Dynamics of the Atmosphere and the Ocean

Lecture 1

Szymon Malinowski

2021-2022 Fall

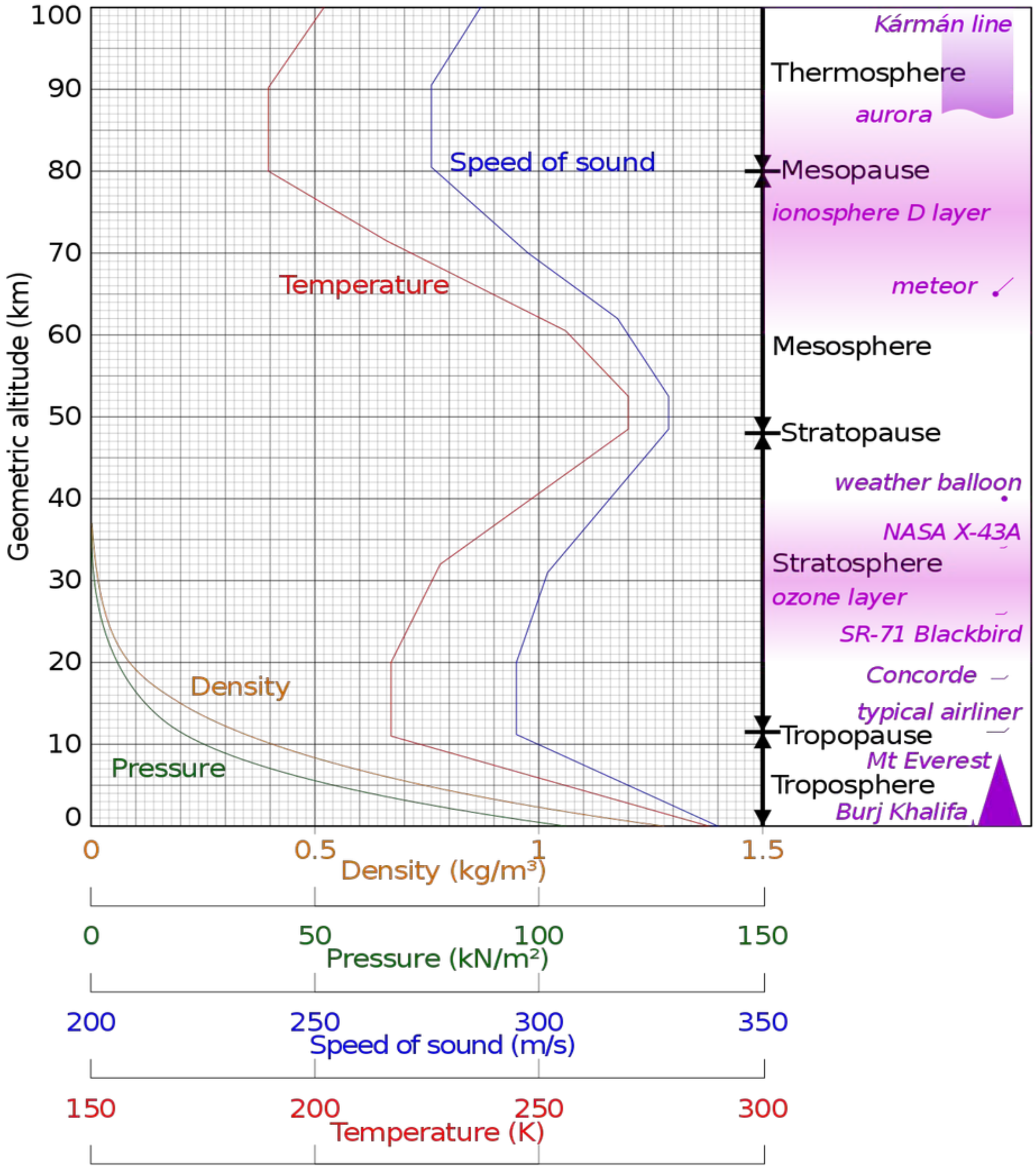
Acknowledgments to G.K. Vallis,
A lot of material in this lecture comes from:
Atmospheric and Oceanic Fluid Dynamics.
Available from www.princeton.edu/gkv/aofd
(and later book published by Cambridge University Press)

James Holton's "An introduction to dynamic meteororlogy" is the must read textbook and should be a companion to this course

Geophysical fluid dynamics,
in its broadest meaning,
refers to the fluid dynamics of naturally occurring flows,
such as lava flows,
oceans, and planetary atmospheres,
on Earth and other planets.

We will focus on the applications of geophysical fluid dynamics to the Earth's atmosphere and ocean.

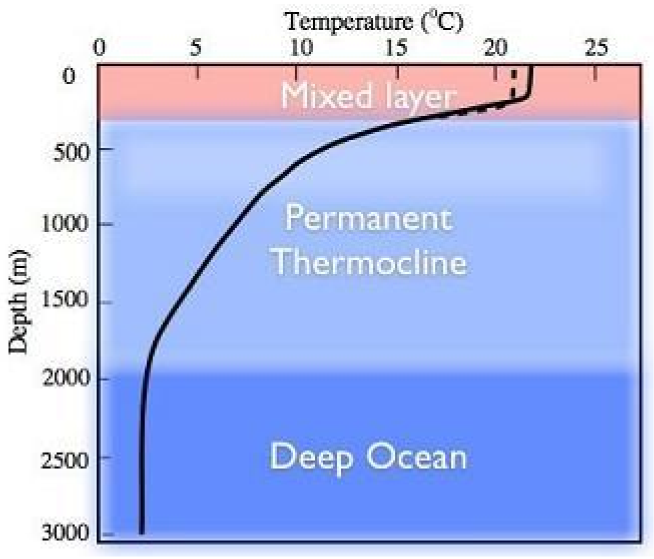
ATMOSPHERE



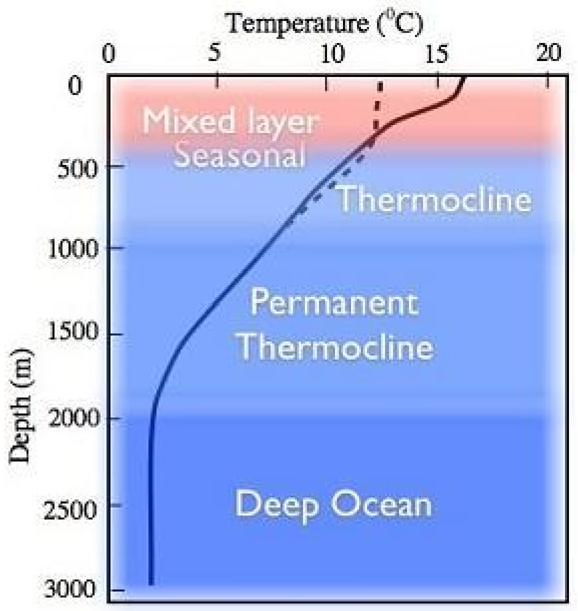
Geometric altitude (km)
100
90
80
70
60
50
40
30
20
10
0
Speed of sound
Temperature
Density
Pressure
Kármán line
Thermosphere
aurora
Mesopause
ionosphere D layer
meteor
Mesosphere
Stratopause
weather balloon
NASA X-43A
Stratosphere
ozone layer
SR-71 Blackbird
Concorde
typical airliner
Tropopause
Mt Everest
Troposphere
Burj Khalifa
0 0.5 1 1.5
Density (kg/m³)
0 50 100 150
Pressure (kN/m²)
200 250 300 350
Speed of sound (m/s)
150 200 250 300
Temperature (K)

OCEAN

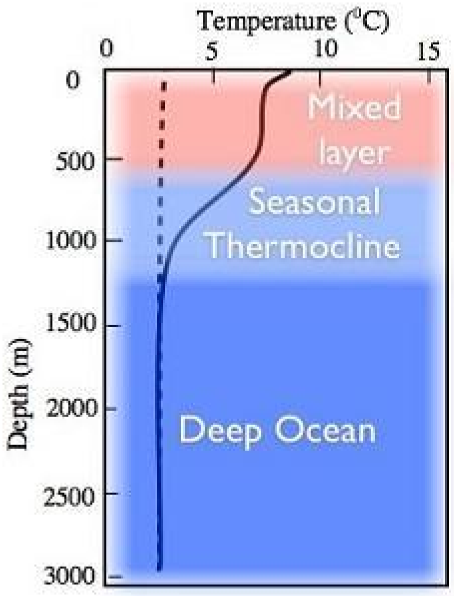
Tropics



Midlatitudes



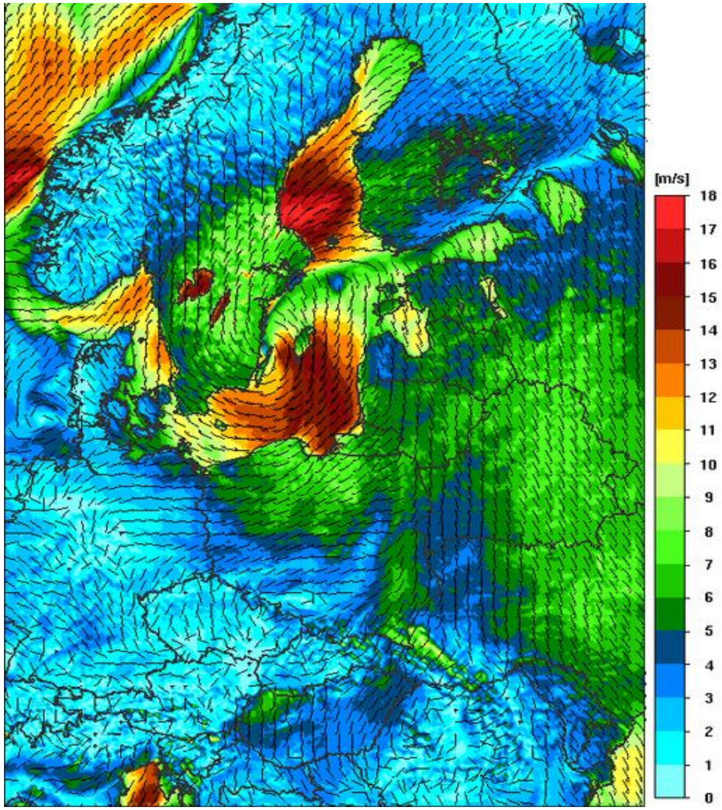
High latitudes

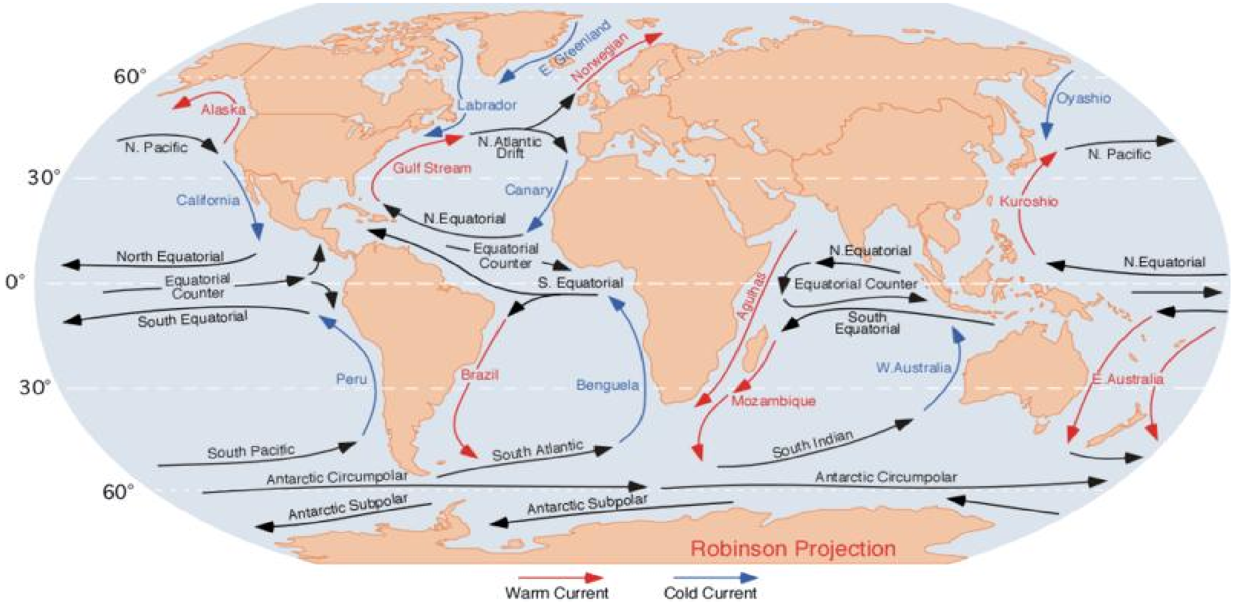


Not only temperature, but salinity as well

Eulerian specification of the flow.

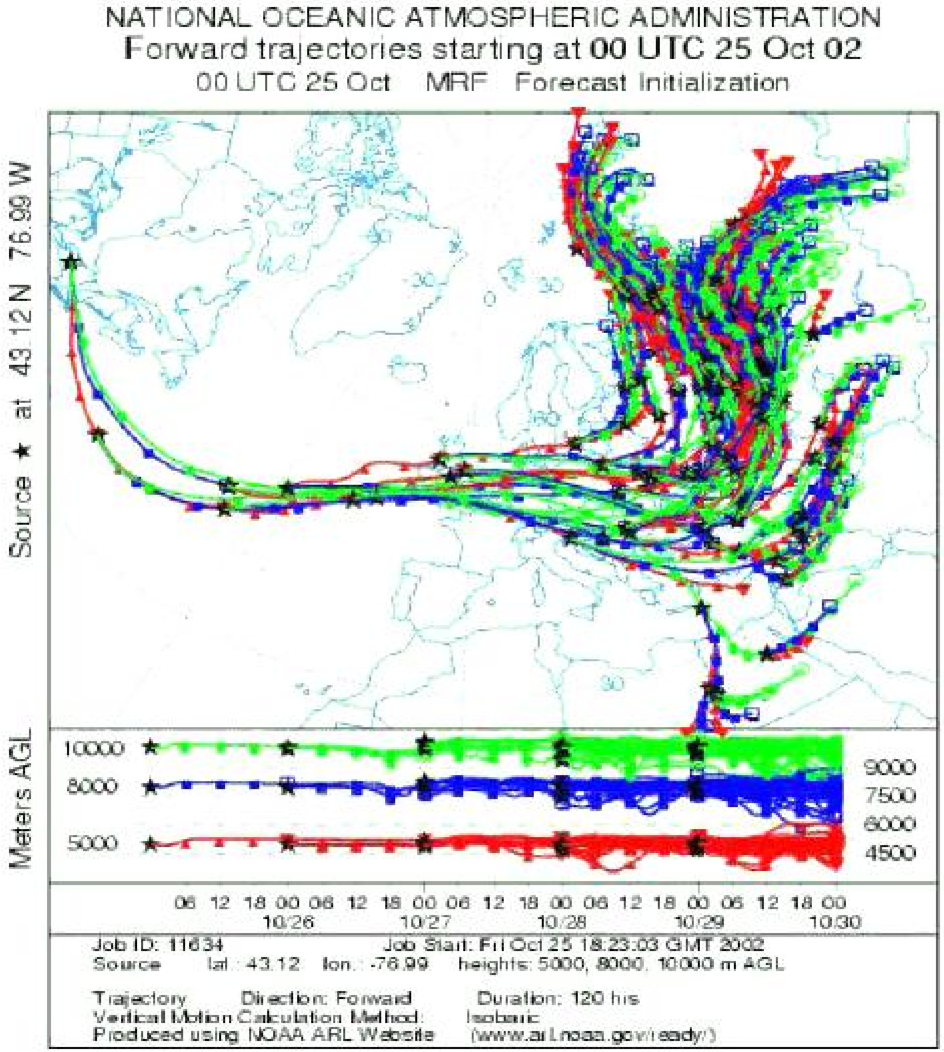
The **Eulerian** specification of the flow field is a way of looking at fluid motion that focuses on specific locations in the space (**X**) through which the fluid flows as time (t) passes. Velocity **V**(**X**,t) characterizes the flow.





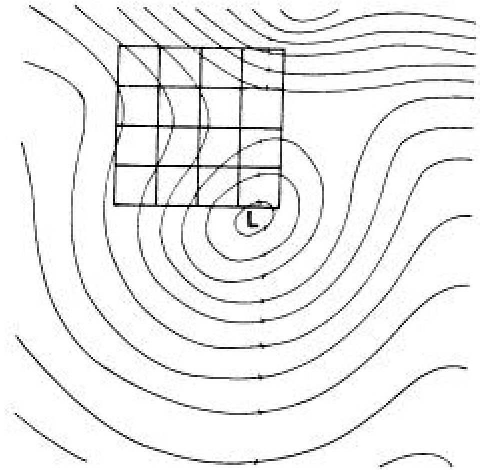
Lagrangian specification of the flow

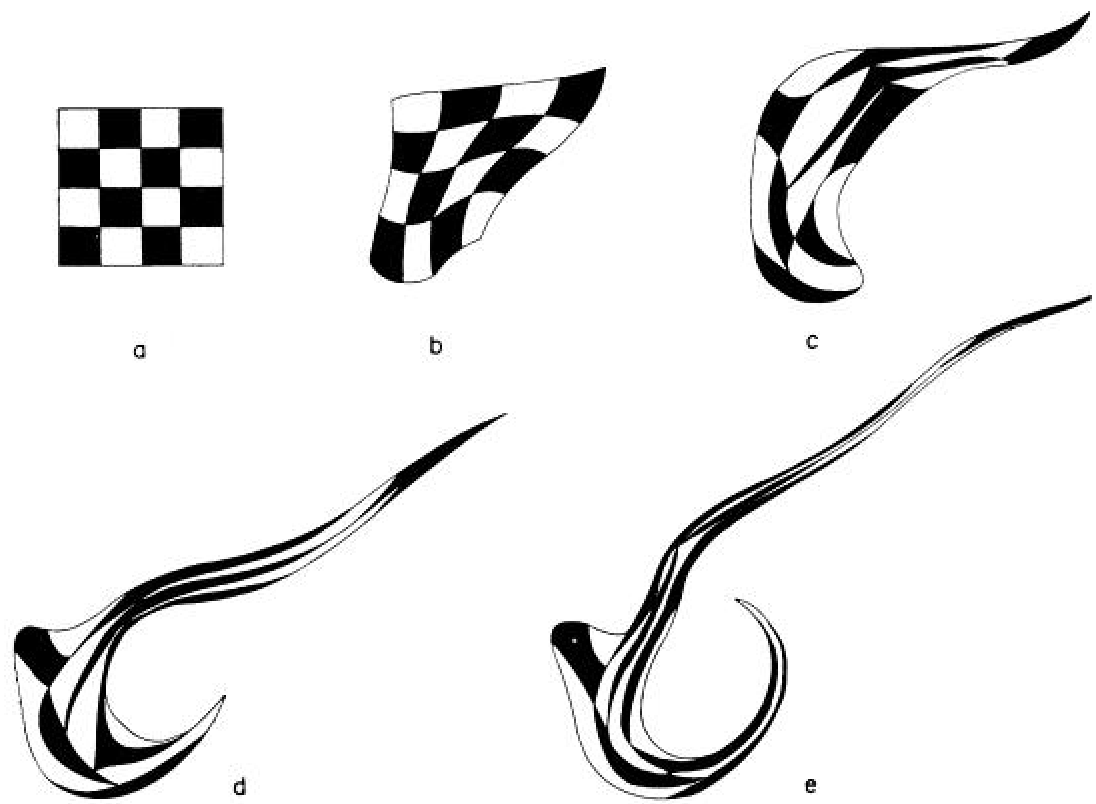
The Lagrangian specification of the flow field is a way of looking at fluid motion where the observer follows an individual fluid parcel as it moves through space and time. Position of the parcel is **X**(t) and properties evolve with time.



Practical problem:

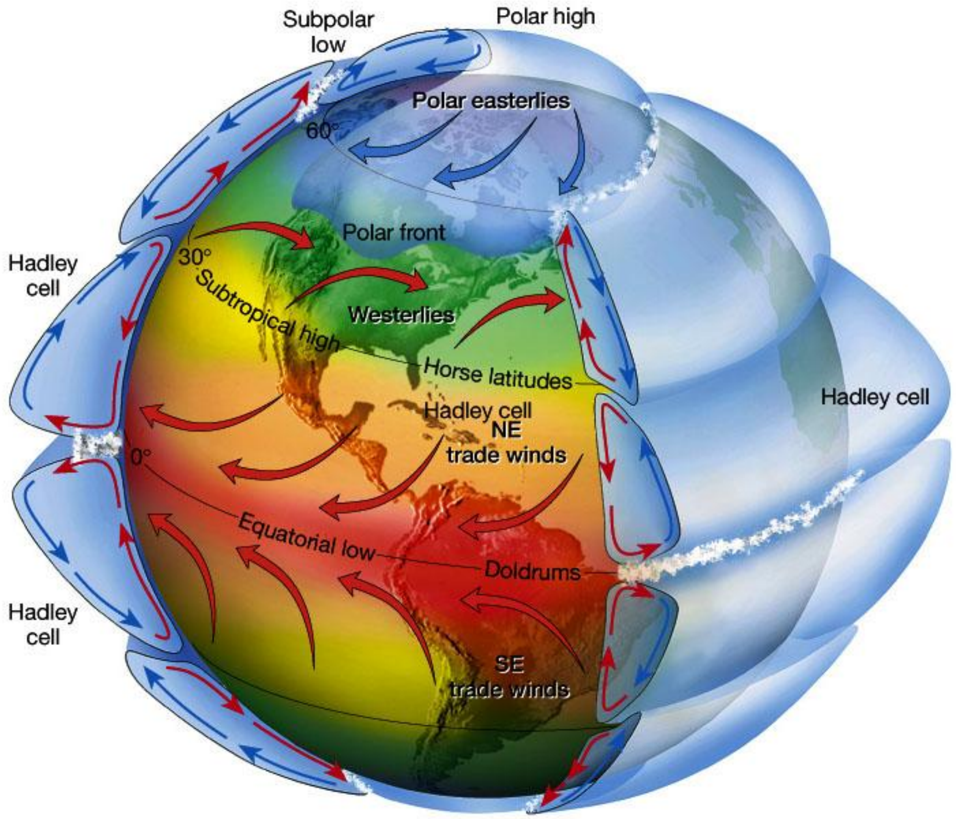
deformation of
Lagrangian parcel
in time...

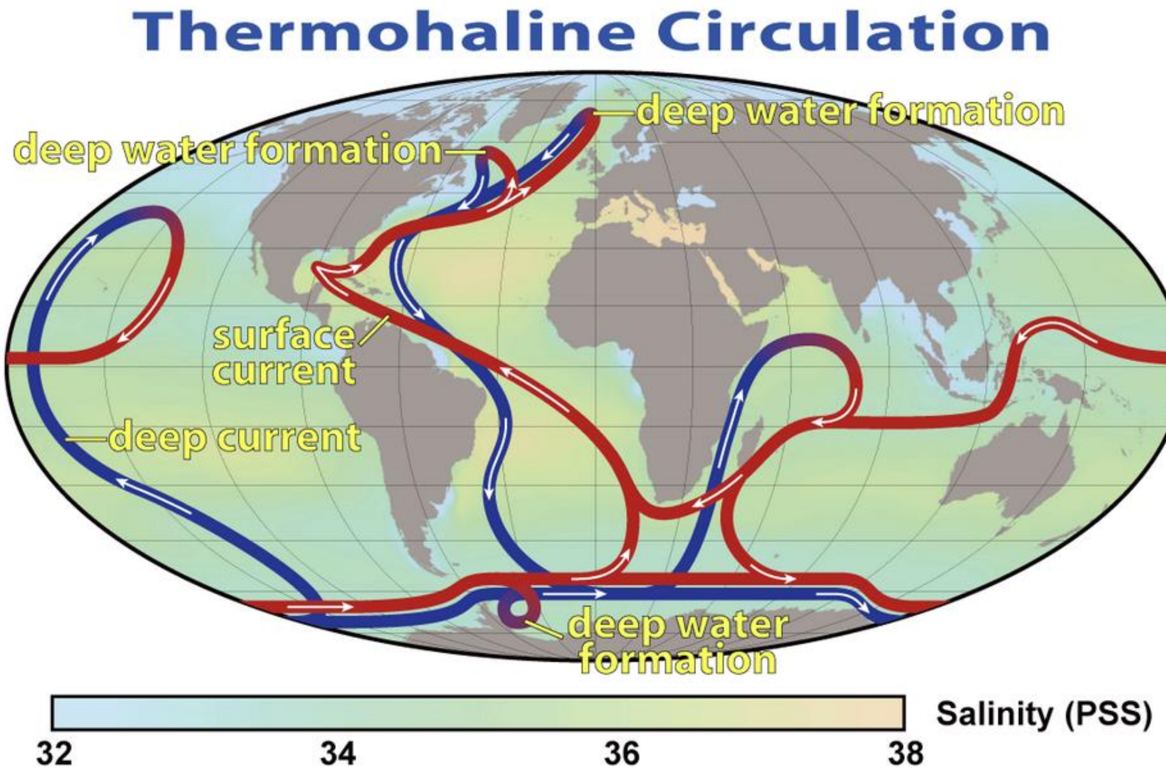




R.A. Pielke, M. Uliasz, Use of meteorological models as input to regional and mesoscale air quality models — limitations and strengths, Atmospheric Environment, Volume 32, Issue 8, 1 April 1998, Pages 1455-1466, ISSN 1352-2310, 10.1016/S1352-2310(97)00140-4.

Global-scale circulations





Both oceanic and atmospheric flows span over a wide range of scales: from global scale down to viscosity scale.

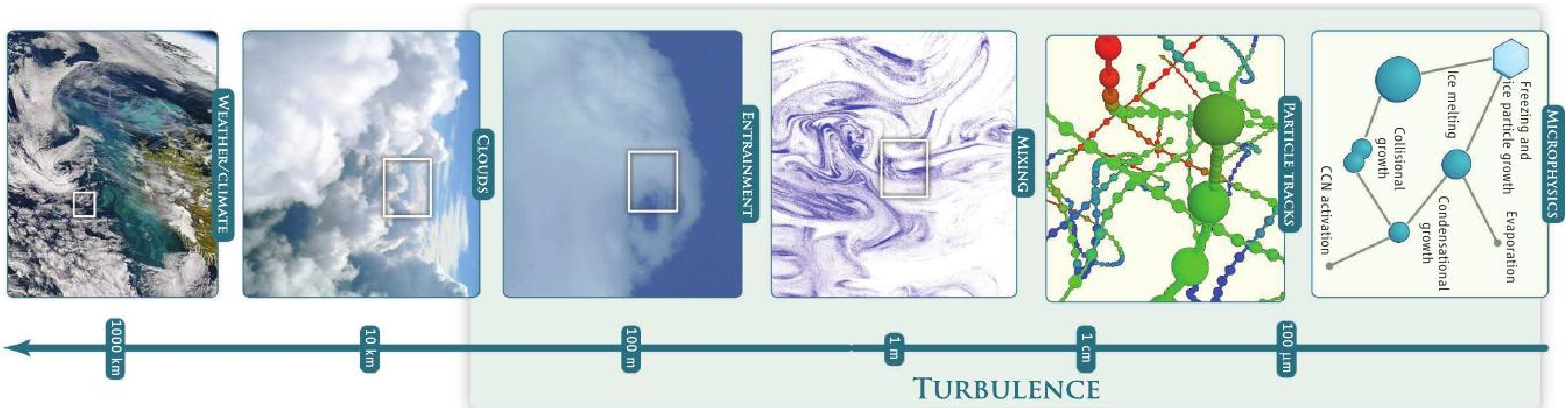
Flow type	Scale (m)	Scale name
	10^{-7}m (0.1 μm) or less	mean free path
viscous flow	10^{-3}m (1mm)	
dissipation-scale eddies	10^{-2}m (1cm)	turbulence
small eddies	10^{-1}m (10cm)	
dust devils, eddies, voritces, surface waves	1-10m	
wind blows, surface waves	10-100m	
tornadoes,	100-1000m	
convective clouds, boundary layer eddies	10^{3}-10^{4}m (1-10km)	mesoscale
mesoscale convective systems, fronts, sea current loops	10^{4}-10^{6}m (10-1000km)	
hurricanes,	10^{5}-10^{6}m (100-1000km)	synoptic scale
low and high pressure systems	10^{6}m (1000km)	
clobal circulation	10^{7}m (10000km)	global scale

ATMOSPHERIC SCIENCE

Can We Understand Clouds Without Turbulence?

Advances at the interface between atmospheric and turbulence research are helping to elucidate fundamental properties of clouds.

E. Bodenschatz,[1,2] S. P. Malinowski,[3] R. A. Shaw,[4] F. Stratmann[5]



Material and Eulerian Derivatives

The material derivative of a scalar (ϕ) and a vector ($\boldsymbol{b}$) field are given by:

$$\frac{\mathrm{D}\phi}{\mathrm{D}t} = \frac{\partial \phi}{\partial t} + \boldsymbol{v} \cdot \nabla \phi, \qquad \frac{\mathrm{D}\boldsymbol{b}}{\mathrm{D}t} = \frac{\partial \boldsymbol{b}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{b}. \tag{D.1}$$

Various material derivatives of integrals are:

$$\frac{\mathrm{D}}{\mathrm{D}t}\int_V \phi \,\mathrm{d}V = \int_V \left(\frac{\mathrm{D}\phi}{\mathrm{D}t} + \phi \nabla \cdot \boldsymbol{v}\right)\mathrm{d}V = \int_V \left(\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \boldsymbol{v})\right)\mathrm{d}V, \tag{D.2}$$

$$\frac{\mathrm{D}}{\mathrm{D}t}\int_V \mathrm{d}V = \int_V \nabla \cdot \boldsymbol{v}\,\mathrm{d}V, \tag{D.3}$$

$$\frac{\mathrm{D}}{\mathrm{D}t}\int_V \rho\phi \,\mathrm{d}V = \int_V \rho \frac{\mathrm{D}\phi}{\mathrm{D}t}\,\mathrm{d}V. \tag{D.4}$$

These formulae also hold if ϕ is a vector. The Eulerian derivative of an integral is:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \phi \,\mathrm{d}V = \int_V \frac{\partial \phi}{\partial t}\,\mathrm{d}V, \tag{D.5}$$

so that

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \mathrm{d}V = 0 \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho\phi \,\mathrm{d}V = \int_V \frac{\partial \rho\phi}{\partial t}\,\mathrm{d}V. \tag{D.6}$$

Governing Equations

Mass continuity in Eulerian approach:

consider the flow of mass in and out of a control volume

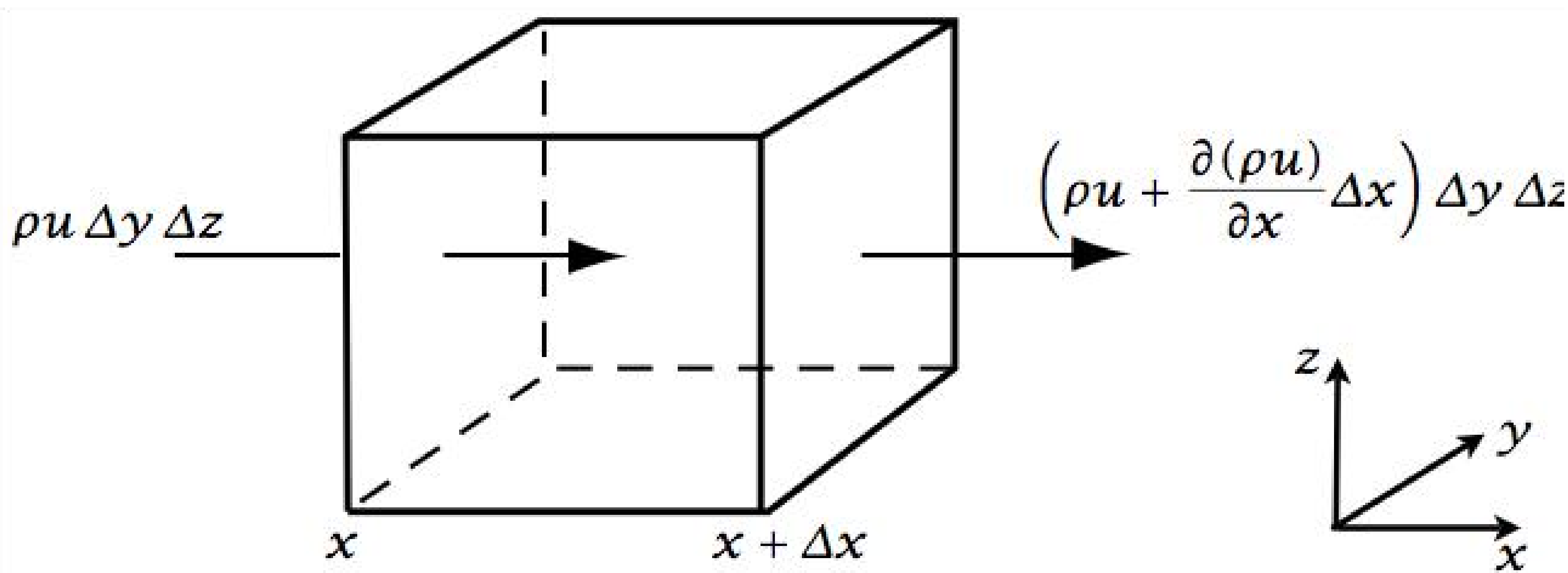


Fig. 1.1 Mass conservation in an Eulerian cuboid control volume.

$$\delta x \delta y \delta z \left[\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} \right] = 0.$$

$$\delta x \delta y \delta z \left[\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} \right] = 0.$$

This [in brackets] equals ZERO, hence....

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0.$$

Continuity equation = conservation of mass, which in classical mechanics is strictly fulfilled.

There are alternative derivations, consult Vallis book!!!

One example is by material derivative:

$$\frac{\mathrm{D}}{\mathrm{D}t}(\rho \Delta V) = 0$$

Mass conserved

Similar conservation laws are valid for any PASSIVE SCALAR carried by the flow:

$$\frac{D\xi}{Dt} = \dot{\xi}.$$

Any scalar with no sources or sinks (r.h.s. =0) is conserved!

E.g. temperature with no heat sources or sinks, dye....

THE ABOVE FORM OF EQUATION IS CRUCIALLY IMPORTANT FOR ALL CONSERVATION LAWS IN FLUID DYNAMICS!

Notice in the l.h.s. the advection operator

$$(\boldsymbol{v} \cdot \nabla)\ \square$$

This stays for anything!!!

Conservation of momentum:

m(x,y,z,t) - the momentum-density field (momentum per unit volume),

$$\mathbf{m} = \rho\, \mathbf{v}$$

The total momentum of a volume of fluid is given by the volume integral.

The rate of change of a momentum is given by the material derivative, and by Newton's second law is equal to the force acting on it:

$$\frac{\mathrm{D}}{\mathrm{D}t}\int_V \rho \boldsymbol{v}\, \mathrm{d}V = \int_V \boldsymbol{F}\, \mathrm{d}V$$

$$\frac{\mathrm{D}}{\mathrm{D}t}\int_V \rho \boldsymbol{v}\, \mathrm{d}V = \int_V \rho \frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t}\, \mathrm{d}V,$$

acceleration of fluid of density ρ

$$\int_V \left(\rho \frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} - \boldsymbol{F}\right) \mathrm{d}V = 0.$$

$$\rho \frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} = \boldsymbol{F}$$

NOTICE THE FORM OF THE ABOVE EQUATION!

NO FORCE = CONSERVATION OF MOMENTUM...

The most common form of **momentum equation**:

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = \frac{\boldsymbol{F}}{\rho}$$

In the r.h.s. F stays for sum of ALL forces!

Which forces ???

1. **Pressure force**:

at the boundary of a fluid the pressure is the normal force per unit area

$$\mathrm{d}\boldsymbol{F}_p = -p\,\mathrm{d}\boldsymbol{S}\,.$$

pressure force — pressure — infinitesimal surface element

$$\boldsymbol{F}_p = -\int_S p\,\mathrm{d}\boldsymbol{S}\,.$$

with application of divergence theorem one gets:

$$\boldsymbol{F}_p = -\int_V \nabla p\,\mathrm{d}V$$

2. **Viscosity force**:

Many textbooks show that, for most Newtonian fluids, the viscous force per unit volume is equal to μΔv, where μ is a coefficient of diffusivity. This is an extremely good approximation for most liquids and gases. With this term, the momentum equation becomes:

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{v}$$

kinematic viscosity, $\nu = \mu/\rho$,
$\nu \sim$(mean free path)*(mean molecular velocity)

	μ ($\mathrm{kg\,m^{-1}\,s^{-1}}$)	ν ($\mathrm{m^2\,s^{-1}}$)
Air	$1.8\ 10^{-5}$	$1.5\ 10^{-5}$
Water	$1.1\ 10^{-3}$	$1.1\ 10^{-6}$
Mercury	$1.6\ 10^{-3}$	$1.2\ 10^{-7}$

3. **Gravity force**.
Let's consider situation with negligible viscosity and important gravity.

Let's focus on a vertical component of the momentum equation (along gravity acceleration).

vertical velocity component — gravity acceleration

$$\frac{\mathrm{D}w}{\mathrm{D}t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

in static situation (no motion) vertical pressure gradient is balanced by gravity acceleration, this is condition for **hydrostatic balance**:

$$\frac{\partial p}{\partial z} = -\rho g,$$

Consider, that in many fluids density and pressure can be related.
In general, equation which allows to calculate density with use of other properties of fluid is called a **CONSTITUTIVE EQUATION** (in a narrow sense), or EQUATION OF STATE

E.g. for air ideal gas equation can be used as a goos approximation of equation of state:

$$p = \rho R T$$

gas constant for air — temperature

For pure water we usually use:

$$\rho = \rho_0[1 - \beta_T(T - T_0)]$$

thermal expansion coefficient

For ocean water relation is more complicated, salinity and pressure should be accounted for.

Energy equation or the first principle of thermodynamics:

$$\mathrm{d}I = T\,\mathrm{d}\eta - p\,\mathrm{d}\alpha + \mu\,\mathrm{d}S$$

internal energy per unit mass (I), entropy (η), specific volume (α), chemical potential (μ), salinity (S)

In fact this is conservation equation for internal energy and can be expressed in the form similar to these discussed earlier.

E.g. assuming that air is the ideal gas we can write:

$$\mathrm{d}Q = \mathrm{d}I + p\,\mathrm{d}\alpha$$

$$\mathrm{d}Q = c_v\,\mathrm{d}T + p\,\mathrm{d}\alpha \qquad\qquad \mathrm{d}Q = c_p\,\mathrm{d}T - \alpha\,\mathrm{d}p$$

$$\alpha = RT/p \text{ and and } c_p - c_v = R$$

Let's consider the material derivative of internal energy:

$$\frac{DI}{Dt} + p\frac{D\alpha}{Dt} = \dot{Q}.$$

$$c_v\frac{DT}{Dt} + p\frac{D\alpha}{Dt} = \dot{Q}, \qquad \text{or} \qquad c_p\frac{DT}{Dt} - \frac{RT}{p}\frac{Dp}{Dt} = \dot{Q}.$$

Finally we get **energy equation** in form:

$$c_v\frac{DT}{Dt} + p\alpha\nabla\cdot\boldsymbol{v} = \dot{Q}$$

Adiabatic processes:

In adiabatic processes there are no sources and sinks of heat:

$$c_p \, \mathrm{d}T = \alpha \, \mathrm{d}p$$

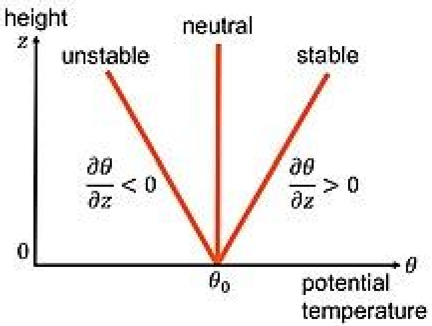
The **potential temperature**, Θ is the temperature that a fluid would have if moved adiabatically to some reference pressure (often 1000 hPa, close to the pressure at the earth's surface). In adiabatic flow the potential temperature of a fluid parcel is conserved, by definition:

$$\frac{\mathrm{D}\theta}{\mathrm{D}t} = 0$$

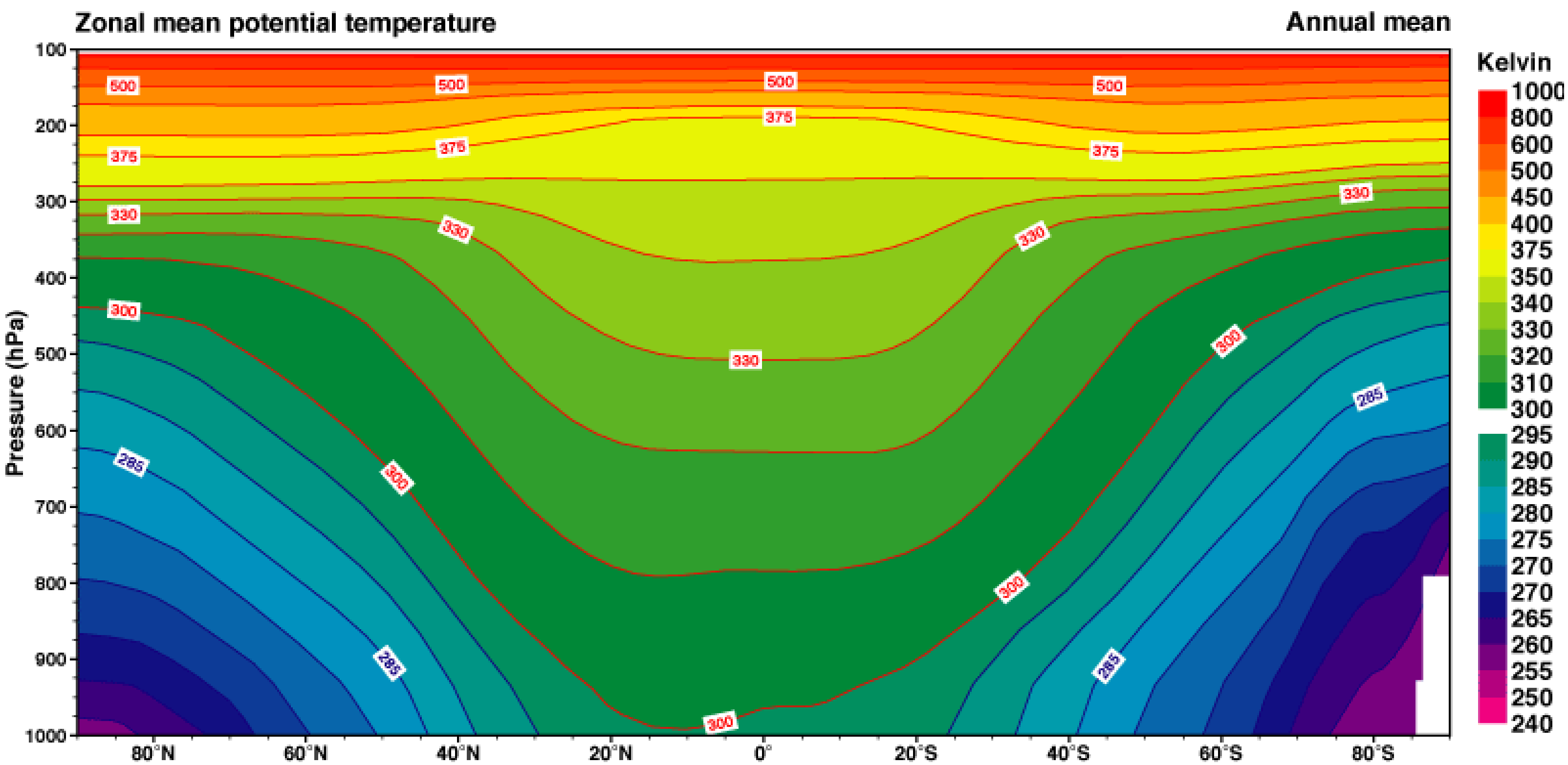
For an ideal gas:

$$\mathrm{d}\eta = c_p \, \mathrm{d} \ln T - R \, \mathrm{d} \ln p$$

$$c_p \, \mathrm{d} \ln \theta = c_p \, \mathrm{d} \ln T - R \, \mathrm{d} \ln p$$



Atmospheric stability
and meridional potential temperature distribution



Consequently: reference pressure $\kappa = R/c_p$

$$\theta = T\left(\frac{p_R}{p}\right)^{\kappa}$$

Finally we may write **a compact form of the energy equation**:

$$c_p \frac{D\theta}{Dt} = \frac{\theta}{T}\dot{Q}$$

Forms of the Thermodynamic Equation

General form

For a parcel of constant composition the thermodynamic equation is

$$T\frac{\mathrm{D}\eta}{\mathrm{D}t} = \dot{Q} \qquad \text{or} \qquad c_p\frac{\mathrm{D}\ln\theta}{\mathrm{D}t} = \frac{1}{T}\dot{Q} \tag{T.1}$$

where η is the entropy, θ is the potential temperature, $c_p \ln\theta = \eta$ and $\dot{Q}$ is the heating rate. Appplying the first law of thermodynamics $T\,\mathrm{d}\eta = \mathrm{d}I + p\,\mathrm{d}\alpha$ gives:

$$\frac{\mathrm{D}I}{\mathrm{D}t} + p\frac{\mathrm{D}\alpha}{\mathrm{D}t} = \dot{Q} \qquad \text{or} \qquad \frac{\mathrm{D}I}{\mathrm{D}t} + RT\nabla\cdot\boldsymbol{v} = \dot{Q} \tag{T.2}$$

where I is the internal energy.

Ideal gas

For an ideal gas $\mathrm{d}I = c_v\,\mathrm{d}T$, and the (adiabatic) thermodynamic equation may be written in the following equivalent, exact, forms:

$$
\begin{aligned}
c_p \frac{\mathrm{D}T}{\mathrm{D}t} - \alpha \frac{\mathrm{D}p}{\mathrm{D}t} &= 0, & \frac{\mathrm{D}p}{\mathrm{D}t} + \gamma p \nabla \cdot \boldsymbol{v} &= 0, \\
c_v \frac{\mathrm{D}T}{\mathrm{D}t} + p\alpha \nabla \cdot \boldsymbol{v} &= 0, & \frac{\mathrm{D}\theta}{\mathrm{D}t} &= 0,
\end{aligned}
\tag{T.3}
$$

where $\theta = T(p_{\mathrm{R}}/p)^{\kappa}$. The two expressions on the second line are usually the most useful in modelling and theoretical work.

Liquids

For liquids we may usefully write the (adiabatic) thermodynamic equation as a conservation equation for potential temperature θ or potential density ρ_{pot} and represent these in terms of other variables. For example:

$$
\frac{\mathrm{D}\theta}{\mathrm{D}t} = 0, \qquad \theta \approx \begin{cases} T & \text{(approximately)} \\ T + (\beta_T g z / c_p) & \text{(with some thermal expansion),} \end{cases} \tag{T.4a}
$$

$$
\frac{\mathrm{D}\rho_{\mathrm{pot}}}{\mathrm{D}t} = 0, \quad \rho_{\mathrm{pot}} \approx \begin{cases} \rho & \text{(very approximately)} \\ \rho + (\rho_0 g z / c_s^2) & \text{(with some compression).} \end{cases} \tag{T.4b}
$$

Unlike (T.3) these are not equivalent forms. More accurate semi-empirical expressions that may also include saline effects are often used for quantitative applications.

The Equations of Motion of a Fluid

For dry air, or for a salt-free liquid, the complete set of equations of motion may be written as follows:

The *mass continuity equation:*

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0. \qquad \text{(EOM.1)}$$

If density is constant this reduces to $\nabla \cdot \boldsymbol{v} = 0$.

The *momentum equation:*

$$\frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \boldsymbol{v} + \boldsymbol{F}, \qquad \text{(EOM.2)}$$

The *thermodynamic equation:*

$$\frac{\mathrm{D}\theta}{\mathrm{D}t} = \frac{1}{c_p}\left(\frac{\theta}{T}\right)\dot{Q}. \qquad \text{(EOM.3)}$$

where $\dot{Q}$ represents external heating and diffusion, the latter being $\kappa \nabla^2 \theta$ where κ is the diffusivity.

The *equation of state:*

$$\rho = g(\theta, p) \qquad \text{(EOM.4)}$$

where g is a given function. For example, for an ideal gas, $\rho = p_{\mathrm{R}}^{\kappa}/(R\theta p^{\kappa-1})$.

The real-scienific-life example:

Numerical Simulation of Cloud–Clear Air Interfacial Mixing

MIROSŁAW ANDREJCZUK

Institute of Geophysics, Warsaw University, Warsaw, Poland

WOJCIECH W. GRABOWSKI

National Center for Atmospheric Research, Boulder, Colorado*

SZYMON P. MALINOWSKI

Institute of Geophysics, Warsaw University, Warsaw, Poland

PIOTR K. SMOLARKIEWICZ

National Center for Atmospheric Research, Boulder, Colorado*

(Manuscript received 25 October 2002, in final form 2 February 2004)

$$D/Dt \equiv \partial/\partial t + \mathbf{v} \cdot \boldsymbol{\nabla}$$

$$B \equiv g\left[\frac{T - T_0}{T_0} + \varepsilon(q_v - q_{v_0}) - q_c\right], \quad (2)$$

$$\frac{D\mathbf{v}}{Dt} = -\boldsymbol{\nabla}\boldsymbol{\pi} + \mathbf{k}B + \nu\nabla^2\mathbf{v}, \quad (1a)$$

$$\boldsymbol{\nabla} \cdot \mathbf{v} = 0, \quad (1b)$$

$$\frac{DT}{Dt} = \frac{L}{c_p}C_d + \mu_T\nabla^2 T, \quad (1c)$$

$$\frac{Dq_v}{Dt} = -C_d + \mu_v\nabla^2 q_v, \quad (1d)$$

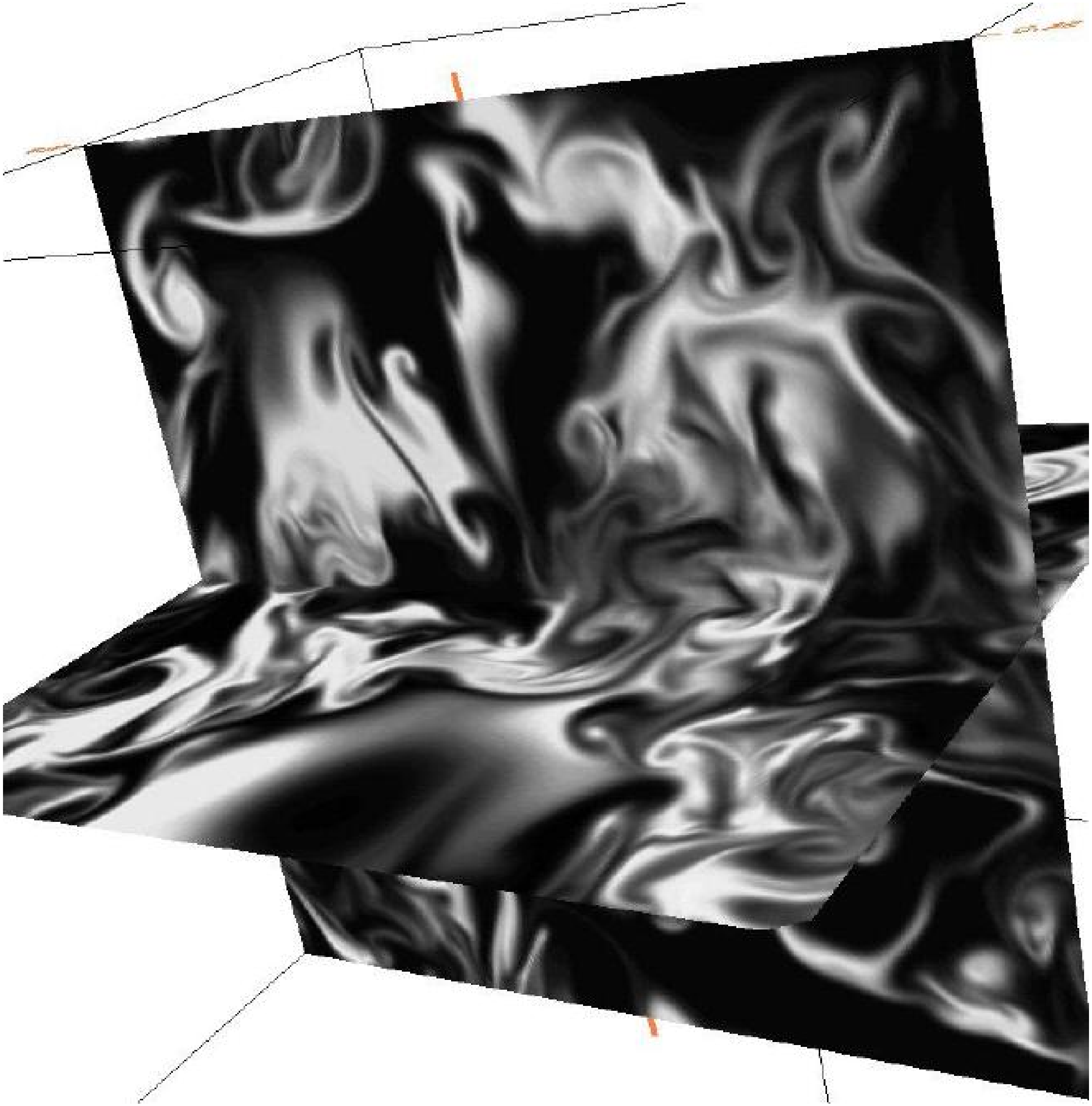
$$\frac{Dq_c}{Dt} = C_d.$$

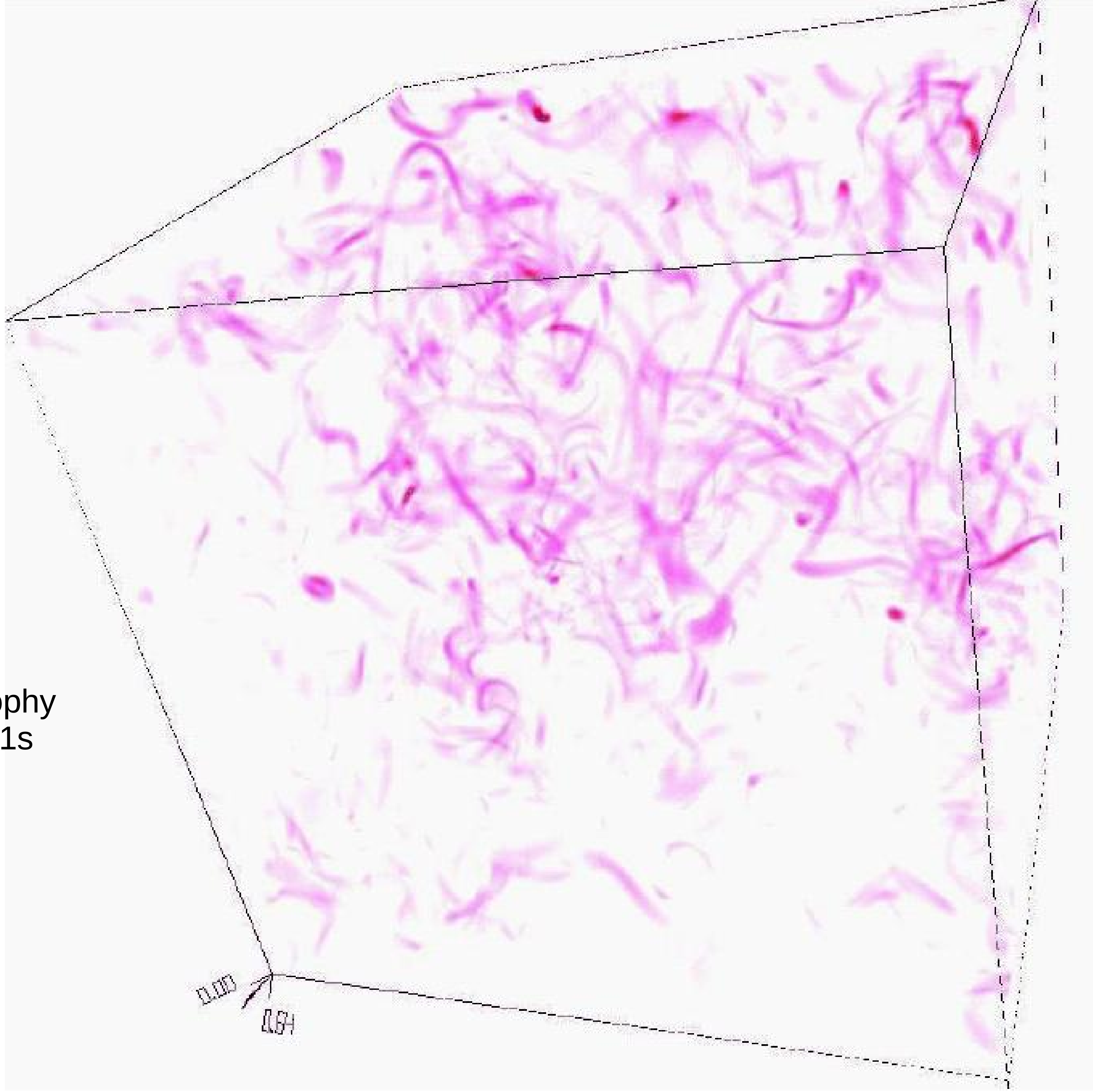
$$C_d = \int f\frac{dm}{dt}\,dr, \quad \frac{D^* f}{D^* t} = -\frac{\partial}{\partial r}\left(f\frac{dr}{dt}\right) + \eta,$$

$$D^*/D^*t \equiv \partial/\partial t + (\mathbf{v} - \mathbf{k}v_t) \cdot \boldsymbol{\nabla}$$

Andrejczuk et al., 2004

Cloud water
after 11s





Enstrophy
after 11s

Similar equations,
another application