Session 11

1. The horizontal wind may be decomposed into geostrophic and ageostrophic parts,

$$\mathbf{u} = \mathbf{u}_g + \mathbf{u}_a.$$

The quasi-geostrophic momentum equation reads

$$\frac{\mathrm{d}_g \mathbf{u}_g}{\mathrm{d}t} = -f_0 \hat{\mathbf{k}} \times \mathbf{u}_a - \beta y \hat{\mathbf{k}} \times \mathbf{u}_g,$$

where:

$$\frac{\mathrm{d}_g}{\mathrm{d}t} \equiv \frac{\partial}{\partial t} + \mathbf{u}_g \cdot \nabla_p.$$

(a) Show that:

$$\frac{\mathrm{d}_g \zeta_g}{\mathrm{d}t} = -f_0 \nabla \cdot \mathbf{u}_a - \beta v_g.$$

(b) Show that:

$$\frac{\mathrm{d}_g}{\mathrm{d}t}(\zeta_g + f) = -f_0 \nabla \cdot \mathbf{u}_a = f_0 \frac{\partial \omega}{\partial p},$$

where $f = f_0 + \beta y$.

2. Show that the geostrophic vorticity $\nabla \times \mathbf{u}_g = \zeta_g \hat{\mathbf{k}}$ and the geopotential Φ are related by

$$\zeta_g = \frac{1}{f_0} \nabla^2 \Phi.$$