Exercise Sheet 7

Questions, comments and corrections: e-mail to Marta.Waclawczyk@fuw.edu.pl

1. **Flow past a symmetric Zhukovsky airfoil**

   In the solution of the irrotational flow past a finite flat plate at angle of attack α, a singularity in the velocity field at the *trailing edge* may be removed by introducing a clockwise circulation Γ. However this circulation still leaves a singularity at the *leading edge*. Let us construct the irrotational flow past an airfoil with round nose but a sharp trailing edge (*symmetric Zhukovsky airfoil*) which is free from singularities.

   (a) Start from an irrotational flow past a cylinder of radius $(a+\lambda)$ with clockwise circulation Γ. Use the appropriate transformations, together with the Zhukovsky transformation

   $$z = z_1 + \frac{a^2}{z_1}, \tag{1}$$

   on the cylinder with center on the real axis at a displaced position $(-\lambda, 0)$, to obtain the flow past a *symmetric Zhukovsky airfoil* at angle of attack α.

   (b) Obtain an expression for the complex velocity $\mathrm{d}w/\mathrm{d}z$ of the flow past the Zhukovsky airfoil in terms of the complex velocity of the irrotational flow around a cylinder (centered at the origin of the complex plane) with clockwise circulation Γ.

   (c) Which value of Γ makes the velocity finite at the **trailing** edge?

   (d) Show that now the **leading** edge does not cause concern and the flow is free from singularities.

   (e) Write the expression for the lift force (per unit length) on the airfoil in terms of the angle of attack α.

2. **Milne-Thomson's circle theorem** Prove that if w_0 is a complex potential in the whole space which has all singularities in the region $|z| > a$, then

   $$w = w_0(z) + \overline{w_0}\left(\frac{a^2}{\bar{z}}\right), \tag{2}$$

   is a complex potential for which $|z| = a$ is a stream line and that w has the same singularities as w_0 for $|z| > a$.

3. Use the Milne-Thomson theorem to find:

   (a) The flow around a cylinder $|z| = a$ in a uniform stream of an ideal fluid. Show that this is not the only flow satisfying the boundary condition at $|z| = a$ and a linear vortex can be added at the center to establish a family of solutions with different circulations, which have the form

$$w(z) = U\left(z + \frac{a^2}{z}\right) - \frac{i\Gamma}{2\pi} \ln z. \tag{3}$$

   (b) The flow around a cylinder when a vortex is placed at point z_0, with $|z_0| > a$