LECTURE OUTLINE

1. Ways of reaching saturation
   - vertical motion

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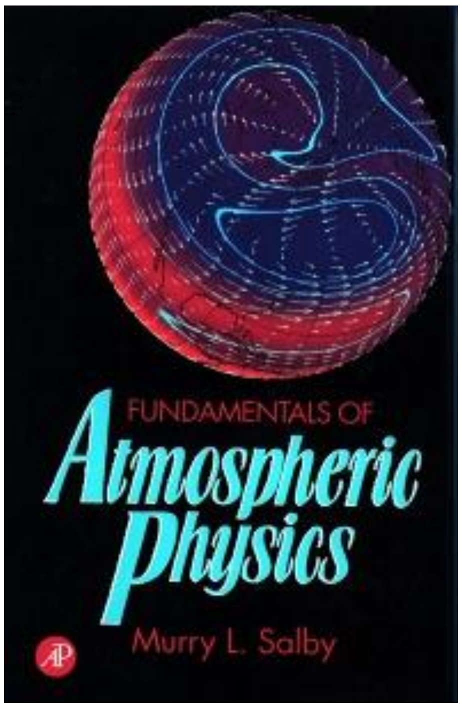
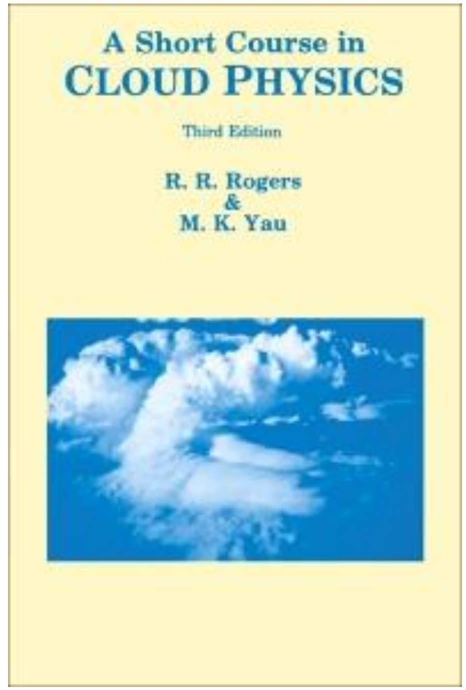
Salby, Chapter 5



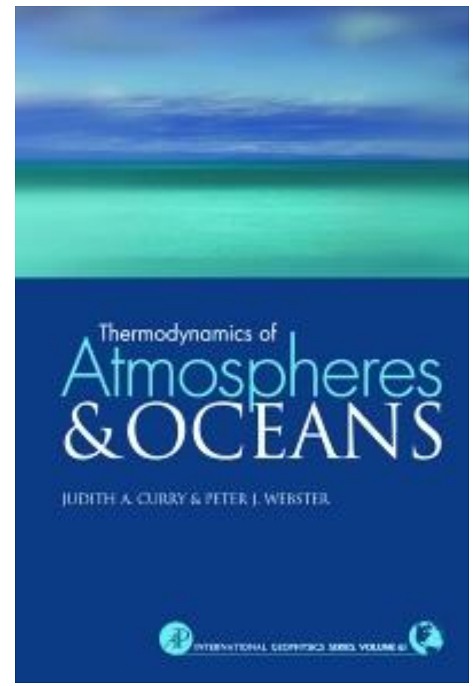
Fundamentals of Atmospheric Physics,
M.L. Salby; Salby

R&Y, Chapter 4

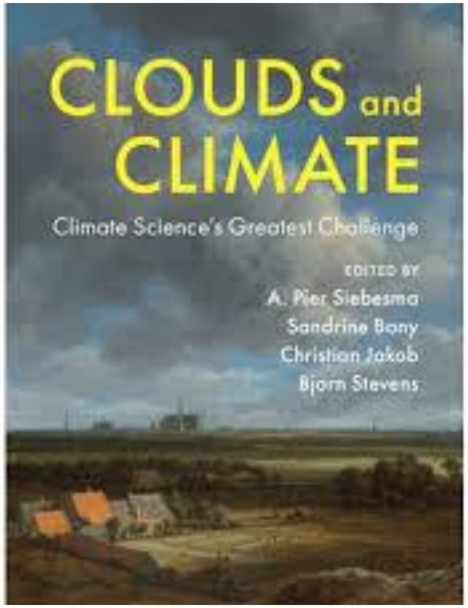


A Short Course in Cloud Physics,
R.R. Rogers and M.K. Yau; R&Y

C&W, Chapter 6



Thermodynamics of Atmospheres
and Oceanes,
J.A. Curry and P.J. Webster; C&W



Chapter 2: Clouds as Fluids

Siebesma, A., Bony, S., Jakob, C., & Stevens, B. (Eds.). (2020). *Clouds and Climate: Climate Science's Greatest Challenge*. Cambridge: Cambridge University Press. doi:10.1017/9781107447738

THERMODYNAMICS IN VERTICAL MOTION

1. Lifting condensation level (LCL)
2. Dew-point temperature variation in vertical motion
3. Pseudo-adiabatic process
   - saturated adiabatic lapse rate
   - water condensed in pseudo-adiabatic process

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An air parcel that moves vertically expands or contracts to preserve its mechanical equilibrium (adjusts its pressure to the environmental pressure).

This results in work being performed.

This work is done at the expense of internal energy, which alters the temperature and, consequently, the saturation vapor pressure.

Saturation vapor pressure depends on temperature (assuming $L_{lv} = const.$):

$$\ln\left(\frac{e_s}{e_{s0}}\right) = -\frac{L_{lv}}{R_v}\left(\frac{1}{T} - \frac{1}{T_0}\right)$$

The saturation specific humidity depends on pressure and temperature:

$$\frac{q_s}{q_{s0}} = \frac{e_s/e_{s0}}{p/p_0} = \frac{\exp\left[-\frac{L_{lv}}{R_v}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]}{\left(\frac{p}{p_0}\right)}$$

$$q_s = \varepsilon\frac{e_s}{p}$$

$$q_{s0} = \varepsilon\frac{e_{s0}}{p_0}$$

$$\frac{q_s}{q_{s0}} = \frac{e_s/e_{s0}}{p/p_0} = \frac{\exp\left[-\frac{L_{lv}}{R_v}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]}{\left(\frac{p}{p_0}\right)}$$

The saturation specific humidity in an air parcel:

- increases as pressure decreases (with increasing altitude)
- decreases sharply as temperature drops, which also occurs during upward motion.

Despite the exponential decrease in pressure with altitude, the dominant impact of adiabatic cooling ensures that the saturation specific humidity of an ascending parcel decreases monotonically.

Lifting condensation level

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Consider a moist (unsaturated) air parcel ascending in thermal convection.

Under unsaturated conditions, the parcel's specific humidity and saturation specific humidity satisfy $q_v < q_s$.

As the parcel rises, it performs work at the expense of its internal energy.

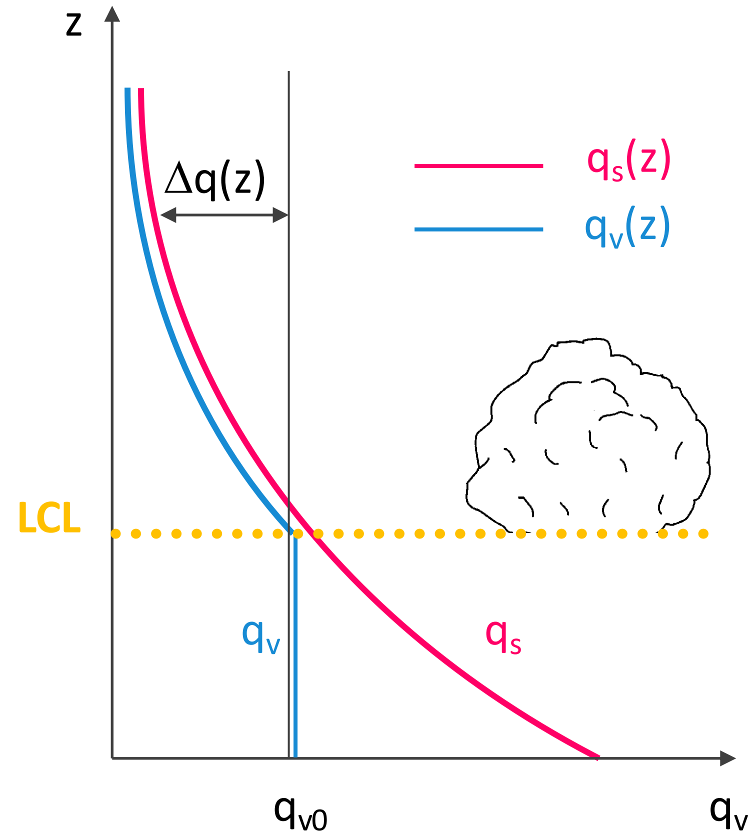
The parcel's temperature decreases at the dry adiabatic lapse rate Γ_d.

This decrease in temperature is accompanied by a reduction in the saturation specific humidity q_s.

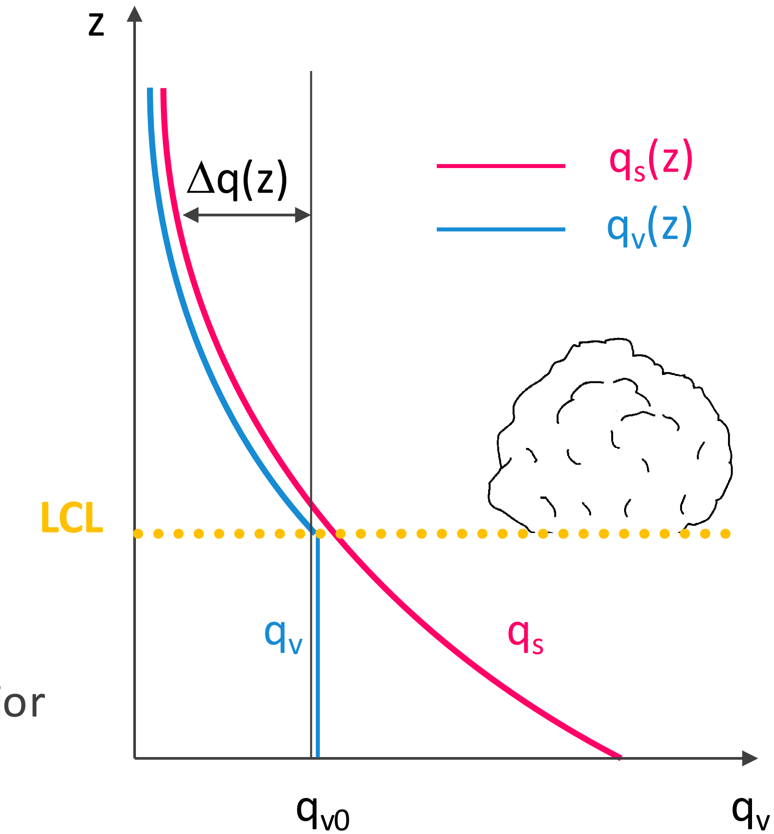
Meanwhile, the parcel's actual specific humidity q_v and potential temperature θ remain constant.

Sufficient upward displacement will reduce the saturation specific humidity, q_s, to the level of the actual specific humidity, q_v .

The elevation where $q_v = q_s$ for the first time is referred to as the **lifting condensation level (LCL).**



The **lifting condensation level** defines the base of cumulus clouds that are fueled by air originating at the surface.



Below the LCL, the parcel's thermodynamic behavior can be regarded as adiabatic.

This is because the timescale for vertical motion (from minutes in cumulus convection to one day in sloping convection) is much shorter compared to the characteristic timescale for heat transfer.

Location of the LCL

As air expands adiabatically and cools, the relative humidity (f) increases as the temperature and saturation mixing ratio decrease.

We will find the thermodynamic coordinates (T_{LCL}, p_{LCL}) of the LCL – the point where the air becomes saturated.

The change in relative humidity satisfies the following equation: $f = \frac{e}{e_s}$

$$d(\ln f) = d(\ln e) - d(\ln e_s)$$

We will determine how e and e_s depend on temperature during an adiabatic ascent.

According to Dalton's law of partial pressures, the total pressure exerted by a mixture of gases is equal to the sum of the partial pressures that would be exerted by each constituent alone if it filled the entire volume at the temperature of the mixture.

$$p_i = \frac{m_i R_i T}{V}$$

$$\frac{dp_i}{p_i} = \frac{dT}{T}$$

Therefore: $d(\ln p) = d(\ln e)$.

The First Law of Thermodynamics for an adiabatic process in enthalpy form:

$$c_p dT = v dp \longrightarrow c_p\, dT = \frac{RT}{p} dp$$

$$d(\ln p) = \frac{c_p}{R} d(\ln T)$$

Because $d(\ln p) = d(\ln e)$:

$$d(\ln e) = \frac{c_p}{R} d(\ln T)$$

Using the Clausius-Clapeyron equation:

$$d(\ln e_s) = \frac{L_{lv}}{R_v T} d(\ln T)$$

The change in relative humidity satisfies the following equation:

$$d(\ln f) = d(\ln e) - d(\ln e_s)$$

$$d(\ln f) = \frac{c_p}{R} d(\ln T) - \frac{L_{lv}}{R_v T} d(\ln T)$$

We will integrate the equation from initial conditions to conditions where saturation is attained, indicated by $f = 1$ and $T = T_{LCL}$. We will assume that $L_{lv} = const$.

$$\int_{f}^{1} d(\ln f') = \int_{T}^{T_{LCL}} \left(\frac{c_p}{R} - \frac{\varepsilon L_{lv}}{RT'}\right) d(\ln T')$$

$$-\ln f = \frac{c_p}{R} \ln \frac{T_{LCL}}{T} + \frac{\varepsilon L_{lv}}{R} \left(\frac{1}{T_{LCL}} - \frac{1}{T}\right)$$

Equation can be solved numerically to obtain T_{LCL}.

The saturation pressure can be derived from the dry adiabat equation:

$$p_{LCL} = p \left(\frac{T_{LCL}}{T}\right)^{c_p/R}$$

An approximate equation for T_{LCL} given initial values of T and f, is given by Bolton (1980).

f	T (°C)	T_{LCL}(°C)	p_{LCL}(hPa)
0,1	0	-16	815
0,1	10	-7	806
0,1	20	2	797
0,1	30	10	788
0,3	0	-8	896
0,3	10	1	891
0,3	20	10	886
0,3	30	19	880
0,5	0	-5	938
0,5	10	5	935
0,5	20	14	932
0,5	30	24	929
0,7	0	-3	968
0,7	10	7	966
0,7	20	17	964
0,7	30	27	962

$$T_{LCL} = \frac{1}{\frac{1}{T-55} - \frac{\ln f}{2840}} + 55, \qquad [T] = K$$

Bolton, D., 1980:The computation of equivalent potential temperature. Mon. Weather Rev., 108, 1046-1053

Variation of dew-point temperature with altitude

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During adiabatic ascent, the water vapor specific humidity, q_v, remains constant until saturation occurs.

The dew-point temperature decreases slightly during the ascent as pressure decreases.

We will calculate how the dew-point temperature changes during the ascent of an unsaturated adiabatic parcel.

The dew-point temperature satisfies the equation: $d\ln e = \frac{\varepsilon L_{lv}}{RT_d^2} dT_d$

The hydrostatic equation: $dp = -\rho g dz \longrightarrow d\ln p = -\frac{g}{RT} dz$

$p = RT\rho$

From the Dalton's law: $d\ln p = d\ln e$ $\quad d\ln e = -\frac{g}{RT} dz$

$$\frac{\varepsilon L_{lv}}{RT_d^2} dT_d = -\frac{g}{RT} dz$$

$$\frac{dT_d}{dz} = -\frac{T_d^2 g}{\varepsilon L_{lv} T} \cdot \frac{c_p}{c_p} \cong -\frac{T_d^2 c_p}{\varepsilon L_{lv} T} \cdot \Gamma_d$$

$$\Gamma_d = \frac{g}{c_{pd}} \approx \frac{g}{c_p}$$

For typical atmospheric values dT_d/dz is approximately 1/6 of the dry adiabatic lapse rate.

$$\frac{T_d^2 c_p}{\varepsilon L_{lv} T} \cong \frac{280^2 \cdot 1004}{0.622 \cdot 2.5 \cdot 10^6 \cdot 300} = 0.166 \approx 1/6$$

At the saturation level, T becomes equal to T_d (and to T_{LCL}).

The lifting condensation level (LCL), z_{LCL}, is the altitude at which an air parcel becomes saturated during adiabatic ascent.

The equation for the change in the dew-point temperature can be expressed in the form of a change in the dew-point deficit: $T - T_d$

$$\frac{dT_d}{dz} = -\frac{T_d^2 c_p}{\varepsilon L_{lv} T} \cdot \Gamma_d \quad \longrightarrow \quad \frac{dT}{dz} - \frac{dT_d}{dz} = -\Gamma_d + \frac{T_d^2 c_p}{\varepsilon L_{lv} T} \cdot \Gamma_d$$

$$\frac{d(T - T_d)}{dz} = -\left(1 - \frac{T_d^2 c_p}{\varepsilon L_{lv} T}\right) \cdot \Gamma_d$$

When $T = T_d$, the saturation level has been reached, and the value of z_{LCL} can be determined by integrating from the initial values $(0, T_0 - T_{d0})$ to the saturation state $(z_{LCL}, 0)$.

$T_0 - T_{d0}$ is the dew-point depression at the surface.

$$\int_{T_0 - T_{d0}}^{0} d(T - T_d) = -\int_{0}^{z_{LCL}} \left(1 - \frac{T_d^2 c_p}{\varepsilon L_{lv} T}\right) \Gamma_d \, dz$$

For a parcel lifted from the surface, the value of z_{LCL} can be estimated

(assuming $dT_d/dz = -1/6\,\Gamma_d$):

$$z_{LCL} \approx 0.12(T_0 - T_{d0}) \quad [\mathrm{km}]$$

Calculating the lifting condensation level provides a good estimate of the cloud base height for clouds that form by adiabatic ascent.

ADIABATIC
AND PSEUDO-ADIABATIC PROCESSES

1. Wet adiabatic lapse rate / saturated adiabatic lapse rate
2. Pseudo-adiabatic process
3. Water condensed in pseudo-adiabatic process

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Wet adiabatic lapse rate

If expansion work occurs rapidly enough, heat exchange with the environment remains negligible.

If no moisture precipitates out, the parcel is closed and its behavior above the LCL is described by a reversible saturated adiabatic process.

$$dh = c_p dT + L_{lv} dq_v$$

First Law of Thermodynamics for adiabatic processes

$$c_p dT + L_{lv} dq_v - v dp = 0$$

$$dh = \cancel{\delta q} + v dp$$

$$dq_v = dq_s = \frac{\partial q_s}{\partial T} dT + \frac{\partial q_s}{\partial p} dp$$

Water vapor is saturated

$$q_v = q_s(T, p)$$

It is convenient to express the partial derivatives of q_s as logarithmic partial derivatives:

$$\frac{\partial q_s}{\partial T} = \frac{q_s}{T}\frac{\partial \ln q_s}{\partial \ln T} \qquad \frac{\partial q_s}{\partial p} = \frac{q_s}{p}\frac{\partial \ln q_s}{\partial \ln p}$$

$$\beta_T = \frac{\partial \ln q_s}{\partial \ln T} \qquad \beta_p = -\frac{\partial \ln q_s}{\partial \ln p}$$

$$dq_s = \frac{q_s}{T}\beta_T dT - \frac{q_s}{p}\beta_p dp$$

$$c_p dT + L_{lv}\frac{q_s}{T}\beta_T dT - L_{lv}\frac{q_s}{p}\beta_p dp - v dp = 0$$

$$\left(c_p + L_{lv}\frac{q_s}{T}\beta_T\right) dT + \left(L_{lv}\frac{q_s}{RT}\beta_p + 1\right) g dz = 0$$

$$dp = -\rho g dz$$

$$v dp = -g dz$$

$$dp = -\frac{p}{RT} g dz$$

Saturated moist adiabatic lapse rate:

$$\Gamma_s = -\frac{dT}{dz} = g\frac{1 + q_s\beta_p\frac{L_{lv}}{RT}}{c_p + q_s\beta_T\frac{L_{lv}}{T}}$$

We will calculate β_p and β_T.

$$q_s = \frac{\varepsilon e_s}{p - (1-\varepsilon) e_s} = \frac{\varepsilon \frac{e_s}{p}}{1 - (1-\varepsilon)\frac{e_s}{p}}$$

β_p

$$\frac{\partial q_s}{\partial p} = -\frac{q_s}{p}\left(1 + \frac{1-\varepsilon}{\varepsilon} q_s\right)$$

$$\beta_p = -\frac{\partial \ln q_s}{\partial \ln p} = \left[1 + \left(\frac{1}{\varepsilon} - 1\right) q_s\right] \approx 1$$

β_T

$$\frac{\partial q_s}{\partial T} = \frac{1}{e_s}\frac{de_s}{dT} q_s + \frac{1-\varepsilon}{\varepsilon}\frac{1}{e_s}\frac{de_s}{dT} q_s^2$$

$$\beta_T = \frac{\partial \ln q_s}{\partial \ln T} = T\frac{d \ln e_s}{dT}\left(1 + \frac{1-\varepsilon}{\varepsilon} q_s\right) = T\frac{d \ln e_s}{dT}\beta_p$$

$$\beta_T = \frac{L_{lv}}{R_v T}\beta_p \approx \frac{5\,400\ K}{T}$$

Clausius-Clapeyron equation

$$\frac{d \ln e_s}{dT} = \frac{L_{lv}}{R_v T^2}$$

Saturated moist adiabatic lapse rate

$$\Gamma_s = g\frac{1 + q_s\beta_T\frac{R_v}{R}}{c_p + q_s\beta_T\frac{L_{lv}}{T}}$$

$$\Gamma_s = \Gamma_d\frac{c_{pd}}{c_p}\frac{1 + q_s\beta_T\frac{R_v}{R}}{1 + q_s\beta_T\frac{L_{lv}}{c_pT}}$$

$$\Gamma_d = \frac{g}{c_{pd}}$$

$$\Gamma_s \equiv -\frac{dT}{dz} = \gamma\Gamma_d$$

$$\gamma = \frac{c_{pd}}{c_p}\frac{1 + q_s\beta_T\frac{R_v}{R}}{1 + q_s\beta_T\frac{L_{lv}}{c_pT}}$$

$$\gamma \leq 1$$

$$c_p = q_dc_{pd} + q_sc_{pv} + q_lc_l$$

$$R = q_dR_d + q_sR_v$$

$$\beta_T = \frac{L_{lv}}{R_vT}\beta_p \approx \frac{5\,400\ K}{T}$$

If no moisture precipitates out, the parcel is closed and its behavior above the LCL is described by a reversible saturated adiabatic process.

This process depends weakly on the amount of condensate present (e.g., on how much of the system's enthalpy is due to the condensate).

Because the condensate exists only in trace amounts, its variation unnecessarily complicates the description of the parcel under saturated conditions.

A simplification known as the pseudo-adiabatic process is often proposed:

The system is treated as open, and any condensate is assumed to be removed from the parcel immediately as it is produced.

A pseudo-adiabatic change of state may be constructed in two steps:

1. Reversible saturated adiabatic expansion (compression), which results in the production (destruction) of condensate of mass dm_c and a commensurate release (absorption) of latent heat to (from) the gas phase.
2. Removal (addition) of the condensate of mass dm_c.

Wet adiabatic vs pseudo-adiabatic lapse rate

$$\Gamma_s = \gamma \Gamma_d$$

$$\gamma = \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \frac{R_v}{R}}{1 + q_s \beta_T \frac{L_{lv}}{c_p T}}$$

$$c_p = q_d c_{pd} + q_s c_{pv} + q_l c_l$$

$$R = q_d R_d + q_s R_v$$

Wet adiabatic lapse rate:

$$q_d + q_s + q_l = 1 \quad \text{and} \quad q_d = const\,, \quad q_s + q_l = const$$

Pseudo-adiabatic lapse rate:

$$q_l = 0, \quad q_d + q_s = 1$$

Simplified version of Γ_s

In many textbooks, a simplified version of Γ_s is presented.

$$R \longrightarrow R_d$$

$$c_p \longrightarrow c_{pd}$$

$$\Gamma_s = \gamma \Gamma_d \qquad \gamma = \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \frac{R_v}{R}}{1 + q_s \beta_T \frac{L_{lv}}{c_p T}} \longrightarrow \gamma = \frac{1 + \frac{q_s L_{lv}}{R_d T}}{1 + \frac{q_s L_{lv}^2}{c_{pd} R_v T^2}}$$

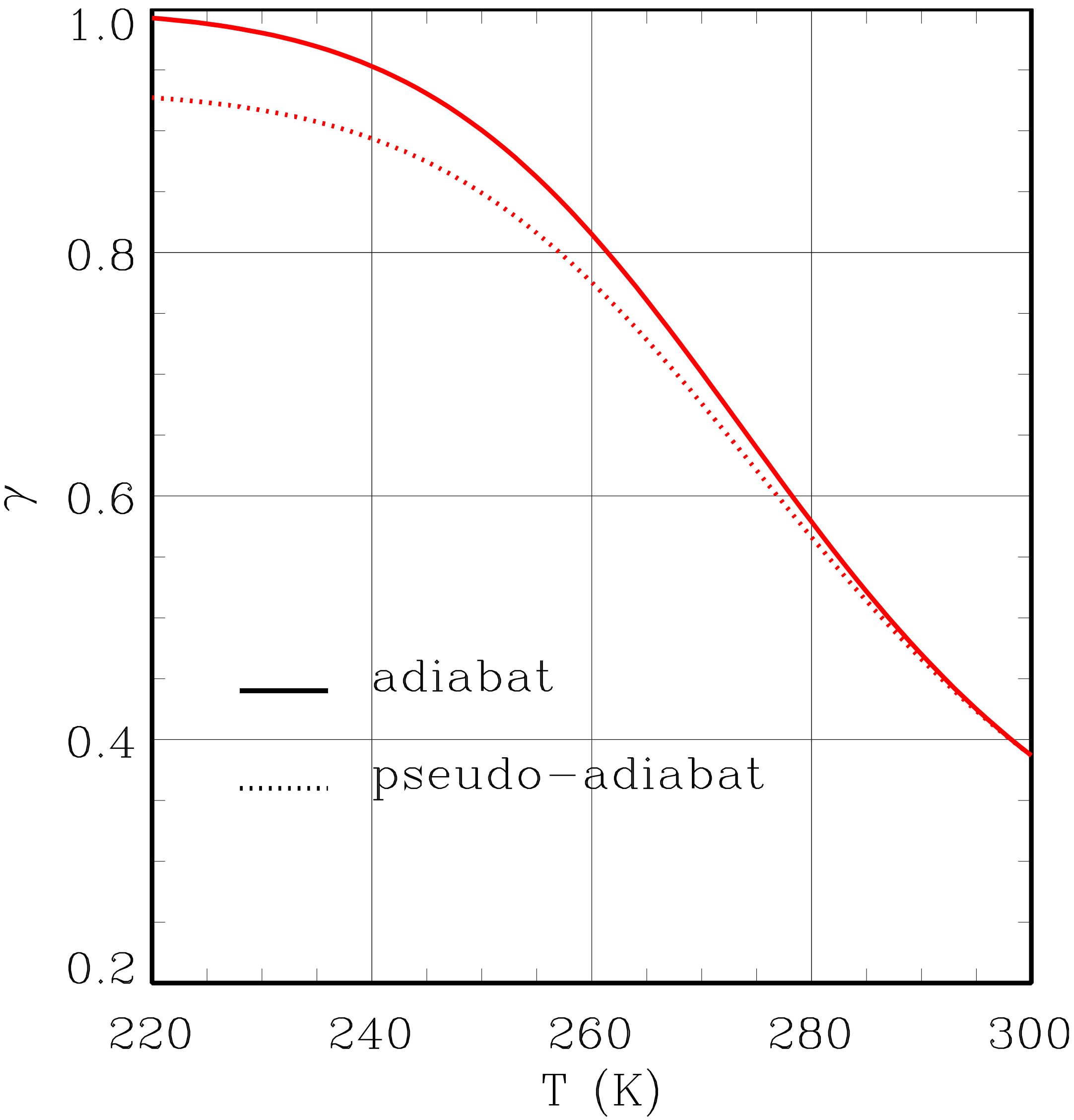
Non-dimensional lapse rate γ

- pressure: 1 000 hPa
- the air initially saturated at 300 K

$$\Gamma_s < \Gamma_d$$

$$\Gamma_s = \gamma \Gamma_d$$

$$\gamma = \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \frac{R_v}{R}}{1 + q_s \beta_T \frac{L_{lv}}{c_p T}}$$



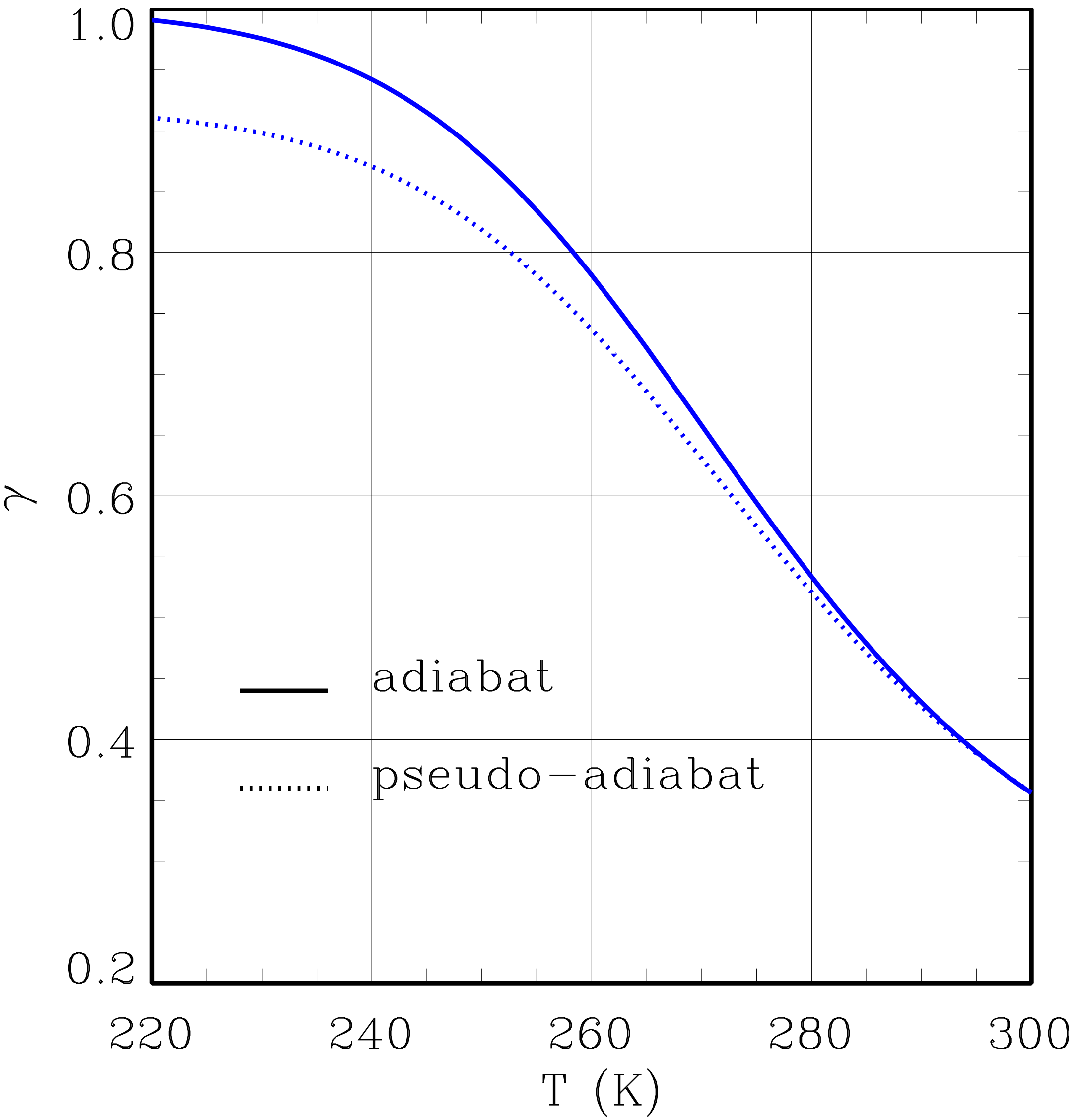
Non-dimensional lapse rate γ

- pressure: 800 hPa
- the air initially saturated at 300 K

$$\Gamma_s < \Gamma_d$$

$$\Gamma_s = \gamma \Gamma_d$$

$$\gamma = \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \dfrac{R_v}{R}}{1 + q_s \beta_T \dfrac{L_{lv}}{c_p T}}$$



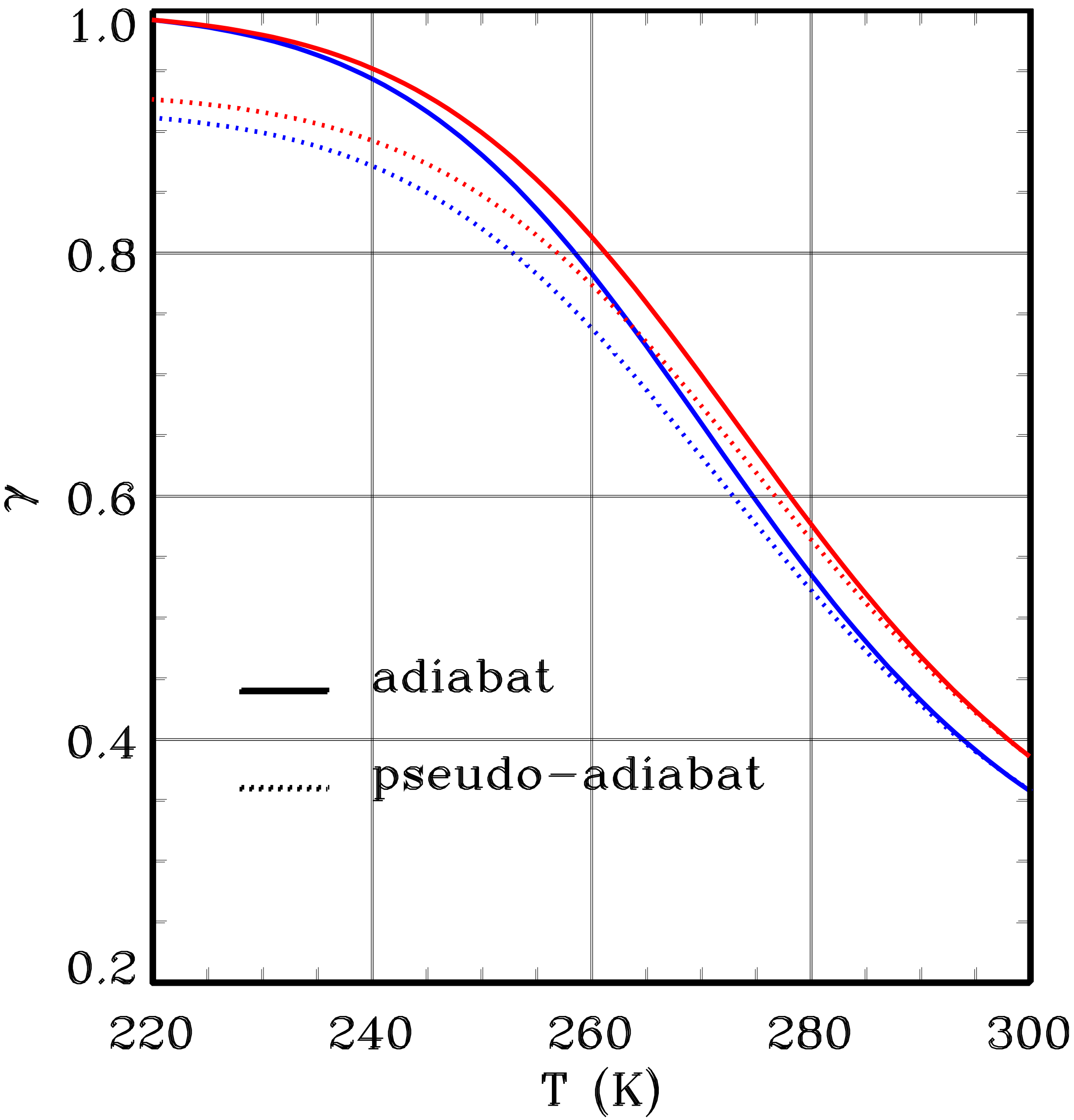
Non-dimensional lapse rate γ

- pressure: 1000 hPa
- pressure: 800 hPa
- the air initially saturated at 300 K

$$\Gamma_s < \Gamma_d$$

$$\Gamma_s = \gamma \Gamma_d$$

$$\gamma = \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \frac{R_v}{R}}{1 + q_s \beta_T \frac{L_{lv}}{c_p T}}$$



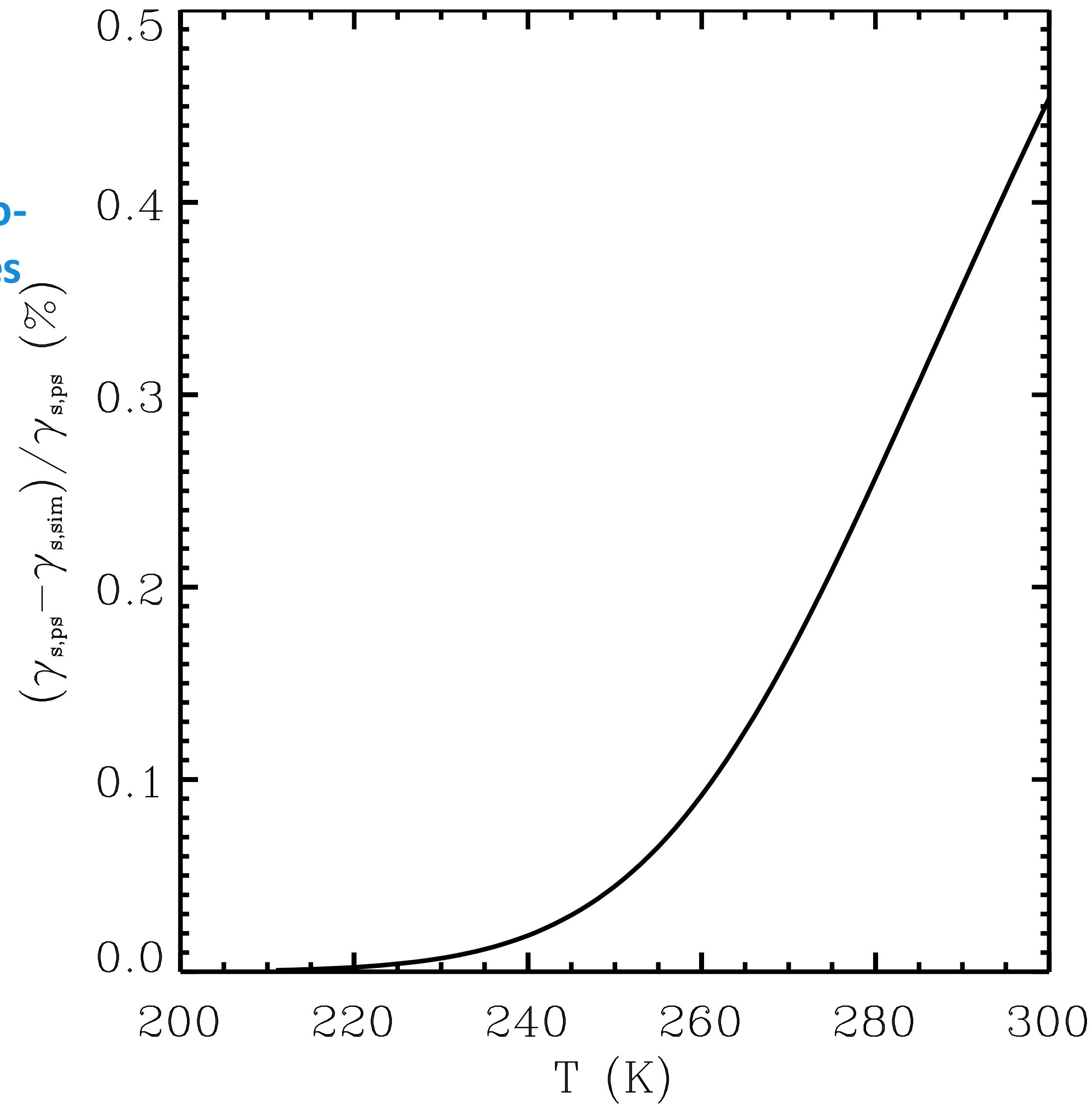
Difference between pseudo-adiabatic and simplified pseudo-adiabatic normalised lapse rates

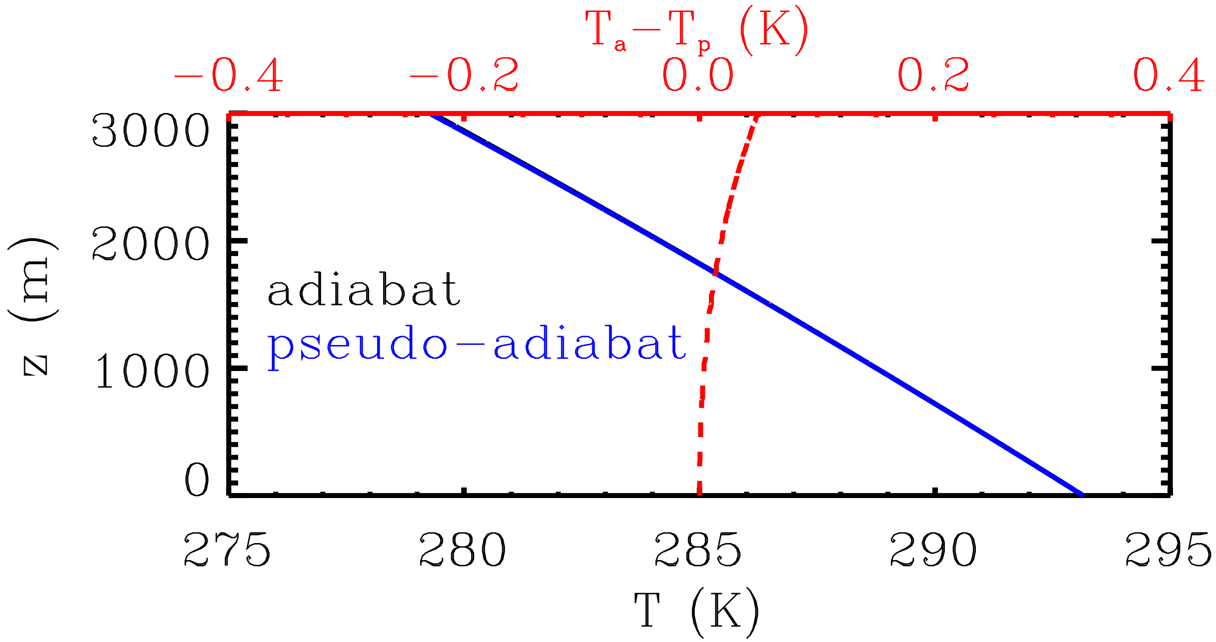
- pressure: 1 000 hPa
- the air initially saturated at 300 K

$$\Gamma_s < \Gamma_d$$

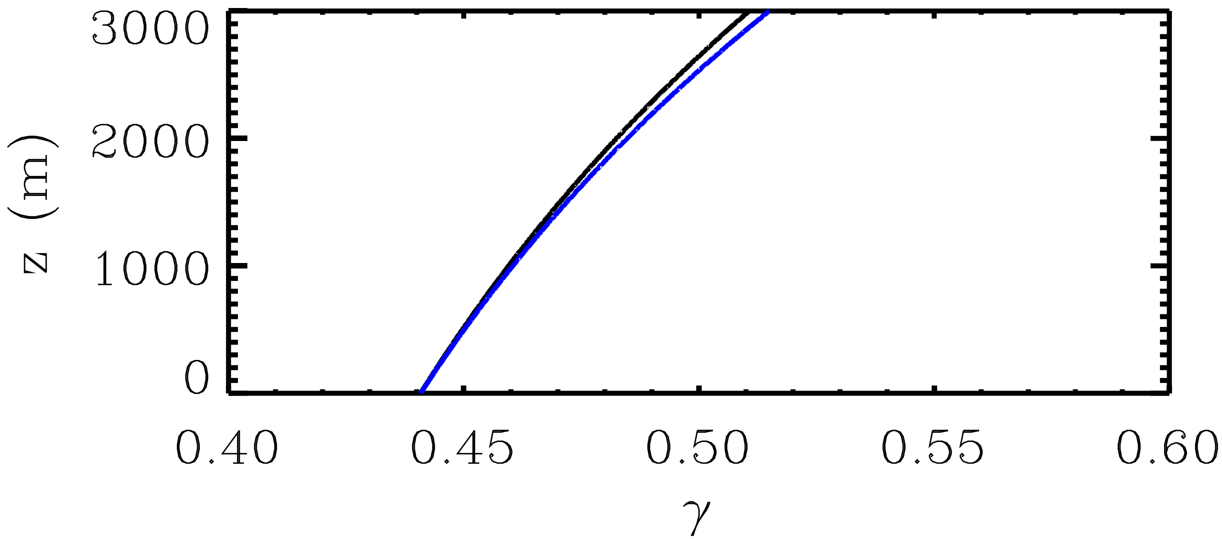
$$\Gamma_s = \gamma \Gamma_d$$

$$\gamma = \frac{c_{pd}}{c_p} \frac{1 + q_s \beta_T \frac{R_v}{R}}{1 + q_s \beta_T \frac{L_{lv}}{c_p T}}$$





$T_a - T_p$ (K)
−0.4
−0.2
0.0
0.2
0.4
3000
2000
1000
0
z (m)
adiabat
pseudo−adiabat
275
280
285
290
295
T (K)



3000
2000
1000
0
z (m)
0.40
0.45
0.50
0.55
0.60
γ

Water condensed in adiabatic process

Adiabatic enthalpy equation for a closed system: dry air, water vapor, and condensed water:

$$dh = c_p dT + L_{lv} dq_s \ ; \quad dh = \cancel{\delta q} + v dp$$

$$0 = c_p dT + L_{lv} dq_s - v dp$$

$$dq_l = -dq_s = \frac{c_p}{L_{lv}} dT - \frac{v}{L_{lv}} dp$$

$$dp = -\frac{g}{v} dz$$

$$dq_l = \frac{c_p}{L_{lv}} dT + \frac{g}{L_{lv}} dz$$

$$dq_l = \frac{c_p}{L_{lv}} \left(\frac{dT}{dz} + \frac{g}{c_p} \right) dz$$

$$\Gamma_d = \frac{g}{c_{pd}} \approx \frac{g}{c_p} \ , \qquad \Gamma_s = -\frac{dT}{dz}$$

$$dq_l \cong \frac{c_p}{L_{lv}} (\Gamma_d - \Gamma_s) dz$$

The amount of water condensed in a rising adiabatic parcel increases with height above the cloud base and with increasing temperature at the cloud base.

For shallow clouds (cloud depth not greater than ca. 300-500 m, e.g., stratocumulus clouds), it can be assumed that the amount of condensed water increases linearly with height above the cloud base (h).

The rate of this increase (c_q) is approximately constant and depends on the temperature and pressure at the cloud base.

$$q_l(h) = c_q(T,p) \cdot h\,; \quad c_q = \frac{c_p}{L_{lv}}(\Gamma_d - \Gamma_s) \quad \left[\frac{g}{kg \cdot m}\right]$$

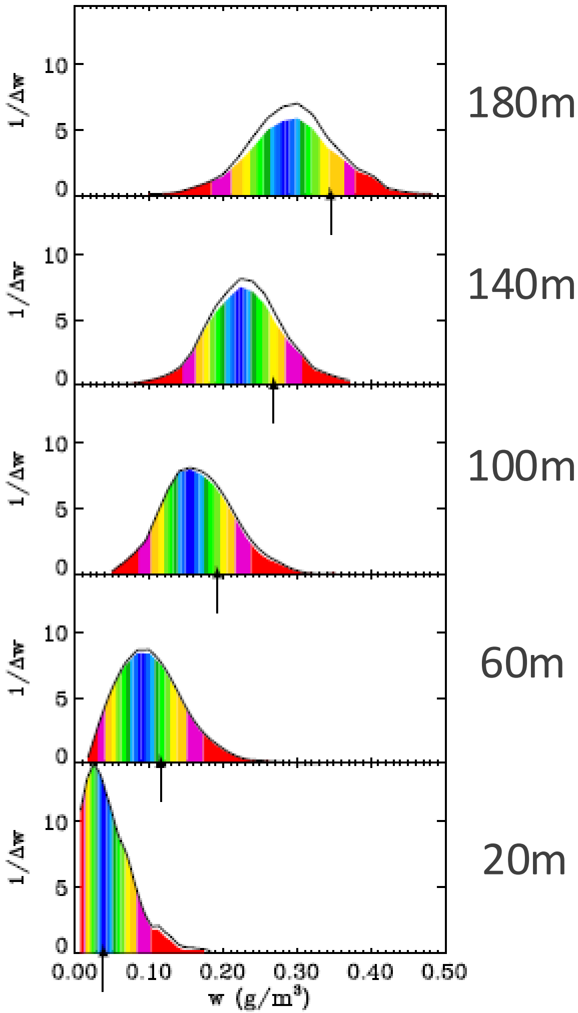
Liquid Water Content (LWC) is the mass of liquid water per unit volume of air:

$$LWC = q_l \cdot \rho\,; \quad \rho \text{ – density of the air}$$

For shallow clouds, it can be assumed that the air density is constant, therefore:

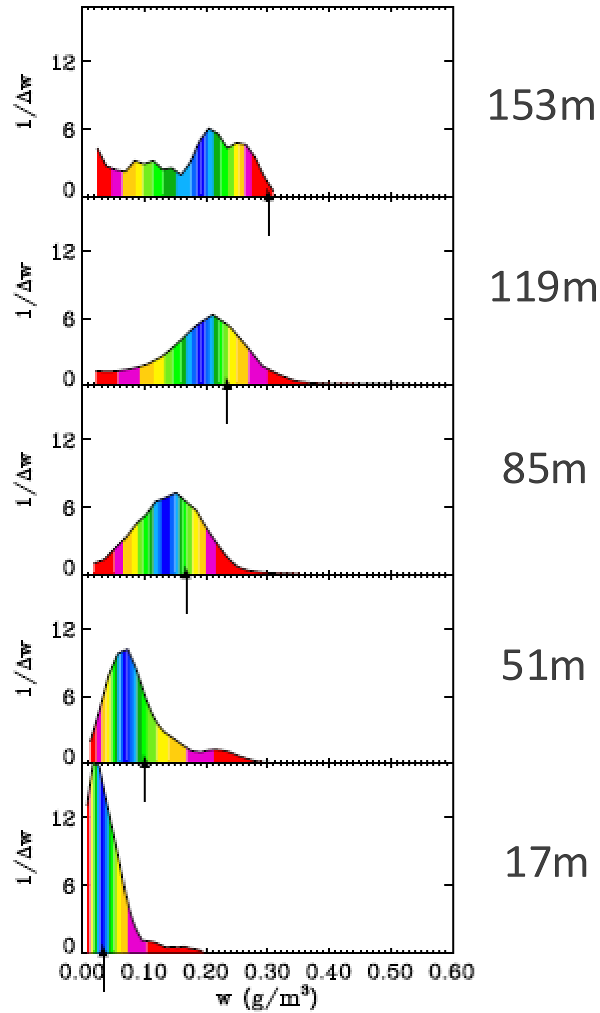
$$LWC = c_w(T,p) \cdot h\,; \quad c_w = \rho \frac{c_p}{L_{lv}}(\Gamma_d - \Gamma_s) \quad \left[\frac{g}{m^4}\right]$$

N=50 cm^{-3}
H_{base}=1278 m



$$c_w = 1.9 \cdot 10^{-3}\,\frac{g}{m^4}$$

N=255 cm^{-3}
H_{base}=844 m



$$c_w = 2 \cdot 10^{-3}\,\frac{g}{m^4}$$

Atmos. Chem. Phys., 13, 8489–8503, 2013
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doi:10.5194/acp-13-8489-2013

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Modeling microphysical effects of entrainment in clouds observed during EUCAARI-IMPACT field campaign

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