Exercise Sheet 5

Questions, comments and corrections: e-mail to Marta.Waclawczyk@fuw.edu.pl

1. Consider the irrotational fluid motion outside a spherical bubble of (a perfect) gas in an incompressible liquid. Neglect the effect of surface tension and mass diffusion. Find an equation for the radius $R(t)$ of the bubble. Assume that the pressure far from the bubble is p_0.

2. Consider the oscillating gas bubble from the task above.

   (a) The pressure at the gas-liquid interface is given by

   $$p(R) = KR^{-3\gamma}, \tag{1}$$

   where $\gamma = C_p/C_v$. Determine the constant K in terms of the equilibrium radius a, the undisturbed pressure p_0.

   (b) Starting from the differential equation obtained in class:

   $$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{K}{\rho}R^{-3\gamma} - \frac{p_o}{\rho}, \tag{2}$$

   assume $R(t) = a + x(t)$, where $x(t)$ is a small deviation from the equilibrium radius. Neglect second order terms and show that the system simplifies to a harmonic oscillator with frequency

   $$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\left(\frac{3\gamma p_0}{\rho a^2}\right)}. \tag{3}$$

   Hint: For the term $(a + x)^{-3\gamma}$, use a Taylor expansion around $x_0 = 0$:

   $$f(x) = f(x_0) + f'(x_0)(x - x_0) + \mathcal{O}(\Delta x^2) \tag{4}$$

   (c) Assume the bubble is air surrounded by water. Take p_0 to be the atmospheric pressure and $a = 10^{-3}$ m. Considering the audible spectrum for humans ranges from 20 Hz to 20 kHz, would you hear the bubble?