Dynamics of the Atmosphere and the Ocean

Lecture 11

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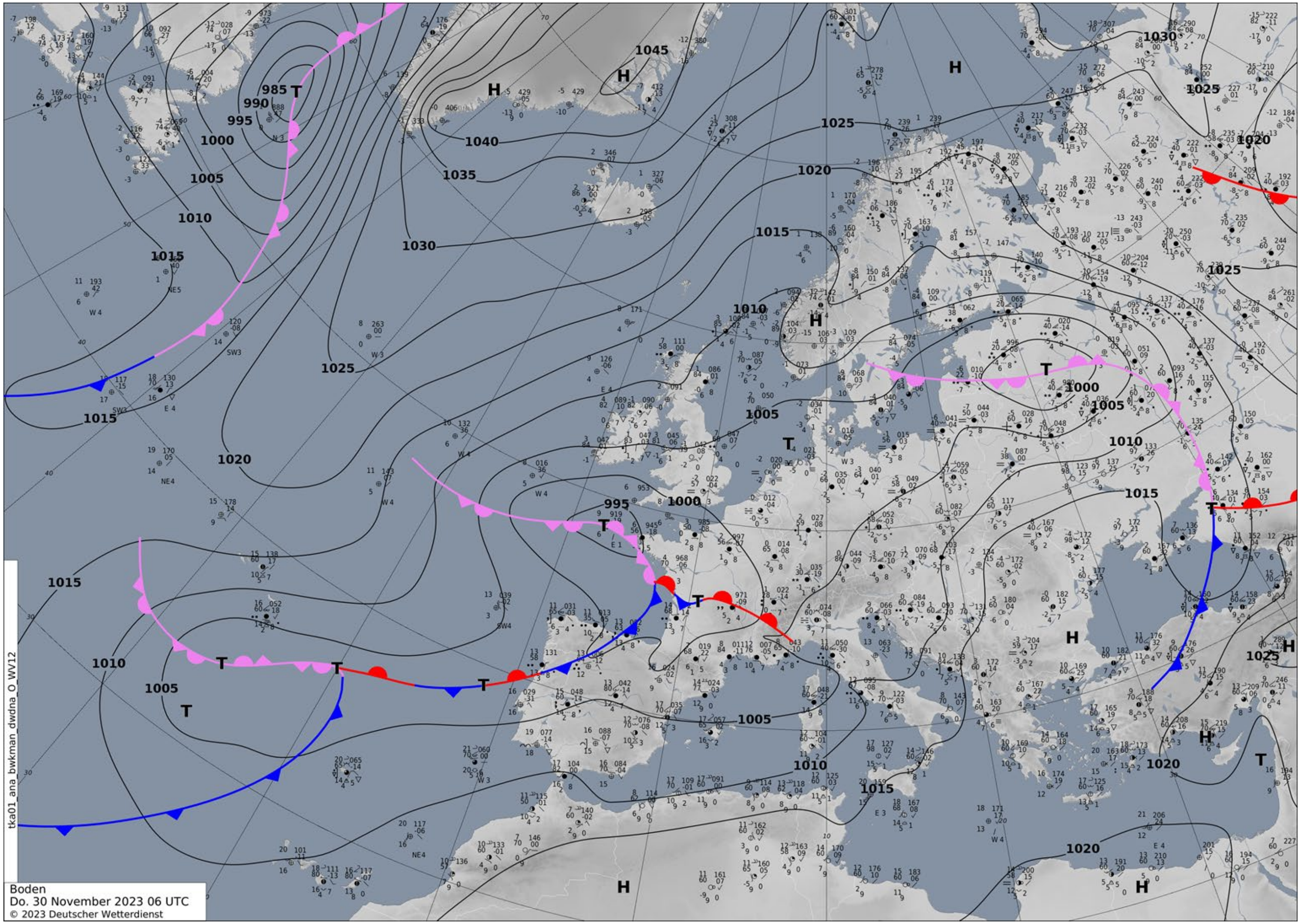
2025-26 Fall



The Quasi-Geostrophic Vorticity Equation

The vertical component of vorticity can be approximated geostrophically:

$$f_0 v_g = \frac{\partial \Phi}{\partial x}, \quad f_0 u_g = -\frac{\partial \Phi}{\partial y}$$

$$\zeta_g = \mathbf{k} \bullet \nabla \times \mathbf{V}_g$$

$$\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} = \frac{1}{f_0}\left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2}\right) = \frac{1}{f_0}\nabla^2 \Phi$$

The above equation can be used to determine $\zeta_g(x, y)$ from a known field $\Phi(x, y)$. Alternatively, it can be solved by inverting the Laplacian operator to determine from a known distribution of ζ_g, provided that suitable conditions on Φ are specified on the boundaries of the region in question.

This invertibility is one reason why vorticity is such a useful forecast diagnostic; if the evolution of the vorticity can be predicted, then inversion of the equation yields the evolution of the geopotential field, from which it is possible to determine the geostrophic wind and temperature distributions.

Since the Laplacian of a function tends to be a maximum where the function itself is a minimum, positive vorticity implies low values of geopotential and vice versa.

The quasi-geostrophic vorticity equation can be obtained from the x and y components of the quasi-geostrophic momentum equation:

$$\frac{D_g u_g}{Dt} - f_0 v_a - \beta y v_g = 0$$

$$\frac{D_g v_g}{Dt} + f_0 u_a + \beta y u_g = 0$$

Taking spatial derivatives and using the fact that the divergence of the geostrophic wind vanishes, yields the vorticity equation:

$$\frac{D_g \zeta_g}{Dt} = -f_0 \left(\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} \right) - \beta v_g$$

$$\mathrm{D}_g f / Dt = \mathbf{V}_g \bullet \boldsymbol{\nabla} f = \beta v_g$$

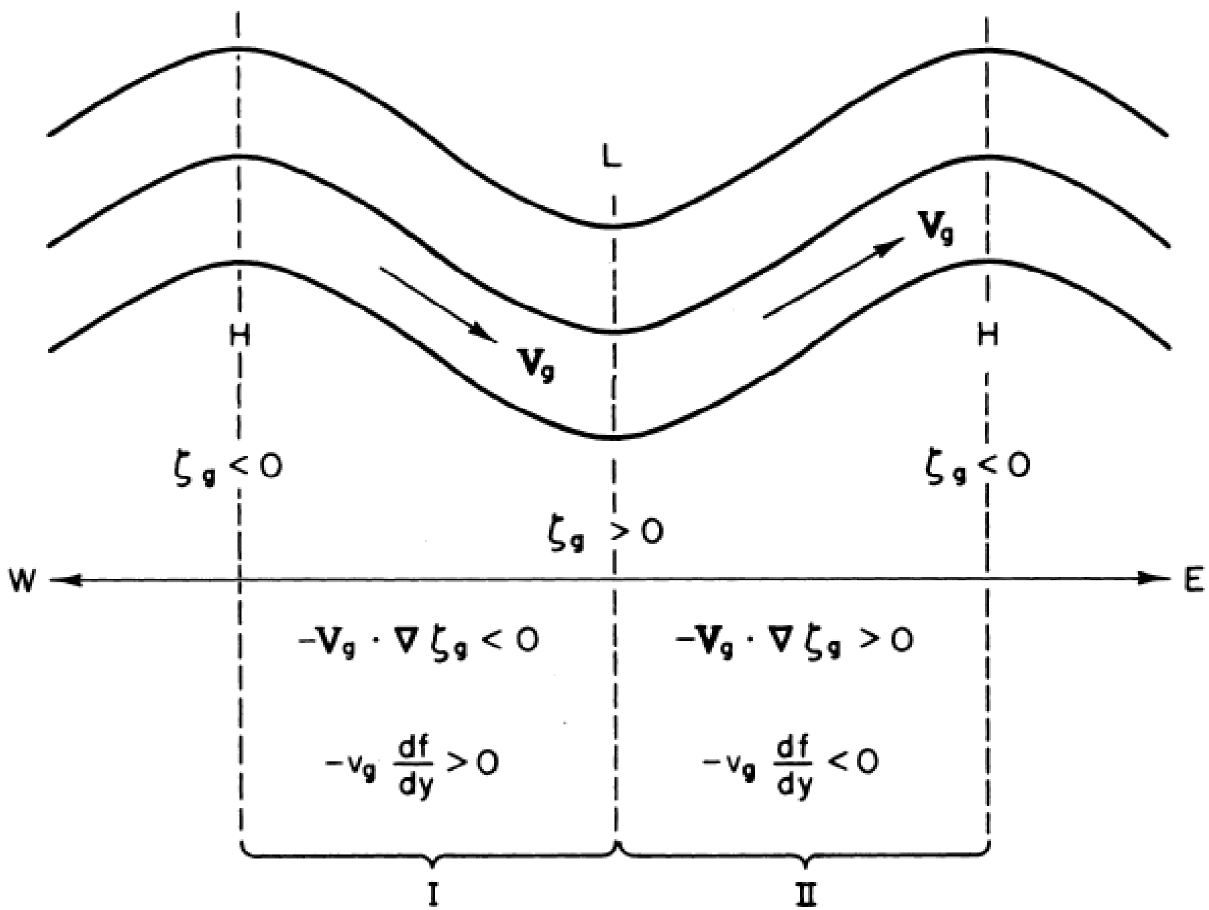
$$\frac{\partial \zeta_g}{\partial t} = -\mathbf{V}_g \bullet \boldsymbol{\nabla} \left(\zeta_g + f \right) + f_0 \frac{\partial \omega}{\partial p}$$

which states that the local rate of change of geostrophic vorticity is given by the sum of the advection of the absolute vorticity by the geostrophic wind plus the concentration or dilution of vorticity by stretching or shrinking of fluid columns (the divergence effect).

The vorticity tendency due to vorticity advection may be rewritten as: $\frac{\partial \zeta_g}{\partial t} = -\mathbf{V}_g \cdot \nabla \left(\zeta_g + f\right) + f_0 \frac{\partial \omega}{\partial p}$

$$-\mathbf{V}_g \cdot \nabla \left(\zeta_g + f\right) = -\mathbf{V}_g \cdot \nabla \zeta_g - \beta v_g$$

The two terms on the right represent the geostrophic advections of relative vorticity and planetary vorticity, respectively. For disturbances in the westerlies, these two effects tend to have opposite signs, as illustrated schematically:



$$\frac{\partial \zeta_g}{\partial t} = -\mathbf{V}_g \cdot \nabla (\zeta_g + f) + f_0 \frac{\partial \omega}{\partial p}$$

In order to investigate details of vorticity advection consider geopotential in sinusoidal form:

$$\Phi(x, y) = \Phi_0 - f_0 U y + f_0 A \sin kx \cos ly \qquad y = a(\phi - \phi_0)$$

The parameters Φ_0 , U , and A depend only on pressure, and the wave numbers k and l are defined as $k = 2\pi/Lx$ and $l = 2\pi/Ly$ with Lx , Ly the wavelengths in the x and y directions, respectively.
The geostrophic wind components are then given by

$$u_g = -\frac{1}{f_0}\frac{\partial \Phi}{\partial y} = U + u'_g = U + lA \sin kx \sin ly$$

$$v_g = \frac{1}{f_0}\frac{\partial \Phi}{\partial x} = v'_g = +kA \cos kx \cos ly$$

and $\left(u'_g,\ v'_g\right)$ is the geostrophic wind due to the synoptic wave disturbance. Then

$$\zeta_g = f_0^{-1} \nabla^2 \Phi = -\left(k^2 + l^2\right) A \sin kx \ \cos ly$$

It can be shown that in this simple case the advection of relative vorticity by the wave component of the geostrophic wind vanishes:

$$u'_g \partial\zeta_g \big/ \partial x + v'_g \partial \zeta_g \big/ \partial y = 0$$

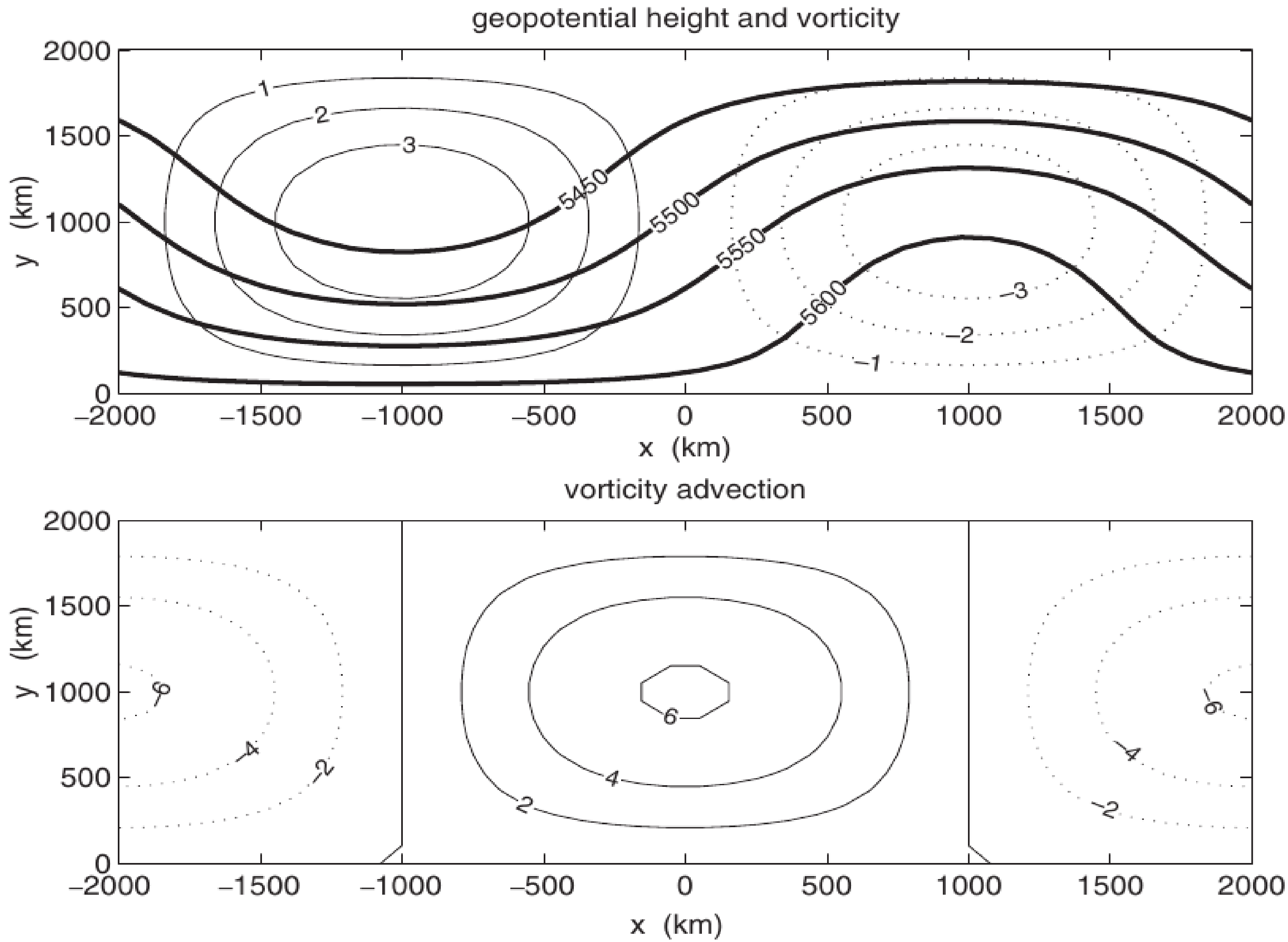
and the advection of relative vorticity is:

$$-u_g \frac{\partial \zeta_g}{\partial x} - v_g \frac{\partial \zeta_g}{\partial y} = -U \frac{\partial \zeta_g}{\partial x} = +kU\left(k^2 + l^2\right) A \cos kx \cos ly$$

Consequently the advection of planetary vorticity can be expressed as

$$\frac{\partial \zeta_g}{\partial t} = -\mathbf{V}_g \cdot \nabla (\zeta_g + f) + f_0 \frac{\partial \omega}{\partial p}$$

$$-\beta v_g = -\beta k A \cos kx \cos ly$$



(Top) Geopotential height in units of m, and relative vorticity in units of 10^{-5} s^{-1} for the sinusoidal disturbance of equation (6.20). Here $\Phi_0 = 5.5 \times 10^4$ m^2 s^{-2}, $f_0 = 10^{-4}$ s^{-1}, $f_0 A = 800$ m^2 s^{-2}, $U = 10$ m s^{-1}, and $k = l = (\pi/2) \times 10^{-6}$ m^{-1}. (Bottom) Advection of relative vorticity in units of 10^{-10} s^{-2} for the disturbance shown above.

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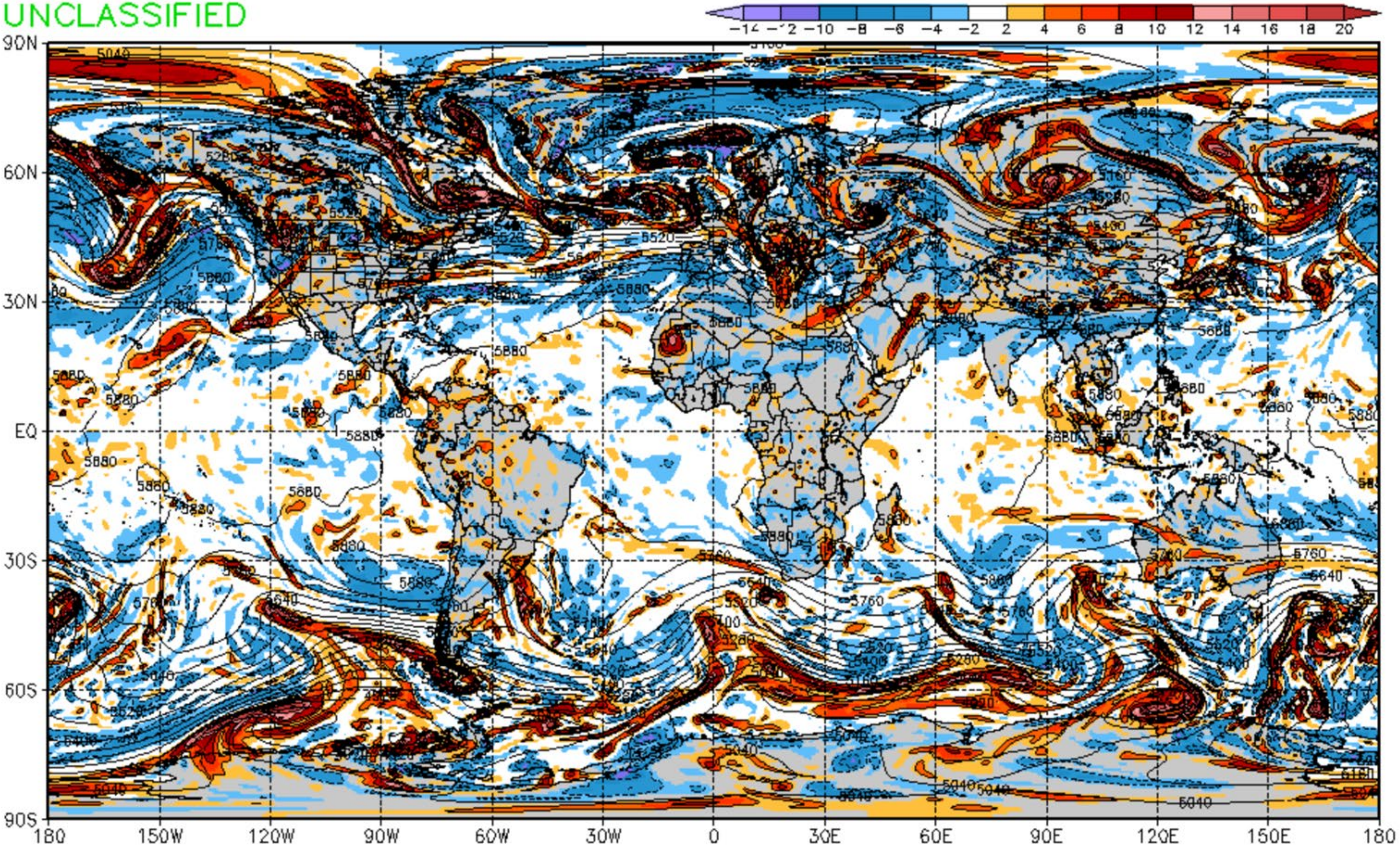

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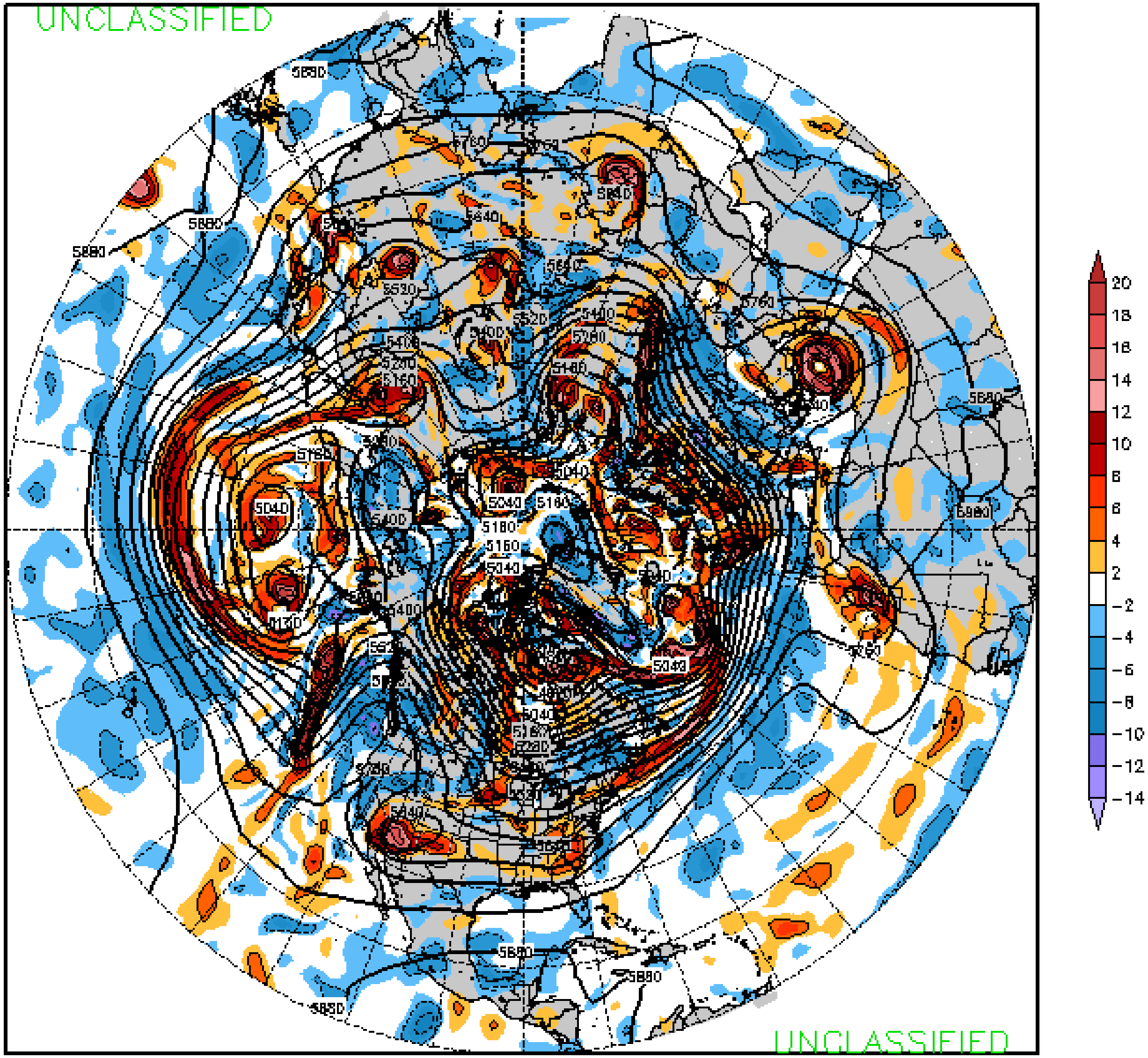

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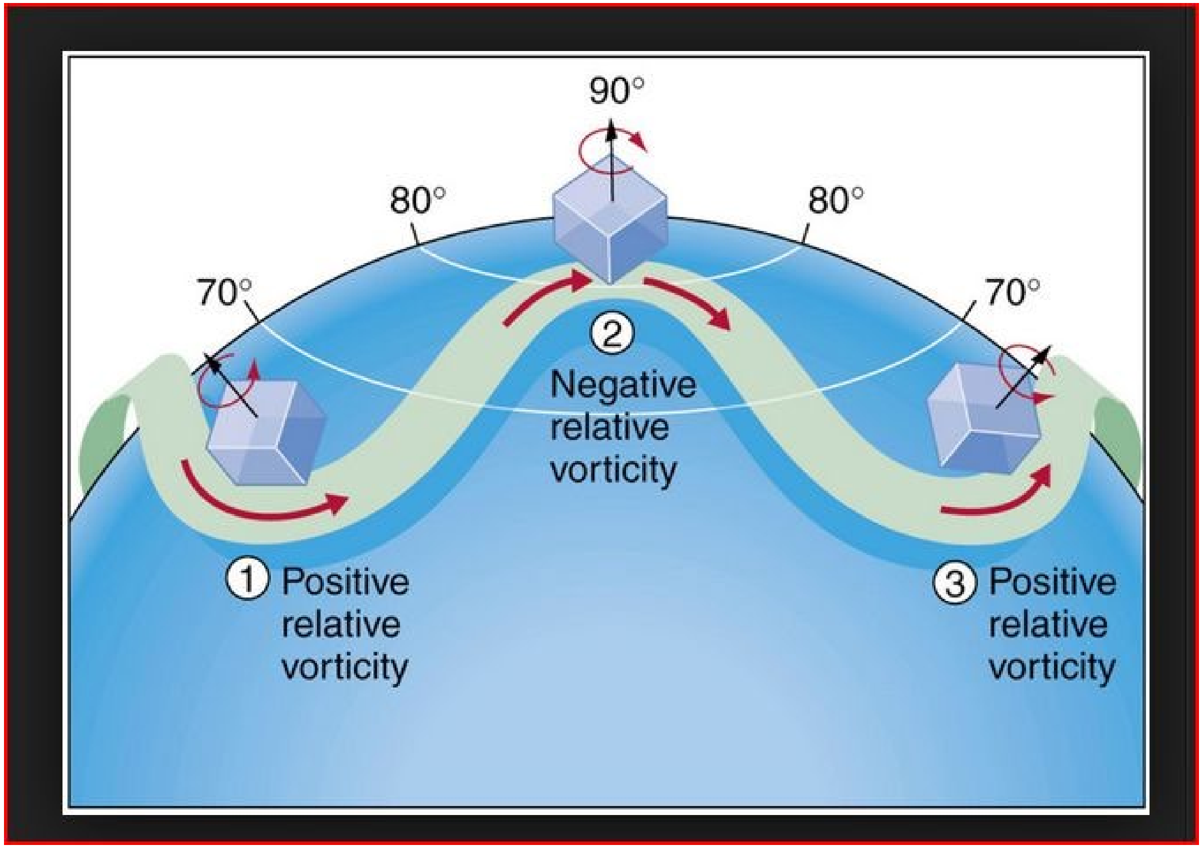
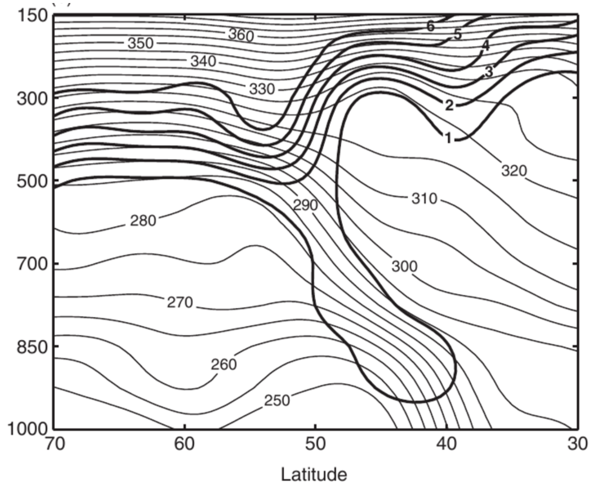
ROSSBY (PLANETARY) WAVES

are of most importance for large-scale meteorological processes in mid latitudes.

In an inviscid **barotropic fluid** of constant depth, the Rossby wave is an absolute vorticity-conserving motion due to the variation of the Coriolis parameter with latitude.
In a **baroclinic atmosphere**, the Rossby wave is a potential vorticity conserving motion that owes its existence to the isentropic gradient of potential vorticity.





Consider a closed chain of **barotropic fluid** parcels along a meridian. The absolute vorticity η is given by $\eta = \zeta + f$. Assume that $\zeta = 0$ at time t_0. Imagine the meridional displacement of a fluid parcel by δy from the original latitude at t_1. Then

$$(\zeta + f)_{t_1} = f_{t_0}$$

which, recalling $\beta = df/dy$ gives

$$\zeta_{t_1} = f_{t_0} - f_{t_1} = -\beta \delta y$$

From the above it is evident that if the chain of parcels is subject to a sinusoidal meridional displacement under absolute vorticity conservation, the resulting perturbation vorticity will be positive for a southward displacement and negative for a northward displacement.

This perturbation induces a meridional velocity field advecting the chain of fluid parcels southward west of the vorticity maximum and northward west of the vorticity minimum. The fluid parcels oscillate back and forth about equilibrium latitude. The pattern of vorticity maxima and minima propagates to the west, constituting a Rossby wave.

Let $\delta y = a \sin [k(x - ct)]$, where a is the maximum northward displacement. Then

$$v = D(\delta y)/Dt = -kca \cos [k(x - ct)]$$

$$\zeta = \partial v / \partial x = k^2 ca \sin [k(x - ct)]$$

$$c = -\beta / k^2$$

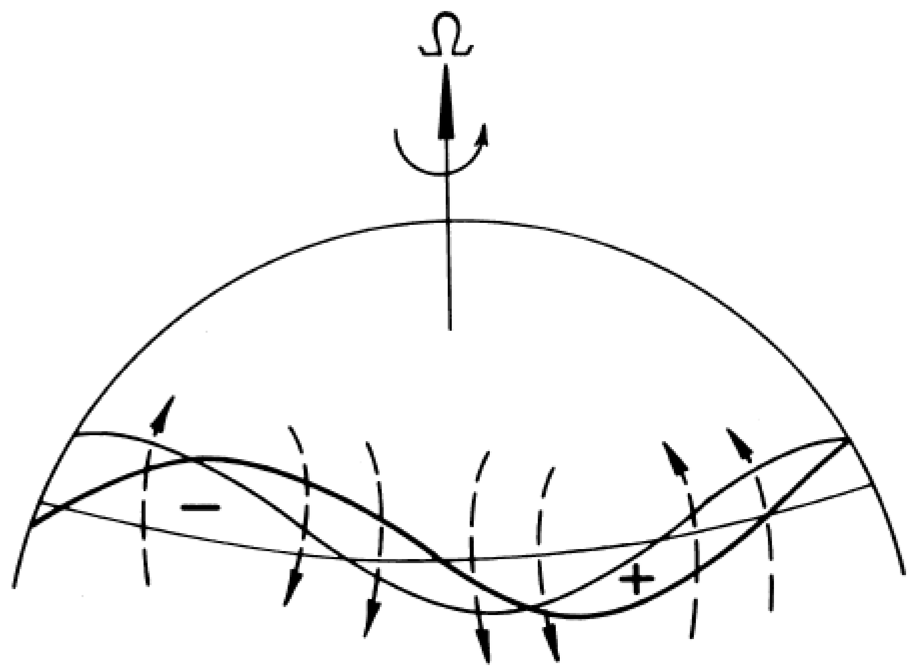


Fig. 7.14 Perturbation vorticity field and induced velocity field (dashed arrows) for a meridionally displaced chain of fluid parcels. Heavy wavy line shows original perturbation position; light line shows westward displacement of the pattern due to advection by the induced velocity.

Free Barotropic Rossby Waves

The dispersion relationship for barotropic Rossby waves may be derived formally by finding wave-type solutions of the linearized barotropic vorticity equation:

$$\frac{D_h\left(\zeta_g + f\right)}{Dt} = 0$$

For a midlatitude β plane this equation has the form:

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\zeta + \beta v = 0$$

Assume that the motion consists of a constant basic state zonal velocity plus a small horizontal perturbation:

$$u = \overline{u} + u', \quad v = v', \quad \zeta = \partial v'/\partial x - \partial u'/\partial y = \zeta'$$

Define perturbation streamfunction:

$$u' = -\partial\psi'/\partial y, \quad v' = \partial\psi'/\partial x$$

$$\zeta' = \nabla^2\psi'.$$

The perturbation form of the vorticity equation is:

$$\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)\nabla^2\psi'+\beta\frac{\partial\psi'}{\partial x}=0$$

We seek a solution of the form:

$$\psi'=\mathrm{Re}\left[\Psi\exp(i\phi)\right] \qquad \phi=kx+ly-\nu t.$$

and find that

$$(-\nu+k\overline{u})\left(-k^2-l^2\right)+k\beta=0$$

$$\nu=\overline{u}k-\beta k/K^2 \qquad K^2\equiv k^2+l^2$$

Recalling that $c = \nu/k$, we find that the zonal phase speed relative to the mean wind is

$$c-\overline{u}=-\beta/K^2$$

The above reduces to $c = -\beta/k^2$ when the mean wind vanishes and $l \rightarrow 0$.

The Rossby wave zonal phase propagation is always westward relative to the mean zonal flow and phase speed depends inversely on the square of the horizontal wavenumber. Therefore, Rossby waves are dispersive waves whose phase speeds increase rapidly with increasing wavelength.

For a typical midlatitude synoptic-scale disturbance, with similar meridional and zonal scales ($l \approx k$) and zonal wavelength of order 6000 km, the Rossby wave speed relative to the zonal flow is approximately $-8\ ms^{-1}$.
Because the mean zonal wind is generally westerly and greater than $8\ ms^{-1}$, synoptic-scale Rossby waves usually move eastward, but at a phase speed relative to the ground that is somewhat less than the mean zonal wind speed.

For longer wavelengths the westward Rossby wave phase speed may be large enough to balance the eastward advection by the mean zonal wind so that the resulting disturbance is stationary relative to the surface of the earth.

The free Rossby wave solution becomes stationary when

$$K^2 = \beta/\overline{u} \equiv K_s^2$$

The zonal group velocity for a Rossby wave may be either eastward or westward relative to the mean flow, depending on the ratio of the zonal and meridional wave numbers (long waves -> westward, short waves -> eastward).

Stationary Rossby modes (i.e., modes with c = 0) have zonal group velocities that are eastward relative to the ground.
Synoptic-scale Rossby waves also tend to have zonal group velocities that are eastward relative to the ground. For these waves, advection by the mean zonal wind is generally larger than the Rossby phase speed so that the phase speed is also eastward relative to the ground, but is slower than the zonal group velocity.
This implies that new disturbances tend to develop downstream of existing disturbances, which is an important consideration for forecasting.

Forced Topographic Rossby Waves

Forced stationary Rossby modes are of primary importance for understanding the planetary scale circulation pattern. They may be forced by longitudinally dependent diabatic heating patterns or by flow over topography, e.g. by flow over the Rockies and the mountains of central Asia.

As the <u>simplest possible</u> dynamical model of topographic Rossby waves, we use the **barotropic potential vorticity equation** for a homogeneous fluid of variable depth:

$$\frac{D_h}{Dt}\left(\frac{\zeta_g + f}{h}\right) = 0$$

We assume that the upper boundary is at a fixed height H , and the lower boundary is at the variable height $h_T(x, y)$ where $|h_T| << H$. We also use quasi-geostrophic scaling so that $|\zeta_g| << f_0$. We can then approximate the above by

$$H\left(\frac{\partial}{\partial t} + \mathbf{V} \bullet \boldsymbol{\nabla}\right)(\zeta_g + f) = -f_0 \frac{Dh_T}{Dt}$$

Which after linearization and use of β-plane approximation gives:

$$\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right)\zeta_g' + \beta v_g' = -\frac{f_0}{H}\overline{u}\frac{\partial h_T}{\partial x}$$

We examine solutions for the special case of a sinusoidal lower boundary:

$$h_T(x, y) = \mathrm{Re}\left[h_0 \exp(ikx)\right] \cos ly$$

and represent the geostrophic wind and vorticity by the perturbation streamfunction

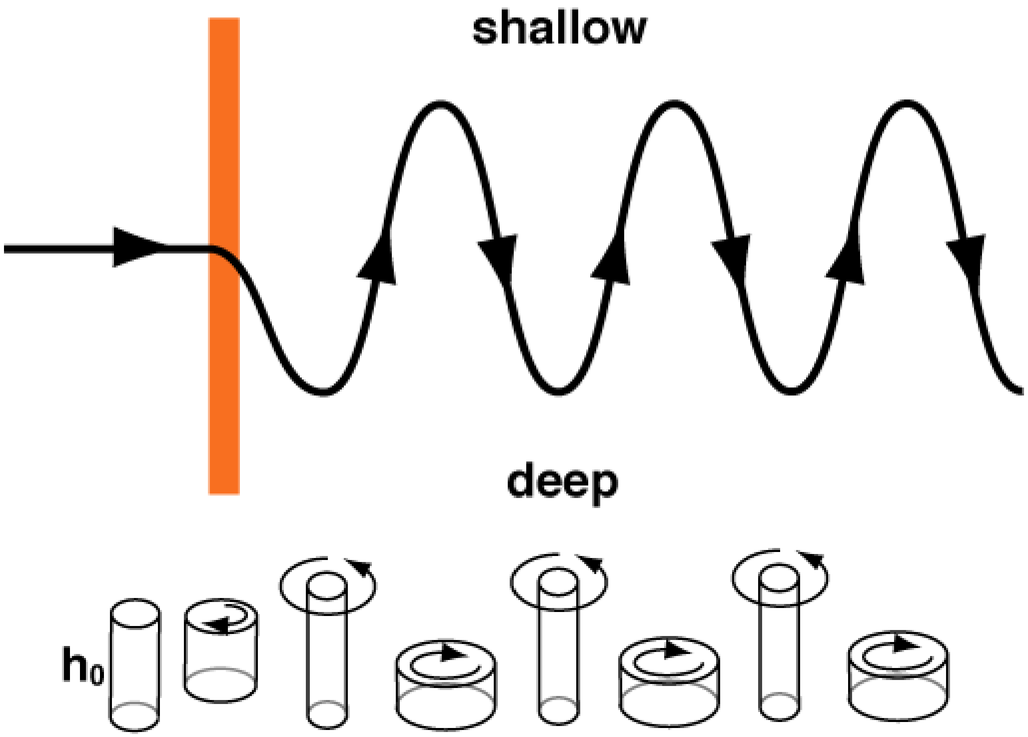
$$\psi(x, y) = \mathrm{Re}\left[\psi_0 \exp(ikx)\right] \cos ly$$

The equation has a steady-state solution with complex amplitude given by:

$$\psi_0 = f_0 h_0 / \left[H\left(K^2 - K_s^2\right)\right]$$

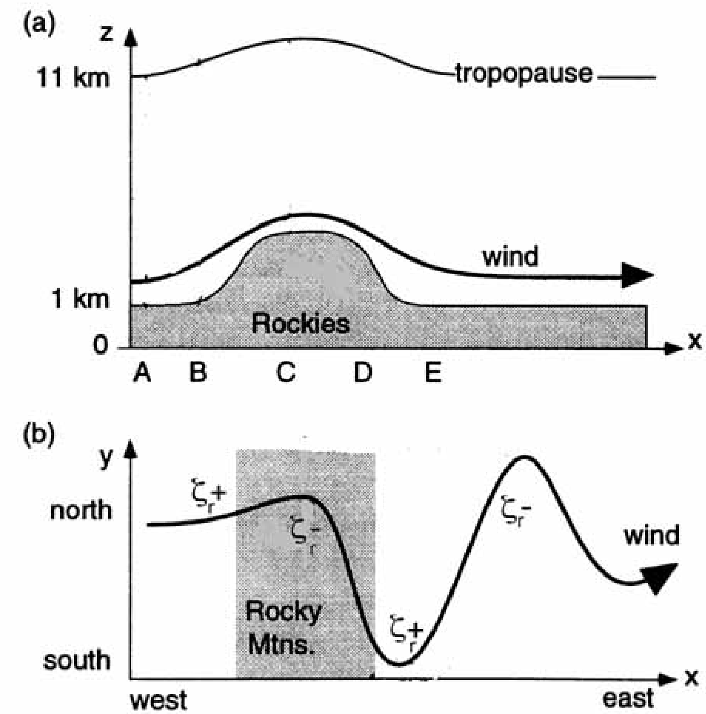
The streamfunction is either exactly in phase (ridges over the mountains) or exactly out of phase (troughs over the mountains), with the topography depending on the sign of $K^2 - K_s^2$.
For long waves ($K < Ks$) the topographic vorticity source is balanced primarily by the meridional advection of planetary vorticity (the β effect).
For short waves ($K > Ks$) the source is balanced primarily by the zonal advection of relative vorticity.
The topographic wave solution has the unrealistic characteristic that when the wave number exactly equals the critical wave number Ks the amplitude goes to infinity.
This singularity occurs at the zonal wind speed for which the free Rossby mode becomes stationary. Thus, it may be thought of as a resonant response of the barotropic system.

Forced Topographic Rossby Waves

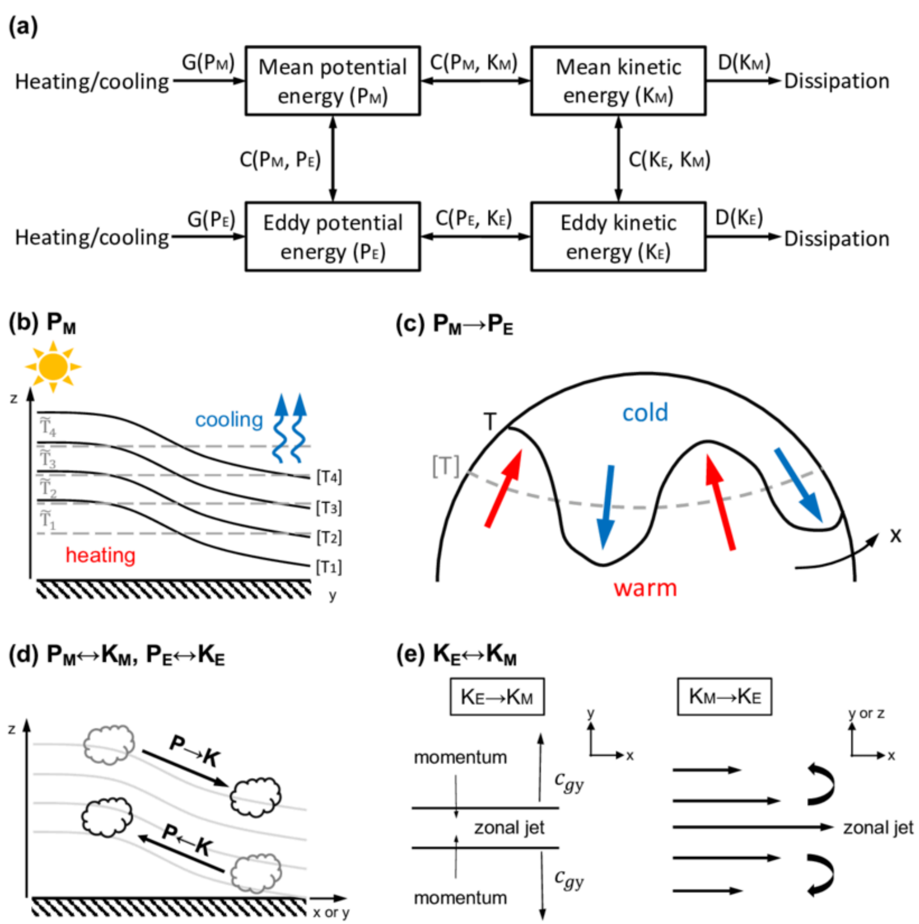


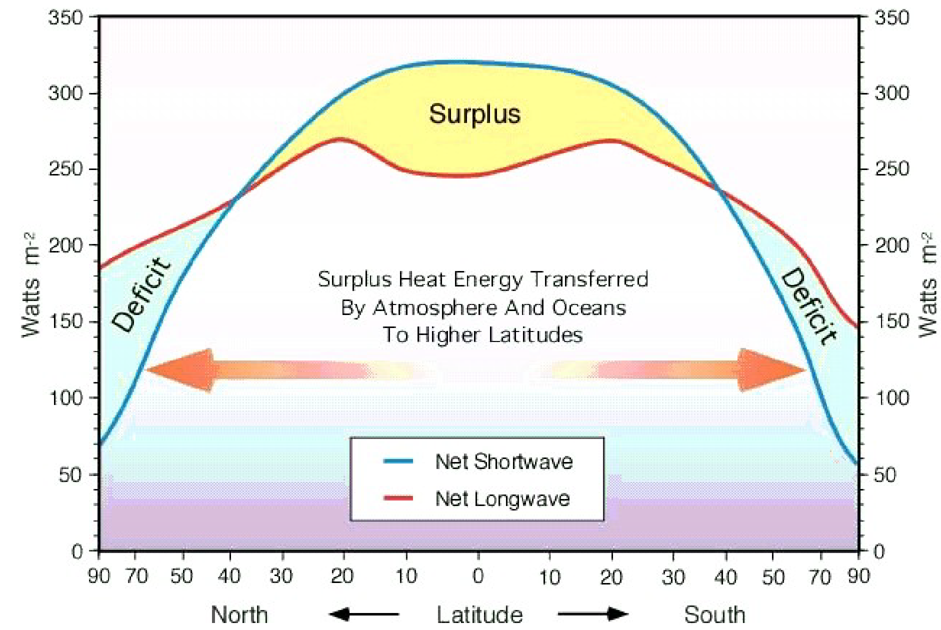
$$PV = \frac{2\Omega + \text{vorticity}}{h} = \frac{2\Omega}{h_0}$$

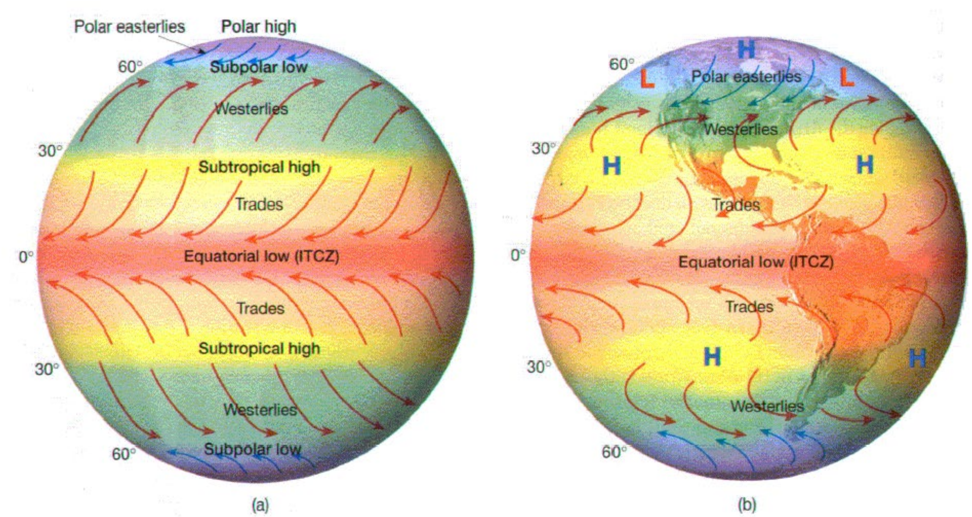
$$\text{vorticity} = 2\Omega\,(h/h_0 - 1)$$



Energy conversion in the atmosphere







Available Potential Energy

If we let dE_I be the internal energy in a vertical section of the column of height dz, then from the definition of internal energy

$$dE_I = \rho c_v T dz$$

$$E_I = c_v \int_0^\infty \rho T dz$$

The gravitational potential energy is just

$$dE_P = \rho g z dz$$

$$E_P = \int_0^\infty \rho g z dz = -\int_{p_0}^0 z dp$$

$$E_P = \int_0^\infty p dz = R \int_0^\infty \rho T dz \rightarrow c_v E_P = R E_I$$

and the total potential energy fulfills

$$E_P + E_I = (c_p/c_v) E_I = (c_p/R) E_P$$

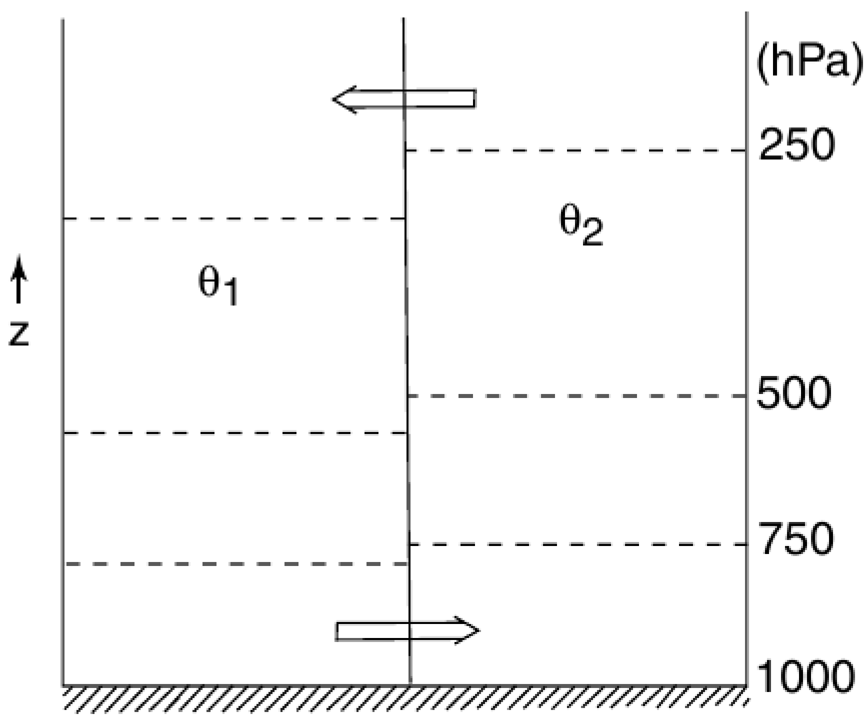
i.e. in hydrostatic atmosphere the total potential energy can be obtained E_I or E_P alone.

Now for an adiabatic process, total energy is conserved: $E_K + E_P + E_I = \text{constant}$
If the air masses are initially at rest $E_K = 0$.
Thus, if we let primed quantities denote the final state

$$E'_K + E'_P + E'_I = E_P + E_I$$

$$E'_K = (c_p/c_v)\,(E_I - E'_I)$$

Because θ is conserved for an adiabatic process, the two air masses cannot mix. It is clear that E_I' will be a minimum (designated by E_I'') when the masses are rearranged so that the air at potential temperature θ_1 lies entirely beneath the air at potential temperature θ_2.



The available potential energy (APE) can now be defined as the difference between the total potential energy of a closed system and the minimum total potential energy that could result from an adiabatic redistribution of mass. Thus, for the idealized model given earlier, the APE, which is designated by the symbol P ,

$$P = (c_p/c_v)\,(E_I - E''_I)$$

This is equivalent to the maximum kinetic energy that can be realized by an adiabatic process.

Lorenz (1960) showed that available potential energy is given approximately by the volume integral over the entire atmosphere of the variance of potential temperature on isobaric surfaces. Thus, letting θ designate the average potential temperature for a given pressure surface and θ' the local deviation from the average, the average available potential energy per unit volume satisfies the proportionality

$$\overline{P} \propto V^{-1} \int \left(\overline{\theta'^2} / \bar{\theta}^2 \right) dV$$

Observations indicate that for the atmosphere as a whole

$$\overline{P} / \left[\left(c_p / c_v \right) \overline{E}_I \right] \sim 5 \times 10^{-3}, \quad \overline{K} / \overline{P} \sim 10^{-1}$$

i.e. only small part of PE is converted to KE.

THE LORENZ ENERGY CYCLE (LEC)

It is useful to examine the exchange of energy between the eddies (e.g. high and low pressure systems) and the mean flow (general circulation, here westerlies).

We limit the analysis to quasi-geostrophic flow on a midlatitude β plane.

Eulerian mean equations in log-pressure coordinates can then be written as

$$\partial \overline{u} / \partial t - f_0 \overline{v} = -\partial \left(\overline{u'v'} \right) / \partial y + \overline{X}$$

$$f_0 \overline{u} = -\partial \overline{\Phi} / \partial y$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \overline{\Phi}}{\partial z}\right) + \overline{w} N^2 = \frac{\kappa}{H}\overline{J} - \frac{\partial}{\partial y}\left(\overline{v' \frac{\partial \Phi'}{\partial z}}\right)$$

$$\partial \overline{v} / \partial y + \rho_0^{-1} \partial \left(\rho_0 \overline{w} \right) / \partial z = 0$$

In order to analyze the exchange of energy between mean flow and eddies, we require a similar set of dynamical equations for the eddy motion. For simplicity we assume that the eddies satisfy the following linearized set of equations :

$$\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)u'-\left(f_0-\frac{\partial \overline{u}}{\partial y}\right)v'=-\frac{\partial \Phi'}{\partial x}+X'$$

$$\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)v'+f_0 u'=-\frac{\partial \Phi'}{\partial y}+Y'$$

$$\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)\frac{\partial \Phi'}{\partial z}+v'\frac{\partial}{\partial y}\left(\frac{\partial \overline{\Phi}}{\partial z}\right)+N^2 w'=\frac{\kappa J'}{H}$$

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{1}{\rho_0}\frac{\partial\left(\rho_0 w'\right)}{\partial z}=0$$

where X' and Y' are the zonally varying components of drag due to unresolved turbulent motions.

We now define a global average

$$\langle\ \rangle \equiv A^{-1} \int_0^{\infty} \int_0^{D} \int_0^{L} (\ )\, dx\,dy\,dz$$

where L is the distance around a latitude circle, D is the meridional extent of the midlatitude β plane, and A designates the total horizontal area of the β plane. Then for any quantity

$$\langle \partial\Psi / \partial x \rangle = 0$$

$$\langle \partial\Psi / \partial y \rangle = 0, \quad \text{if } \Psi \text{ vanishes at } y = \pm D$$

$$\langle \partial\Psi / \partial z \rangle = 0, \quad \text{if } \Psi \text{ vanishes at } z = 0 \text{ and } z \rightarrow \infty$$

After some algebra, defining zonal-mean and eddy kinetic and avaliable potential energies as

$$\overline{K} \equiv \left\langle \rho_0 \frac{\overline{u}^2}{2} \right\rangle, \quad K' \equiv \left\langle \rho_0 \frac{\overline{u'^2} + \overline{v'^2}}{2} \right\rangle,$$

$$\overline{P} \equiv \frac{1}{2} \left\langle \frac{\rho_0}{N^2} \left(\frac{\partial \overline{\Phi}}{\partial z} \right)^2 \right\rangle, \quad P' \equiv \frac{1}{2} \left\langle \frac{\rho_0}{N^2} \overline{\left(\frac{\partial \Phi'}{\partial z} \right)^2} \right\rangle$$

and defining energy transformations

$$\left[\overline{P}\bullet\overline{K}\right]\equiv\left\langle\rho_0\overline{w}\frac{\partial\overline{\Phi}}{\partial z}\right\rangle,\quad\left[P'\bullet K'\right]\equiv\left\langle\rho_0\overline{w'\frac{\partial\Phi'}{\partial z}}\right\rangle,$$

$$\left[K'\bullet\overline{K}\right]\equiv\left\langle\rho_0\overline{u'v'}\frac{\partial\overline{u}}{\partial y}\right\rangle,\quad\left[P'\bullet\overline{P}\right]\equiv\left\langle\frac{\rho_0}{N^2}\overline{v'\frac{\partial\Phi'}{\partial z}}\frac{\partial^2\overline{\Phi}}{\partial y\partial z}\right\rangle$$

and defining energy sources and sinks

$$\overline{R}\equiv\left\langle\frac{\rho_0}{N^2}\frac{\kappa\overline{J}}{H}\frac{\partial\overline{\Phi}}{\partial z}\right\rangle,\quad R'\equiv\left\langle\frac{\rho_0}{N^2}\overline{\frac{\kappa J'}{H}\frac{\partial\Phi'}{\partial z}}\right\rangle,$$

$$\overline{\varepsilon}\equiv\left\langle\rho_0\overline{u}\,\overline{X}\right\rangle,\quad\varepsilon'\equiv\left\langle\rho_0\left(\overline{u'X'}+\overline{v'Y'}\right)\right\rangle$$

energy equations may be expressed in the following form:

$$d\overline{K}/dt = [\overline{P} \bullet \overline{K}] + [K' \bullet \overline{K}] + \overline{\varepsilon}$$
$$d\overline{P}/dt = -[\overline{P} \bullet \overline{K}] + [P' \bullet \overline{P}] + \overline{R}$$
$$dK'/dt = [P' \bullet K'] - [K' \bullet \overline{K}] + \varepsilon'$$
$$dP'/dt = -[P' \bullet K'] - [P' \bullet \overline{P}] + R'$$

Adding above we obtain an equation for the rate of change of total energy

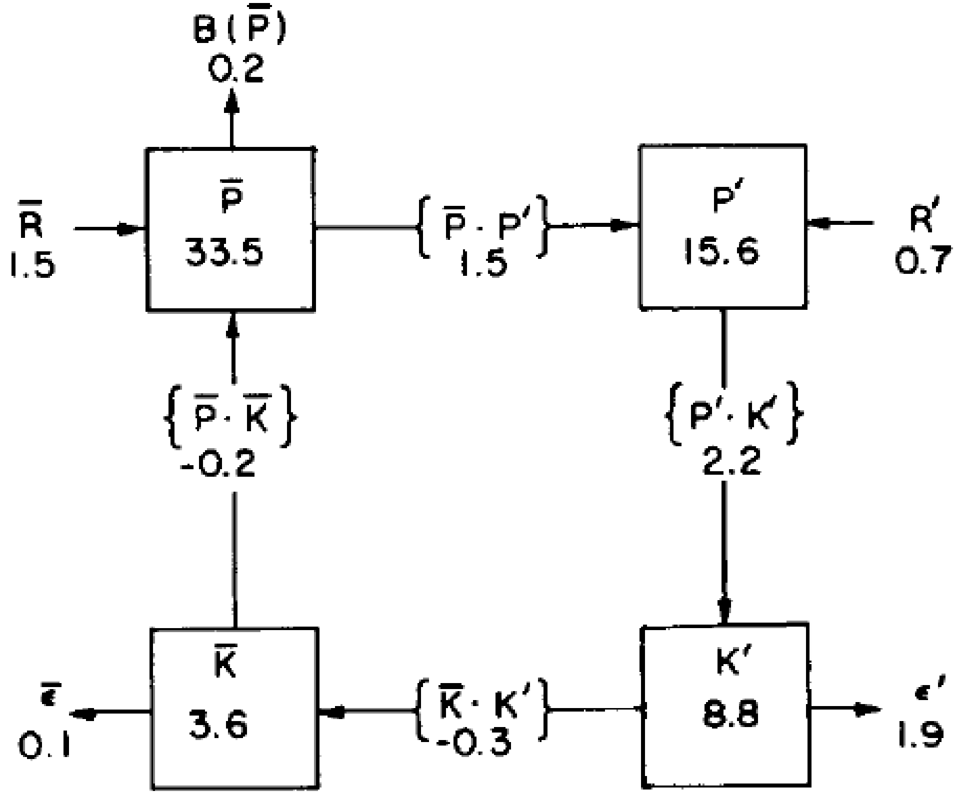
$$d\left(\overline{K} + K' + \overline{P} + P'\right)/dt = \overline{R} + R' + \overline{\varepsilon} + \varepsilon'$$

For adiabatic inviscid flow the right side vanishes and the total energy K + K' + P +P' is conserved. In this system the zonal-mean kinetic energy does not include a contribution from the mean meridional flow because the zonally averaged meridional momentum equation was replaced by the geostrophic approximation.

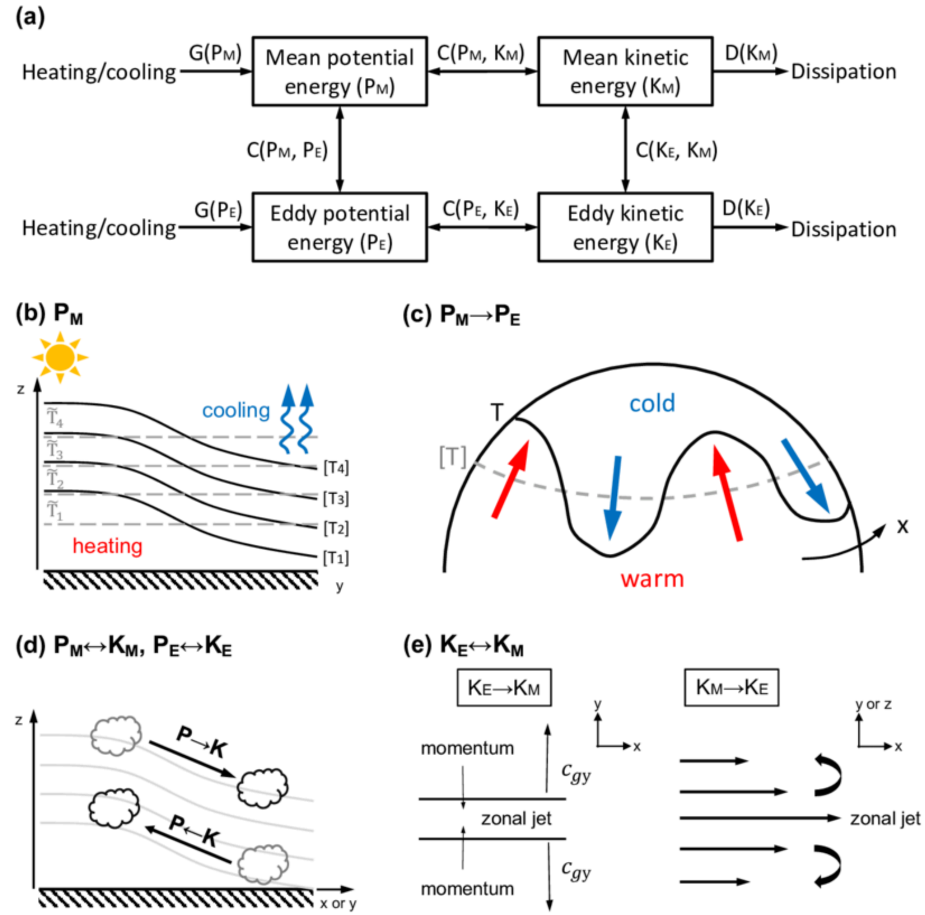
In the long-term the production of available potential energy by zonal-mean and eddy diabatic processes must balance the mean plus eddy kinetic energy dissipation:

$$\overline{R} + R' = -\overline{\varepsilon} - \varepsilon'$$

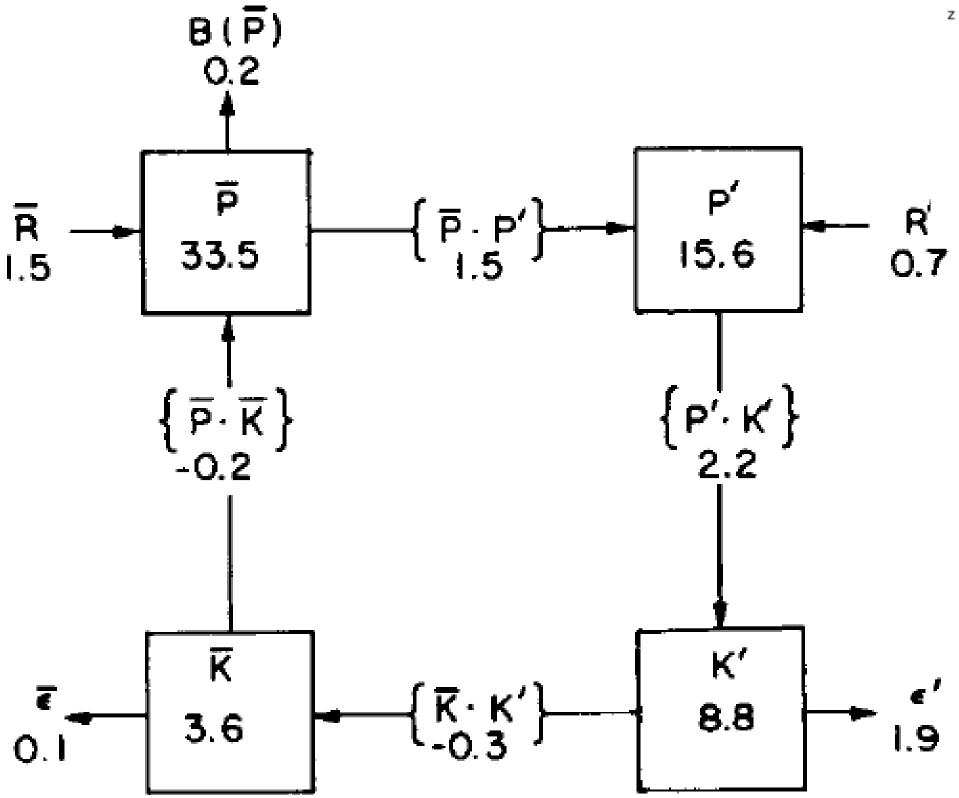
Because solar radiative heating is a maximum in the tropics, where the temperatures are high, it is clear that R, the generation of zonal-mean potential energy by the zonal-mean heating, will be positive.
For a dry atmosphere in which eddy diabatic processes are limited to radiation and diffusion R , the diabatic production of eddy available potential energy should be negative because the thermal radiation emitted to space from the atmosphere increases with increasing temperature and thus tends to reduce horizontal temperature contrasts in the atmosphere.
For the earth's atmosphere, however, the presence of clouds and precipitation greatly alters the distribution of R . Present estimates (suggest that in the Northern Hemisphere R is positive and nearly half as large as R. Thus, diabatic heating generates both zonal-mean and eddy available potential energy.



The observed mean energy cycle for the Northern Hemisphere. Numbers in squares are energy amounts in units of $10^5 Jm^{-2}$. Numbers next to arrows are energy transformation rates in units of W m^{-2} . B(p) represents a net energy flux into the Southern Hemisphere. (Adapted from Oort and Peixoto, 1974.)



(a)
Heating/cooling
G(PM)
Mean potential energy (PM)
C(PM, KM)
Mean kinetic energy (KM)
D(KM)
Dissipation
C(PM, PE)
C(KE, KM)
Heating/cooling
G(PE)
Eddy potential energy (PE)
C(PE, KE)
Eddy kinetic energy (KE)
D(KE)
Dissipation
(b) PM
cooling
heating
[T4]
[T3]
[T2]
[T1]
(c) PM→PE
T
[T]
cold
warm
x
(d) PM↔KM, PE↔KE
P→K
P←K
x or y
(e) KE↔KM
KE→KM
KM→KE
momentum
zonal jet
momentum
zonal jet



B(P̄)
0.2
R̄
1.5
P̄
33.5
{P̄ · P′}
1.5
P′
15.6
R′
0.7
{P̄ · K̄}
-0.2
{P′ · K′}
2.2
ε̄
0.1
K̄
3.6
{K̄ · K′}
-0.3
K′
8.8
ε′
1.9