vectors at a particular heights, and the curve traces out the continuous variation of the velocity. The values on the curve are of the nondimensional variable z/d, where $d = (2A/f)^{1/2}$, and v_g is chosen to be zero.

Figure 2.11 Solutions for a bottom Ekman layer with a given flow in the fluid interior (left), and for a top Ekman layer with a given surface stress (right), both with $d = 1$. On the left we have $u_g = 1$, $v_g = 0$. On the right we have $u_g = v_g = 0$, $\tilde{\tau}_y = 0$ and $\sqrt{2}\tilde{\tau}_x/(fd) = 1$.



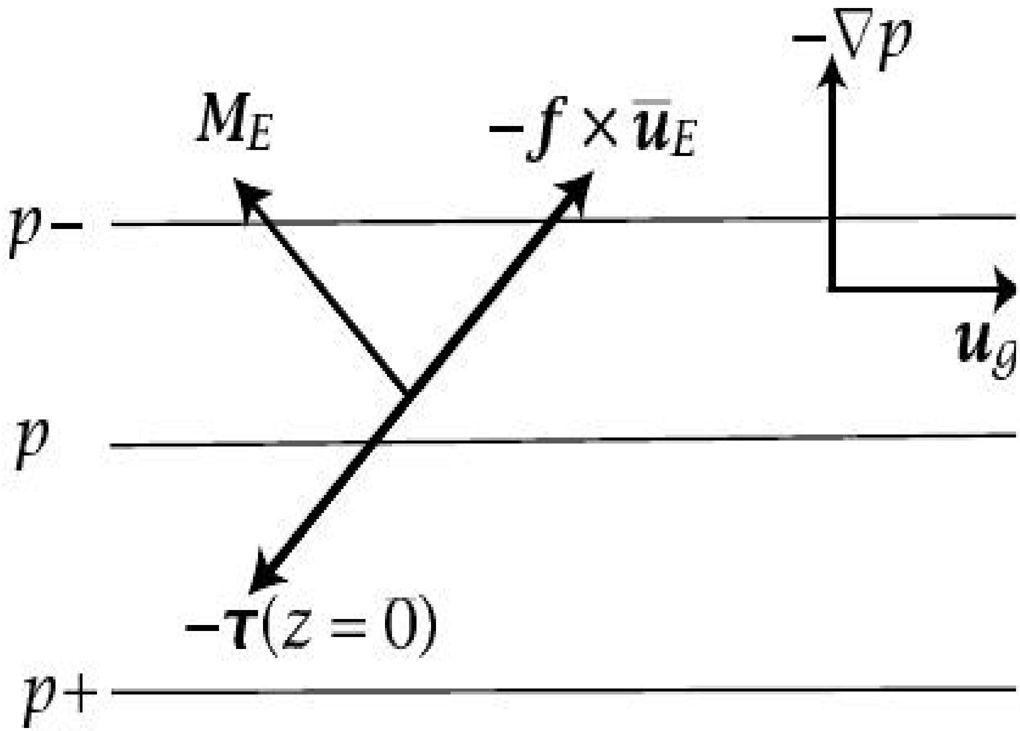
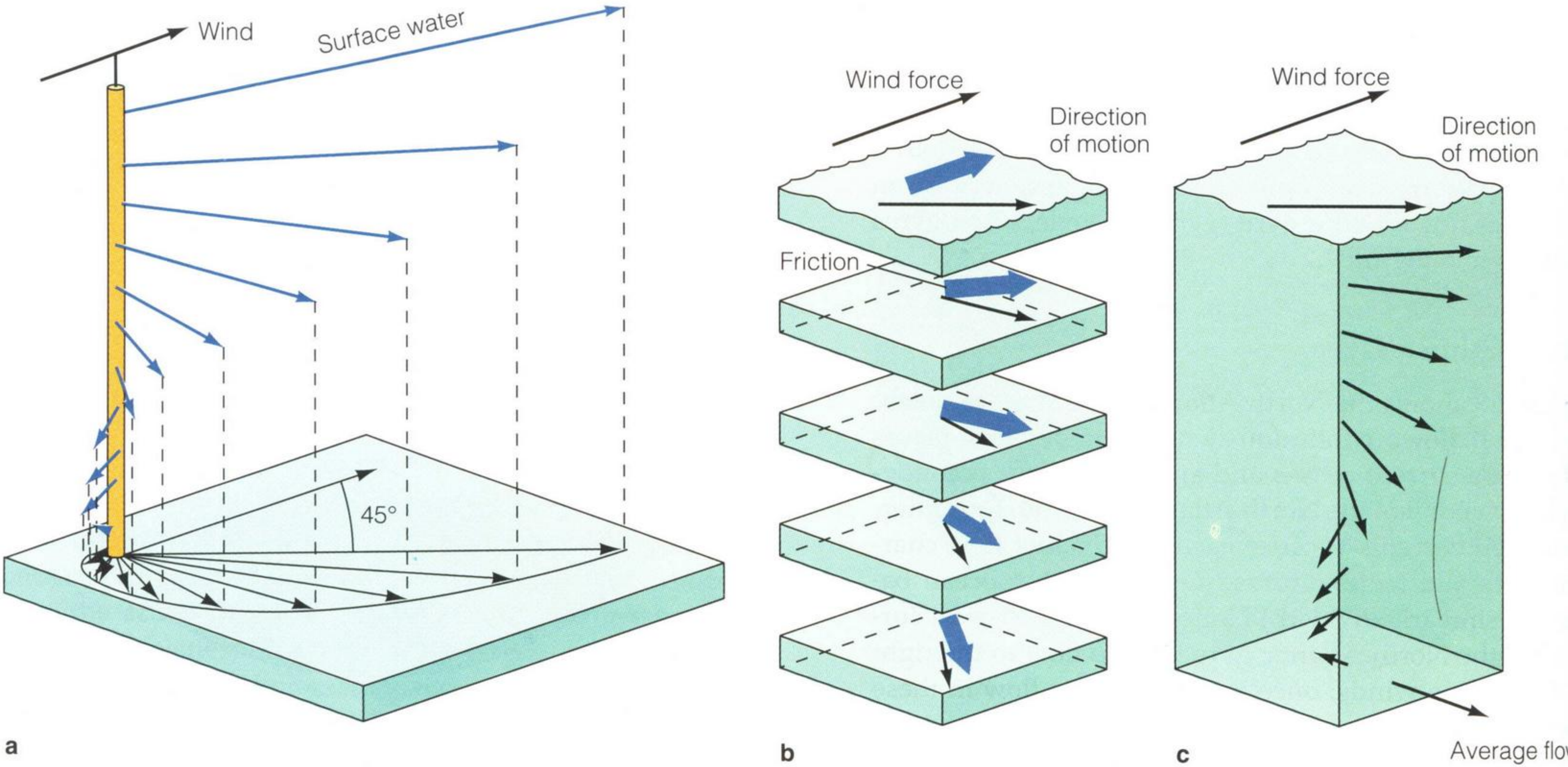


Figure 2.12 A bottom Ekman layer, generated from an eastwards geostrophic flow above. An overbar denotes a vertical integral over the Ekman layer, so that $-f \times \overline{u}_E$ is the Coriolis force on the vertically integrated Ekman velocity. M_E is the frictionally induced boundary layer transport, and τ is the stress.

OCEAN



From UCLA

Ekman pumping/suction

- Convergence/divergence of the Ekman transport drives vertical motions:

$$\frac{\partial w}{\partial z} = -\nabla_h \cdot \mathbf{u}_e$$

assume w=0 at z=0

$$w_{ek} = \frac{1}{\rho_{ref}} \nabla_h \cdot \mathbf{M}_e$$

w_{ek} vertical velocity at the beneath the Ekman layer

$$w_{ek} = \frac{1}{\rho_{ref}} \hat{z} \cdot \nabla \times \left(\frac{\tau_{wind}}{f} \right)$$

Vertical motions associated with the *curl of the wind-stress*

- When $w_{ek} < 0$ the Ekman vertical velocity is known as the *Ekman pumping*
- When $w_{ek} > 0$ the Ekman vertical velocity is known as the *Ekman suction*

From Stanford

In the atmosphere:

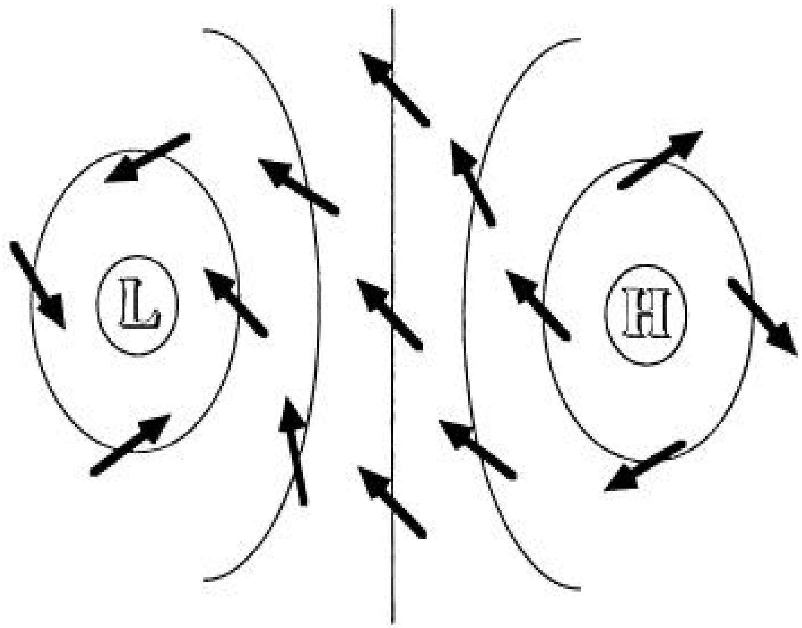


Fig. 5.6 Schematic surface wind pattern (arrows) associated with high- and low-pressure centers in the Northern Hemisphere. Isobars are shown by thin lines, and L and H designate high- and low-pressure centers, respectively. After Stull (1988).

(from Holton)

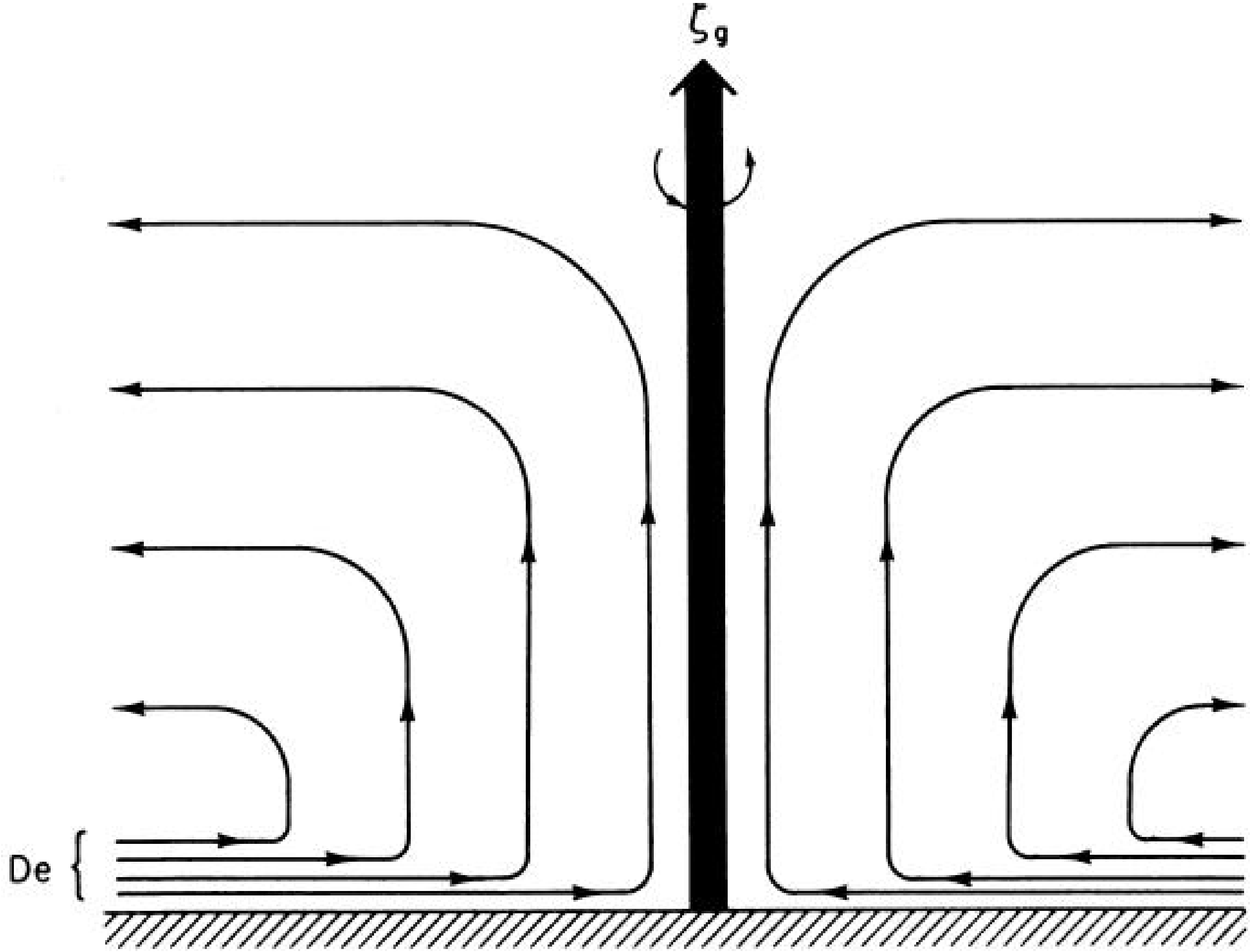


Fig. 5.7 Streamlines of the secondary circulation forced by frictional convergence in the planetary boundary layer for a cyclonic vortex in a barotropic atmosphere. The circulation extends throughout the full depth of the vortex.

In the atmosphere Ekman pumping in low pressure systems is an effective mechanism of vertical momentum transport slowing down atmospheric circulations.

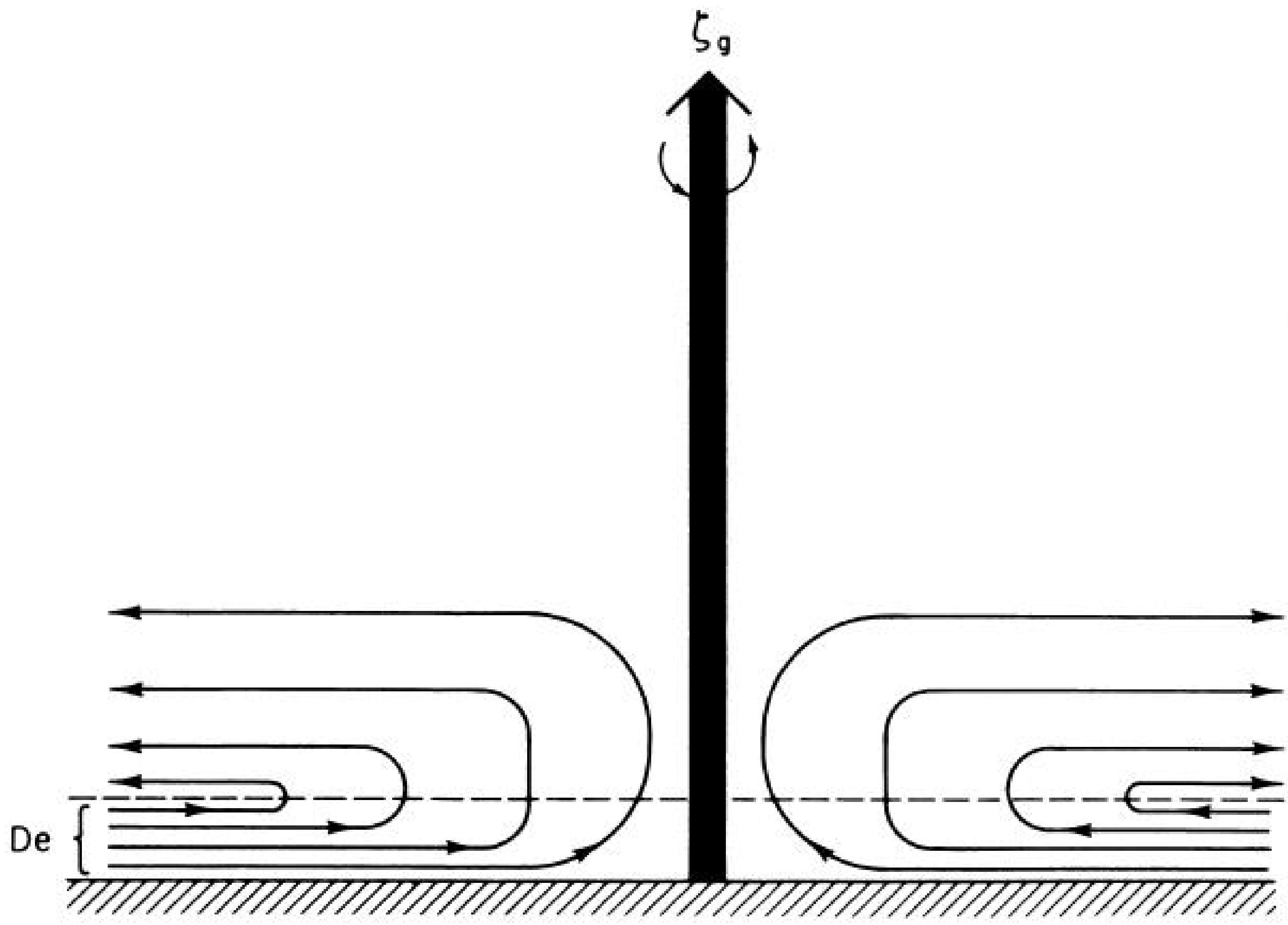


Fig. 5.8 Streamlines of the secondary circulation forced by frictional convergence in the planetary boundary layer for a cyclonic vortex in a stably stratified baroclinic atmosphere. The circulation decays with height in the interior.

Efficiency and range of this effects depends on the static stability of the troposphere.