Planetary boundary layer and atmospheric turbulence.

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Lecture 05

TKE changes only by viscous dissipation. Of course this in unsustainable - a source of kinetic energy is needed. TKE sources (shear production, buoyant production) are NOT isotropic and homogeneous. We sidestep this contradiction by assuming that for large Reynolds numbers, although isotropy and homogeneity are violated by the mechanism producing the turbulence, they still hold at small scales and away from boundaries. Then the turbulence production can be represented simply by a forcing term F, assumed to be isotropic and homogeneous:

$$\frac{d}{dt}E = -\epsilon + F \tag{4.4}$$

TKE production

TKE changes

TKE dissipation

In stationary turbulence production is balanced by dissipation

Kołmogorov (1941) therory of uniform isotropic and stationary turbulence
- after Frisch(1990):

The Navier–Stokes equations for incompressible fluid flow possess a number of symmetries (invariance groups). When boundaries are ignored, the symmetries include: space and time translations, rotations, parity (space and velocity reversal) and galilean transformations. If the viscosity $\nu = 0$, an infinite class of additional symmetries appears, the scaling transformations:

$$\boldsymbol{r} \rightarrow \lambda \boldsymbol{r}, \quad \boldsymbol{v} \rightarrow \lambda^h \boldsymbol{v}, \quad t \rightarrow \lambda^{1-h} t, \quad \lambda \in \boldsymbol{R}_+. \tag{1}$$

Here, t, $\boldsymbol{r}$ and $\boldsymbol{v}$ are, respectively, the time, position and velocity variables. It is assumed that pressure has been eliminated from the Navier–Stokes equation through use of the incompressibility constraint. The different scaling groups are labelled by the scaling exponent $h \in \boldsymbol{R}$. (..............)

I shall present the reformulation in the form of numbered hypotheses.

H1. *In the limit of infinite Reynolds numbers, all the possible symmetries of the Navier–Stokes equation, usually broken by the mechanisms producing the turbulent flow, are restored in a statistical sense at small scales and away from boundaries.*

The words 'small scales' can be technically defined by considering velocity increments over a distance l small compared to the integral scale l_0:

velocity differences on a distance l

$$\delta \boldsymbol{v}(\boldsymbol{r}, \boldsymbol{l}) = \boldsymbol{v}(\boldsymbol{r}+\boldsymbol{l}) - \boldsymbol{v}(\boldsymbol{r}). \tag{2}$$

We may then define, for example, statistical invariance under space-translations (homogeneity) by:

$$\delta \boldsymbol{v}(\boldsymbol{r}+\boldsymbol{q}, \boldsymbol{l}) \stackrel{L}{=} \delta \boldsymbol{v}(\boldsymbol{r}, \boldsymbol{l}), \quad q \ll l_0, \tag{3}$$

where $\stackrel{L}{=}$ means 'equality in law' (identical statistical properties).

Since there is an infinity of different possible scaling exponents h, additional assumptions are needed.

H2. *Under the same assumptions as in H1, the turbulent flow is assumed to be self-similar at small scales, i.e. to possess a single scaling exponent* h.

The value of h is obtained from

H3. *Under the same assumptions as in H1, the turbulent flow is assumed to have a finite non-vanishing mean rate of dissipation* ϵ *per unit mass.*†

From H2 and H3, the value of the scaling exponent can be readily obtained. Indeed, Kolmogorov (1941*c*) has derived the following relation from the Navier–Stokes equation, under the sole assumptions of homogeneity, isotropy and finite mean energy dissipation:

3rd order structure function

$$S_3(l) \equiv \langle (\delta v_\parallel(\boldsymbol{r}, l))^3 \rangle = -\tfrac{4}{5}\epsilon l. \tag{4}$$

Here, $\delta v_\parallel$ denotes the component of the velocity increment parallel to the displacement vector l. The function S_3 is called the third order (longitudinal) structure function. The increment l is assumed by Kolmogorov to be small compared to the integral scale l_0. With the assumption H2, under rescaling of the increment l by a factor λ, the left-hand side of (4) changes by a factor λ^{3h} while the right-hand side changes by a factor λ. Hence,

$$h = \tfrac{1}{3}. \tag{5}$$

universal exponent

Under the assumption that moments of arbitrary integer order p of the velocity increment exist (there is considerable experimental evidence for this assumption), the self-similarity hypothesis implies scaling laws for structure functions of arbitrary order:

$$S_p(l) \equiv \langle (\delta v_{\parallel}(\boldsymbol{r}, l))^p \rangle = C_p\, \epsilon^{\frac{1}{3}p}\, l^{\frac{1}{3}p}. \tag{6}$$

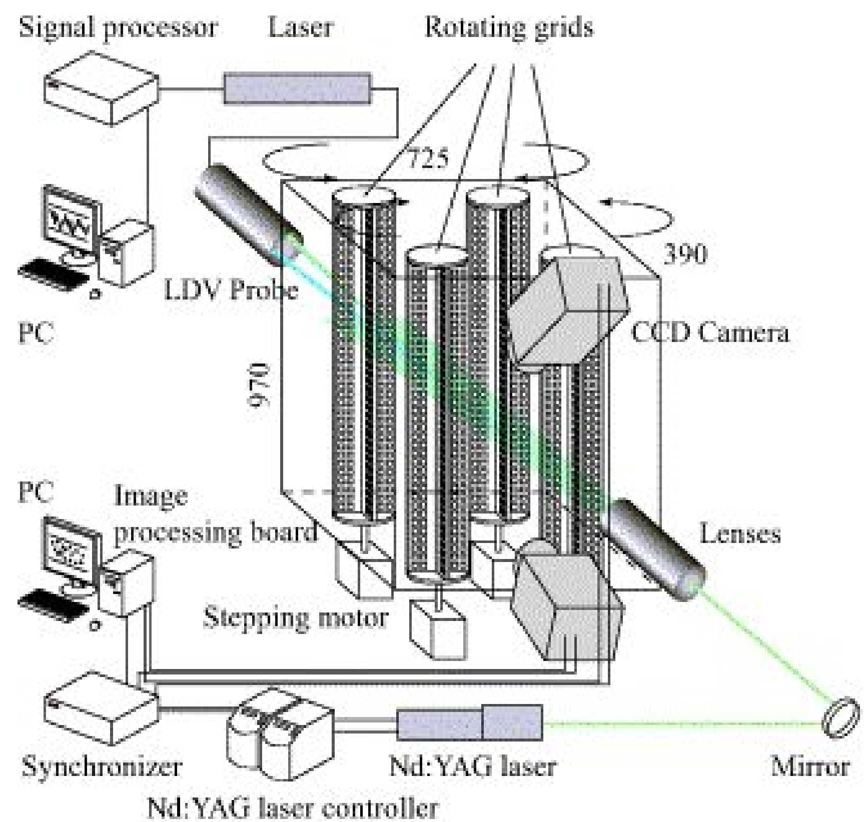
The presence of the factors $\epsilon^{\frac{1}{3}p}$ in the right-hand side ensures that the C_ps are dimensionless. The C_ps cannot depend on the Reynolds number, since the limit of infinite Reynolds number is assumed. For $p = 3$, it follows from (4) that $C_3 = -\frac{4}{5}$, which is clearly universal. All the C_ps, except for $p = 3$, must, however, depend on the detailed geometry of the production of turbulence. In other words, they cannot be universal.

Notice that for p=2 this is the 2nd order structure function, dimension velocity squared e.g. equivalent to turbulent kinetic energy per unit mass!!!,

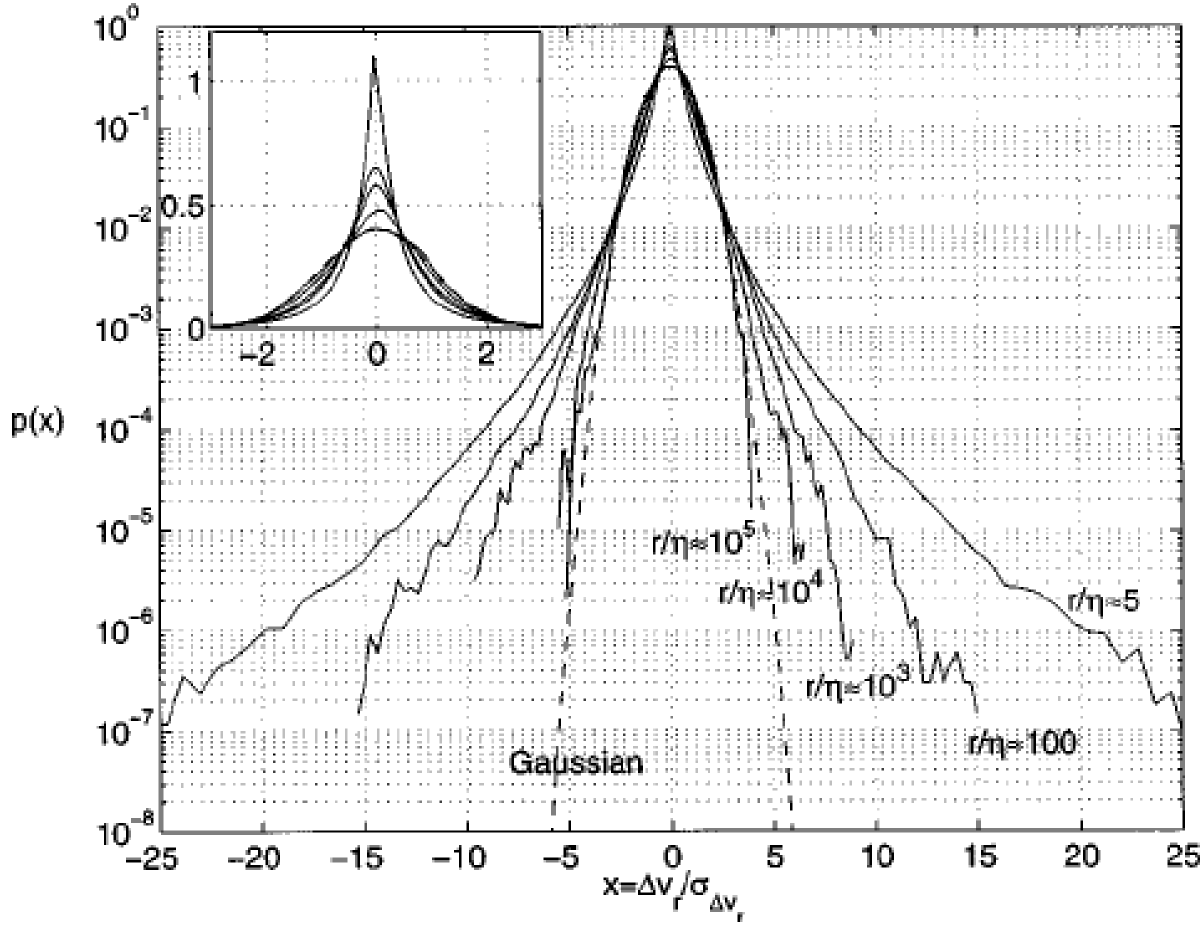
Thus, 2nd order structure fuction can be interpreted in terms of energy, and the whole eq. (6) as relation (dependency) of TKE on scale *l*, on condition that the scale is substantially smaller than the integral scale l_0.

In the other words: TKE in scale l is proportional to this scale in power 2/3.

Goettingen turbulence facility:
GTF3 generates high Reynolds number turbulent water flows between two counter-rotating baffled disks. Large glass windows provide access for LPT or PIV measurements.



Oshima Lab, Tokyo:
Generation of nearly isotropic homogeneous turbulence using rotating grids



Remark:

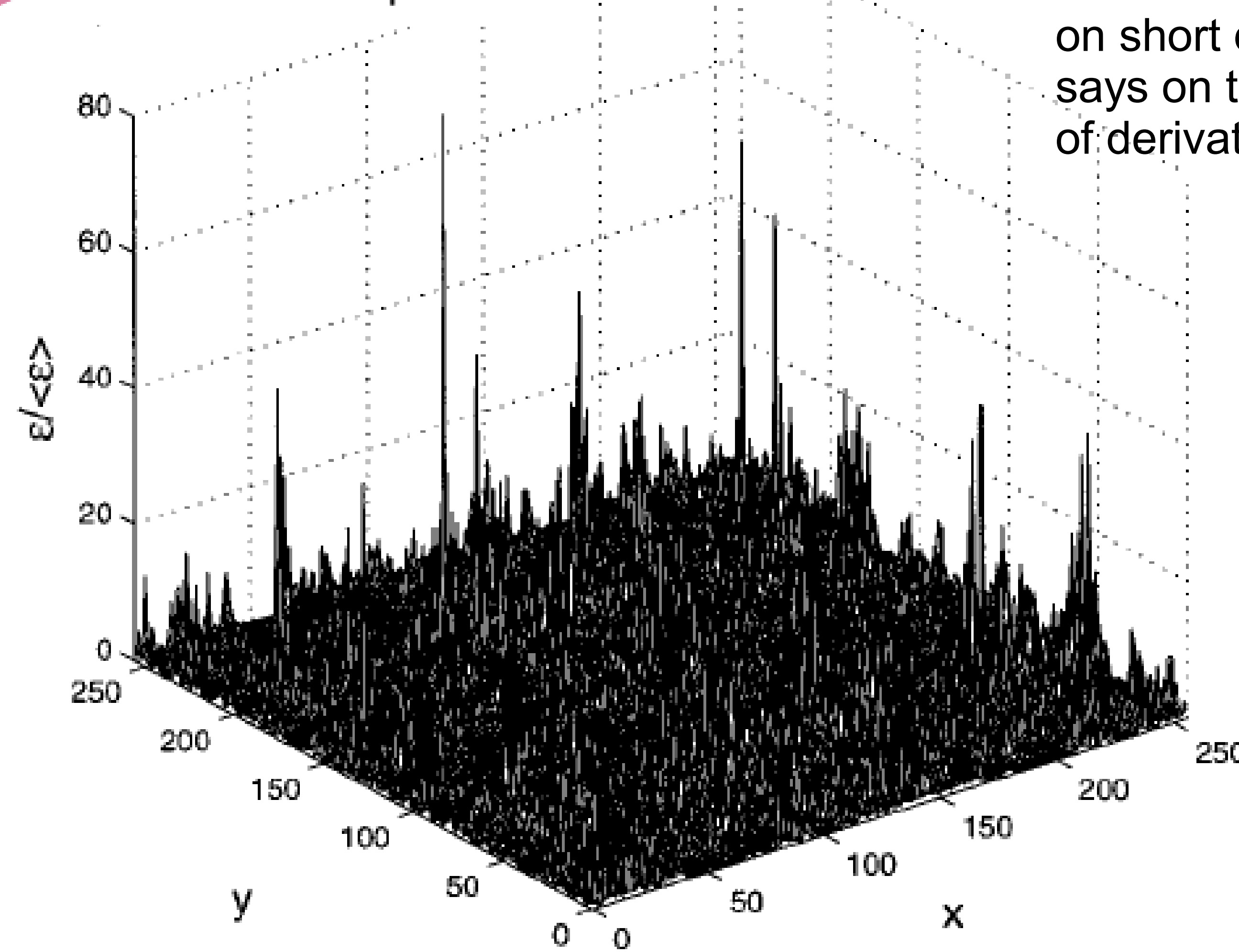
Differences of velocity fluctuations on short distances are closely related to velocity derivatives!

The probability density functions, of differences of velocity fluctuations, obtained in atmospheric turbulence about 30 m above the ground. The ordinate is logarithmic in the main figure and linear in the inset. Each curve is for a different separation distance (using Taylor's hypothesis). The smallest separation distance (about 2.5 mm) is only five times the Kolmogorov scale, while the largest (about 50 m) is comparable to the height of the measurement point. For small separation distances, very large excursions (even as large as 25 standard deviations) occur with nontrivial frequency; they are far more frequent than is given by a Gaussian distribution (shown by the full line), which is approached only for large separation distances. Extended tails over a wide range of scales is related to the phenomenon of small-scale intermittency (that is, uneven distribution in space of the small scales). (Sreeinivasan, 1999)

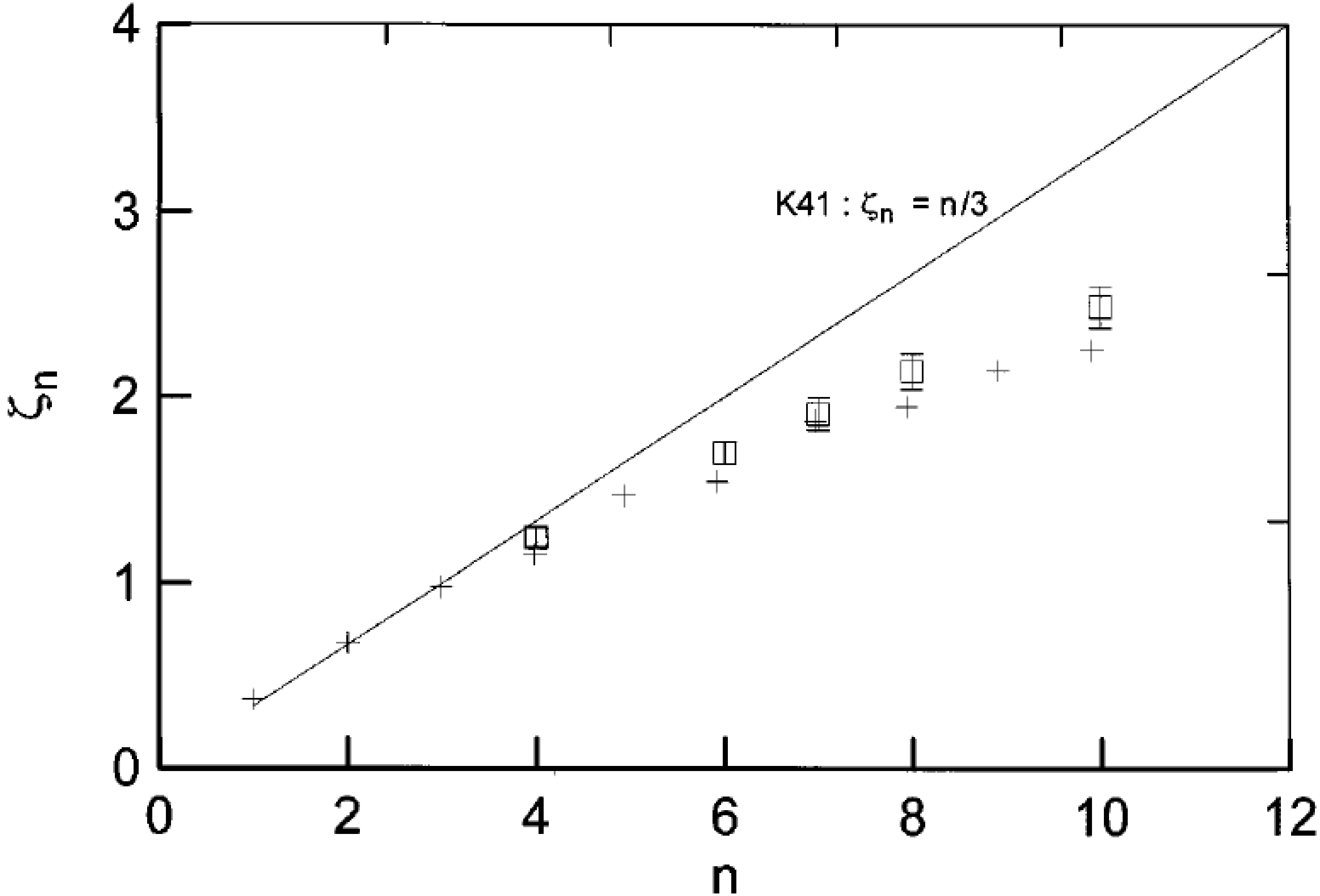
$$\epsilon = \nu \overline{\left(\frac{\partial u'_i}{\partial x_j}\right)^2}$$

ε - dissipation

Remark: 2nd order structure function on short distances says on the square of derivative!



Planar cuts of the three-dimensional fields of energy dissipation in a box of homogeneous and isotropic turbulence. The data are obtained by solving the Navier-Stokes equations on a computer. Not uncommon are amplitudes much larger than the mean; these large events become stronger with increasing Reynolds number.



The scaling exponents for the velocity increments with the separation distance in the inertial range. The unfilled squares are determined for shearless turbulence by the ESS method, using $|\Delta u_r|^3$ as the reference structure function The crosses are for a boundary layer. The full line is K 41.

Alternative approach: Fourier decomposittion (no Frisch anymore).

For a flow which is homogeneous in space (i.e. statistical properties are independent of position), a spectral description is very appropriate, allowing us to examine properties as a function of wavelength. The total kinetic energy, given by

$$E = 1/2 \int u_i(\mathbf{x}) u_i(\mathbf{x}) d\mathbf{x} \tag{4.5}$$

can be written in terms of the spectrum $\phi_{i,j}(\mathbf{k})$

$$E = \frac{1}{2} \int \phi_{i,i}(\mathbf{k}) d\mathbf{k} = \int E(\mathbf{k}) d\mathbf{k} \tag{4.6}$$

where $\phi_{i,j}(\mathbf{k})$ is the Fourier transform of the velocity correlation tensor $R_{i,j}(\mathbf{r})$:

$$\phi_{i,j}(\mathbf{k}) = \frac{1}{(2\pi)^3} \int exp(-i\mathbf{k}.\mathbf{r}) R_{i,j}(\mathbf{r}) d\mathbf{r} \ ; \ R_{i,j}(\mathbf{r}) = \int u_j(\mathbf{x}) u_i(\mathbf{x}+\mathbf{r}) d\mathbf{x} \tag{4.7}$$

$R_{i,j}(\mathbf{r})$ tells us how velocities at points separated by a vector $\mathbf{r}$ are related. If we know these two point velocity correlations, we can deduce $E(\mathbf{k})$. Hence the energy spectrum has the information content of the two-point correlation.

Notice that in 4.7 there are velocities in points x and x+r, which is similar to the 2nd order structure function. In this equation, al well as in 4.5 there is velocity in second power!!!!i.

$E(\mathbf{k})$ contains directional information. More usually, we want to know the energy at a particular scale $k = \sqrt{\mathbf{k}.\mathbf{k}}$ without any interest in separating it by direction. To find $E(k)$, we integrate over the spherical shell of radius k (in 3-dimensions):

$$E = \int E(\mathbf{k}) d\mathbf{k} = \int_0^\infty \oint E(\mathbf{k}) d\sigma dk = \int_0^\infty E(k) dk \tag{4.8}$$

Then

$$E(k) = \oint E(\mathbf{k}) d\sigma = \frac{1}{2} \oint \phi_{i,i}(\mathbf{k}) d\sigma \tag{4.9}$$

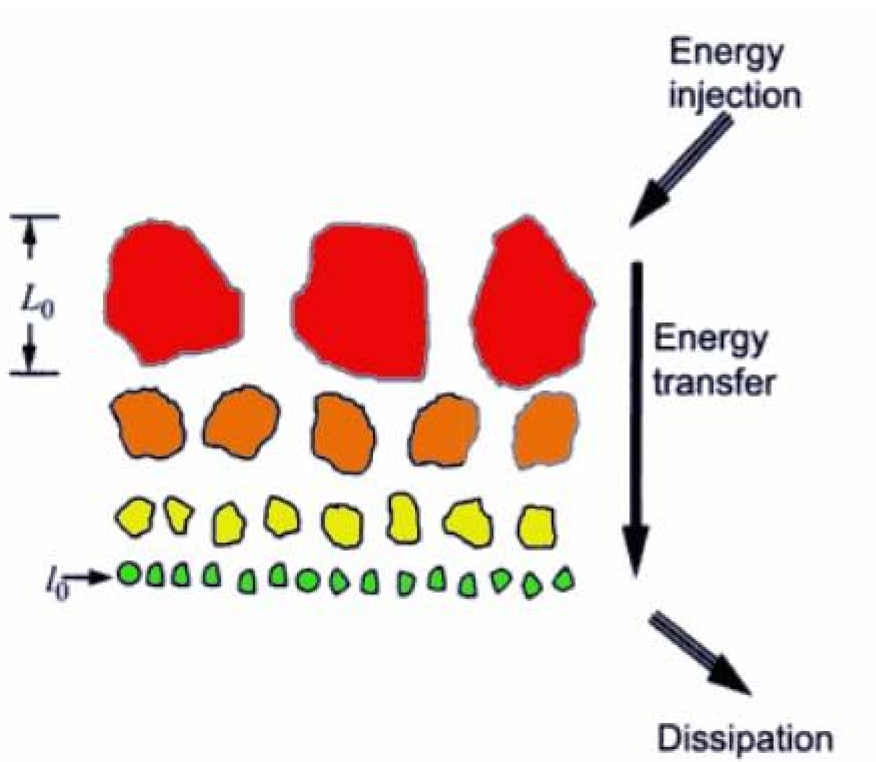
Assuming isotropy:

$$E(k) = 2\pi k^2 \phi_{i,i}(k) \tag{4.10}$$

where $\phi_{i,i}(\mathbf{k}) = \phi_{i,i}(k)$ for all $\mathbf{k}$ such that $\sqrt{\mathbf{k}.\mathbf{k}} = k$.

Balance of energy in phase space.

We have an equation for the evolution of the total kinetic energy E. Equally interesting is the evolution of $E(k)$, the energy at a particular wavenumber k. This will include terms which describe the transfer of energy from one scale to another, via nonlinear interactions.



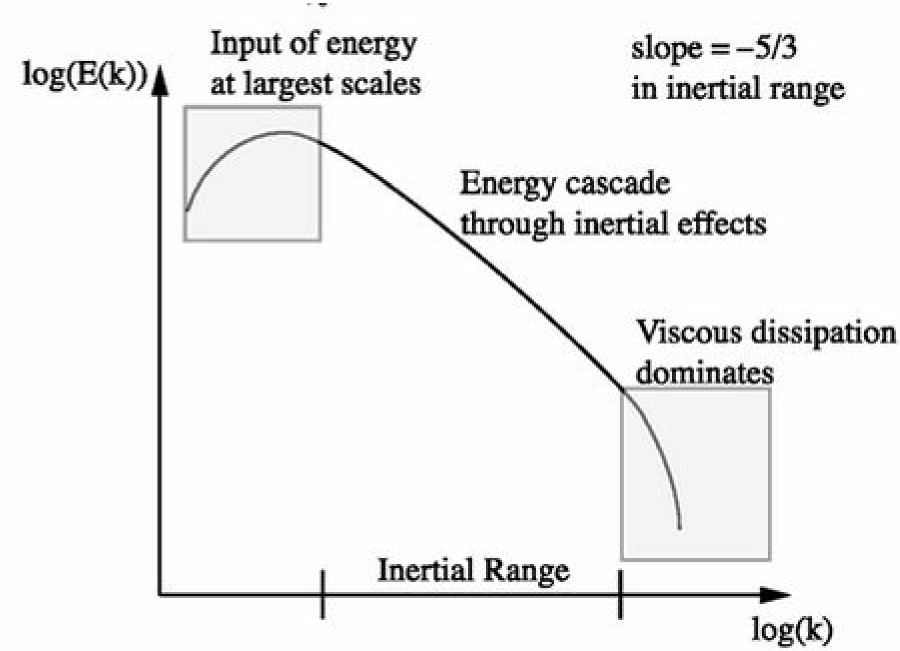
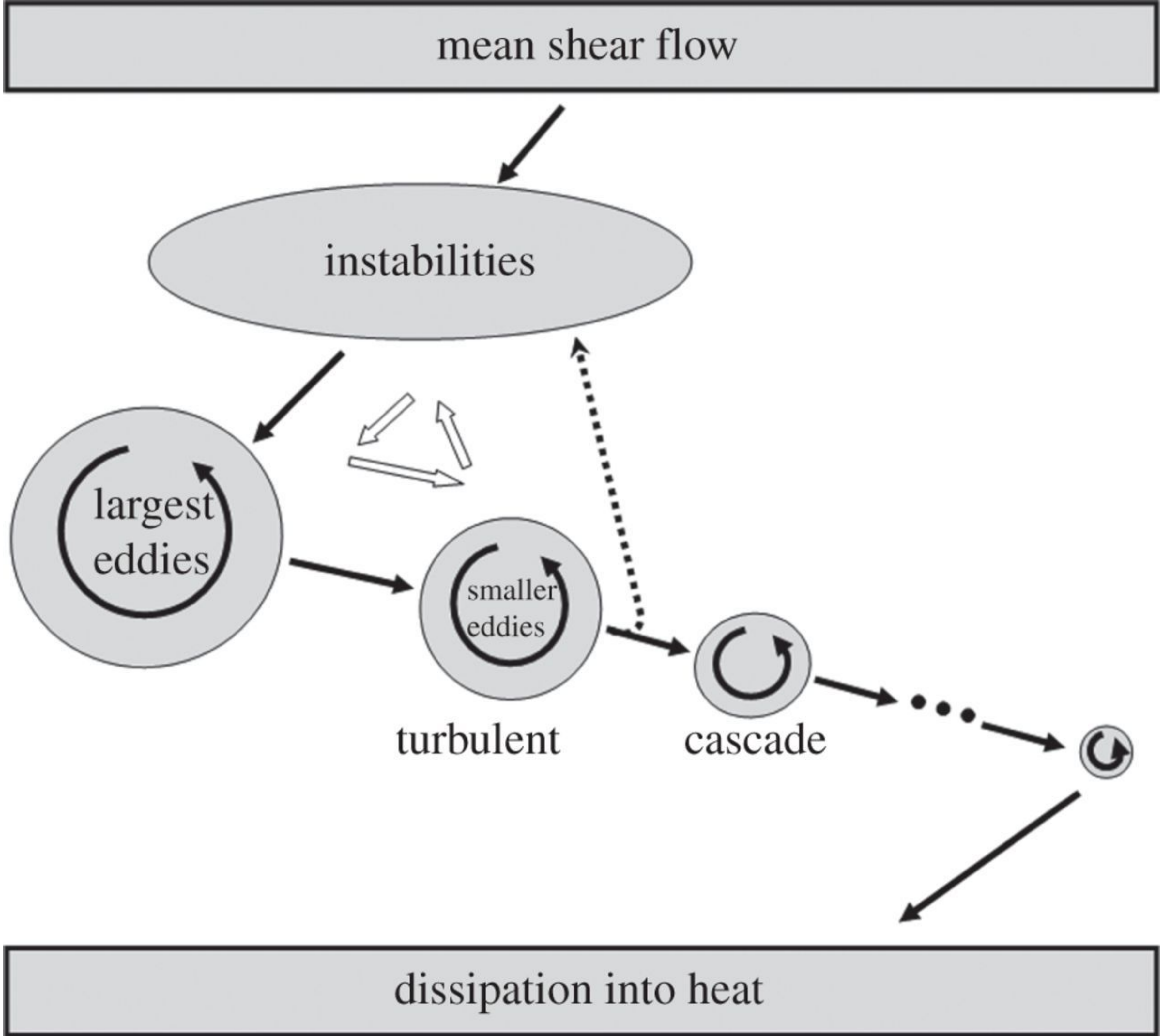


Fig. 2.1. A depiction of the observed energy cascade


mean shear flow
instabilities
largest eddies
smaller eddies
turbulent
cascade
dissipation into heat

Kolmogorov's 1941 theory for the energy spectrum makes use of the result that ϵ, the energy injection rate, and dissipation rate also controls the flux of energy. Energy flux is independent of wavenumber k, and equal to ϵ for $k > k_i$. Kolmogorov's theory assumes the injection wavenumber is much less than the dissipation wavenumber ($k_i << k_d$, or large Re). In the intermediate range of scales $k_i < k < k_d$ neither the forcing nor the viscosity are explicitly important, but instead the energy flux ϵ and the local wavenumber k are the only controlling parameters. Then we can express the energy density as

$$E(k) = f(\epsilon, k) \tag{4.25}$$

Now using dimensional analysis:

Quantity	Dimension	
Wavenumber k	$1/L$	
Energy per unit mass E	$U^2 \sim L^2/T^2$	we find
Energy spectrum $E(k)$	$EL \sim L^3/T^2$	
Energy flux ϵ	$E/T \sim L^2/T^3$	

$$E(k) = C_K \epsilon^{2/3} k^{-5/3} \tag{4.26}$$

C_K is a universal constant known as the Kolmogorov constant. The region of parameter space in k where the energy spectrum follows this $k^{-5/3}$ form is known as the **Inertial range**. In this range, energy **cascades** from the larger scales where it was injected ultimately to the dissipation scale. The theory assumes that the spectra at any particular k depends only on spectrally local quantities - i.e. has no dependence on k_i for example. Hence the possibility for long-range interactions is ignored.

We can also derive the Kolmogorov spectrum in the following manner (after Obukhov): Define an eddy turnover time $\tau(k)$ at wavenumber k as the time taken for a parcel with energy $E(k)$ to move a distance $1/k$. If $\tau(k)$ depends only on $E(k)$ and k then, from dimensional analysis

$$\tau(k) \sim \left[k^3 E(k)\right]^{-1/2} \tag{4.27}$$

The energy flux can be defined as the available energy divided by the characteristic time τ. The available energy at a wavenumber k is of the order of $kE(k)$. Then we have

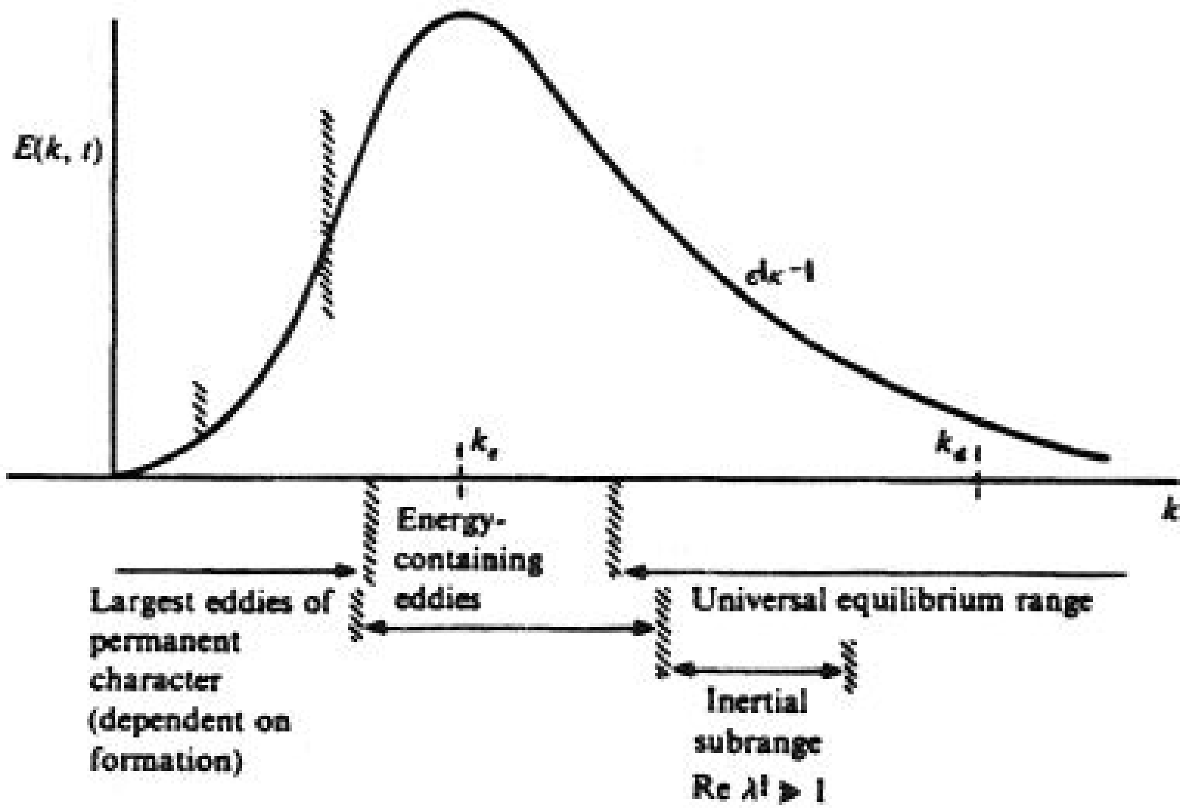
$$\epsilon \sim \frac{kE(k)}{\tau(k)} \sim k^{5/2} E(k)^{3/2} \tag{4.28}$$

and hence

$$E(k) \sim \epsilon^{2/3} k^{-5/3} \tag{4.29}$$

Scales and the Energy Spectrum

The largest length scales in a turbulent flow are set by the dimensions of the flow field or the size of the body generating the flow disturbance. If the characteristic dimension and velocity are L and U respectively, a mean flow advection time scale is L/U. The characteristic time for viscous diffusion across a length L is L^2/v and the ratio of these times is the Reynolds number, $Re_L = UL/v$. The smallest scales, η and η^2/v, are set by the dissipation rate of turbulent energy.



Distribution of turbulent energy in wave number space.

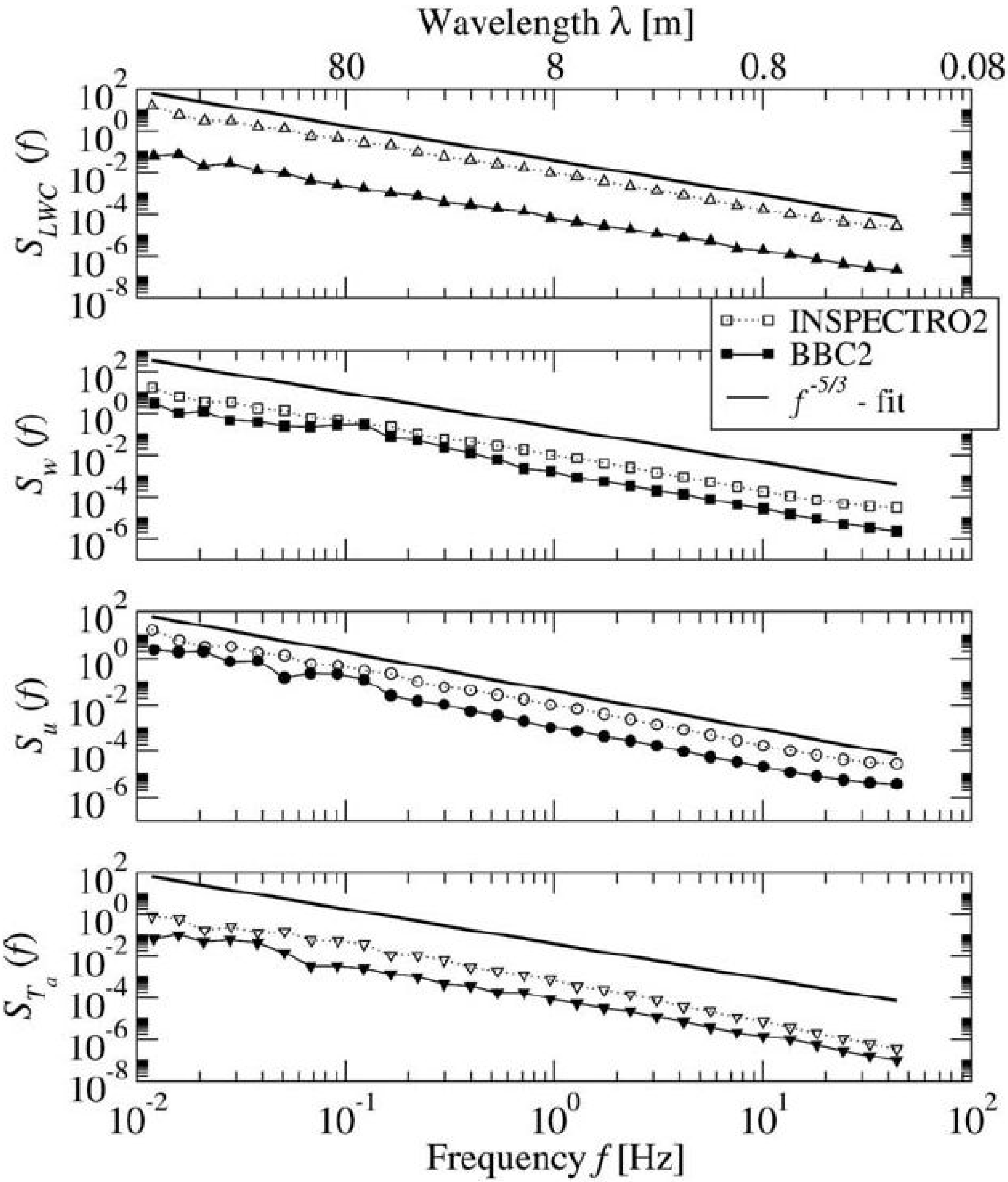
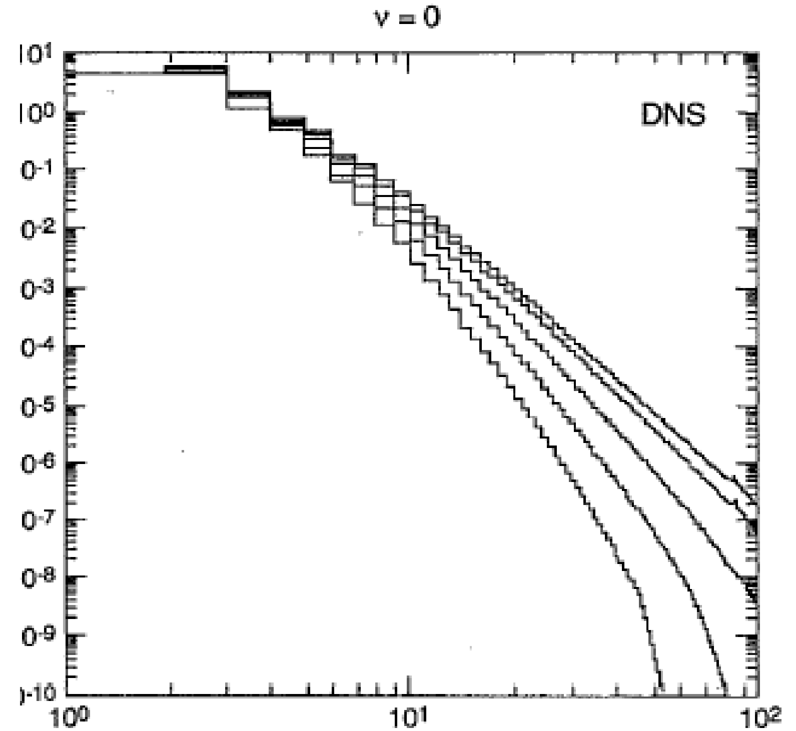


FIG. 5. Power spectral densities $S(f)$ of the same data as presented in Figs. 3 and 4. All spectra are in units of their variance per frequency; spectra of BBC data are divided by a factor of 10 for better resolution. For the top panel the frequencies are converted into wavelength assuming a constant horizontal wind speed of 8 m s^{-1}.



. DNS $E(k,t)$ for $t=(0.150, 0.175, 0.200, 0.225, 0.250)$, for initial
on (7), $\nu=0$.

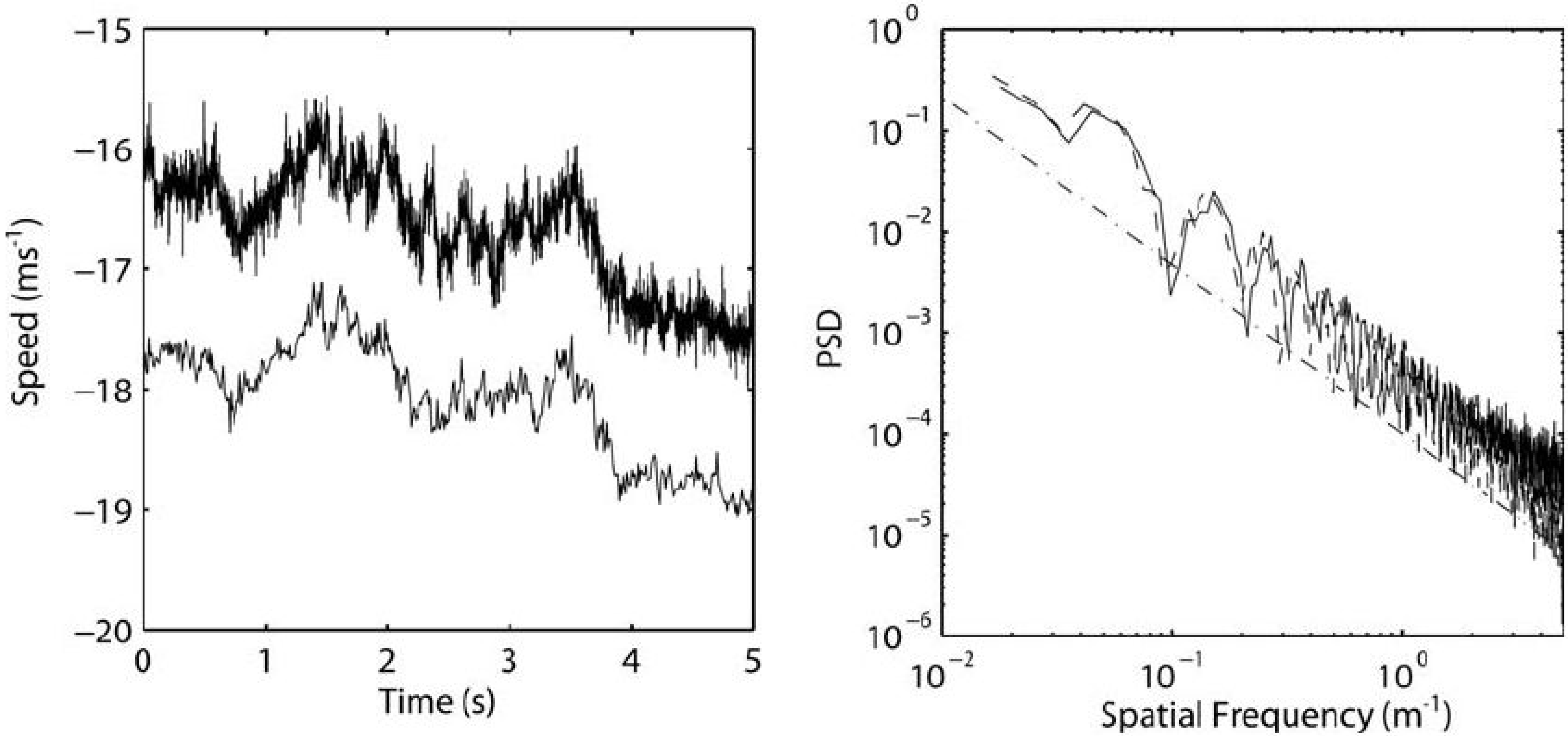
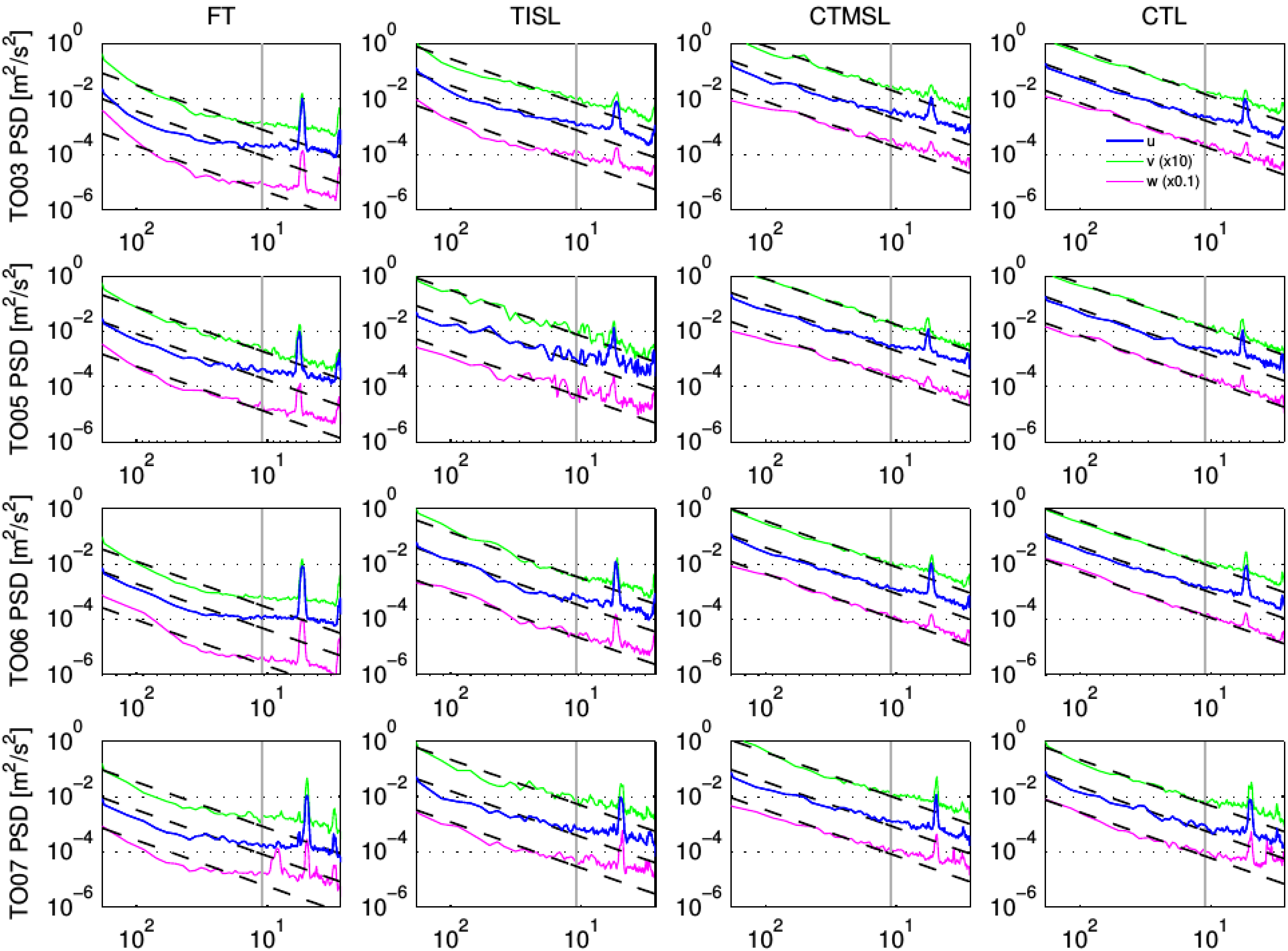
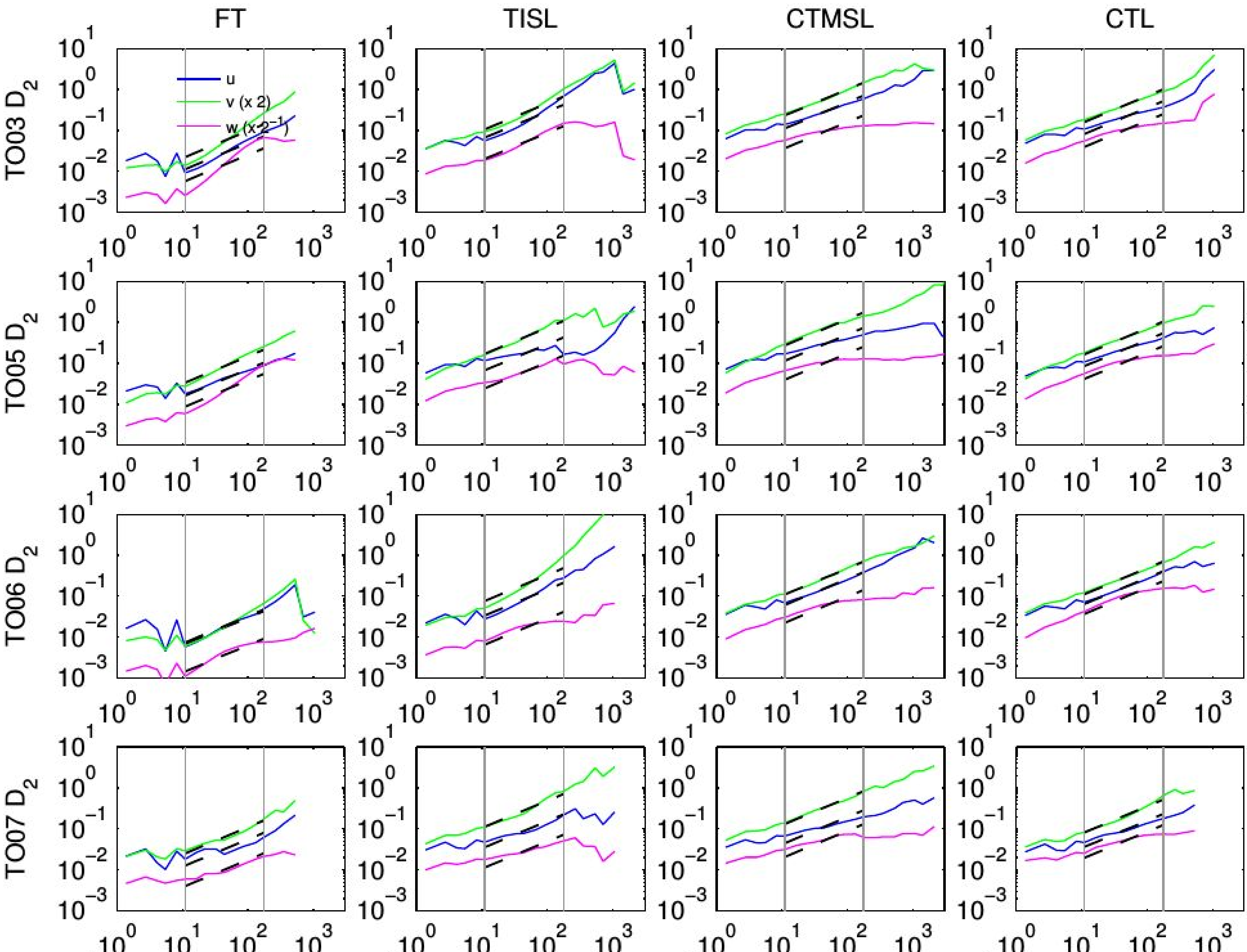


FIG. 12. Left panel: Measured velocity versus flight time for F/PDI (top curve) and a sonic anemometer (bottom curve). The offset of approximately 1.5 m s^{-1} is due to the location of F/PDI being closer to the stagnation point of the measurement platform than the sonic. Right panel: Power spectral density (PSD) versus spatial frequency for flow velocity measurements from F/PDI (solid) and the sonic (dashed); a line with slope −5/3 is included for reference (dotted).

Chuang et al., 2008


FT
TISL
CTMSL
CTL
TO03 PSD [m²/s²]
TO05 PSD [m²/s²]
TO06 PSD [m²/s²]
TO07 PSD [m²/s²]
u
v (x10)
w (x0.1)



FT
TISL
CTMSL
CTL
TO03 D_2
TO05 D_2
TO06 D_2
TO07 D_2
u
v (x 2)
w (x 2^{-1})

Kolmogorov scale and other characteristic scales of turbulence

Above a certain wavenumber k_d, viscosity will become important, and $E(k)$ will decay more rapidly than in the inertial range. The regime $k > k_d$ is known as the **dissipation range**. An estimate for k_d can be made by assuming

$$E(k) = C_K \epsilon^{2/3} k^{-5/3} \,:\, k_i < k < k_d$$
$$E(k) = 0 \,:\, k > k_d \quad (4.30)$$

and substituting in eqn 4.22, and integrating between k_i and k_d. Then we have

$$k_d \sim \left(\frac{\epsilon^{1/4}}{\nu^{3/4}} \right) \quad (4.31)$$

The inverse $l_d = 1/k_d$ is known as the **Kolmogorov scale**, the scale at which dissipation becomes important.

$$l_d \sim \left(\frac{\nu^{3/4}}{\epsilon^{1/4}} \right) \quad (4.32)$$

Komogorov scale is often denoted as η

At the other end of the spectrum, the important lengthscale is l_i, the integral scale, the scale of the energy-containing eddies. $l_i = 1/k_i$. We can also evaluate l_i in terms of ϵ. We can write

$$\overline{u^2} = U^2 = \int_0^\infty E(k)dk \tag{4.33}$$

and substituting for $E(k)$ from eqn 4.26

$$U^2 = \int_0^\infty C_K \epsilon^{2/3} k^{-5/3} dk \tag{4.34}$$

Assume that 1/2 of the energy is contained at scales $k > k_i$. Then

$$U^2 = 6 C_K \epsilon^{2/3} k_i^{-2/3} \tag{4.35}$$

and

$$k_i \sim \frac{\epsilon}{U^3} \tag{4.36}$$

so that $l_i \sim U^3/\epsilon$. Then the ratio of maximum and minimum dynamically active scales

$$\frac{l_i}{l_d} = \frac{k_d}{k_i} \sim \frac{U^3}{\epsilon^{3/4}\nu^{3/4}} \sim \left(\frac{U l_i}{\nu}\right)^{3/4} \sim Re_{l_i}^{3/4} \tag{4.37}$$

where Re_{l_i} is the **Integral Reynolds number**. Hence the range of scales goes as the Reynolds number to the power 3/4. This information is useful in estimating numerical resolution necessary to simulate turbulence down to the Kolmogorov scale at a chosen Reynolds number.

Taylor microscale.

A third length scale often used to characterise turbulence is the **Taylor microscale**:

$$\lambda = \left(\frac{\overline{u_i^2}}{\overline{(\partial u_i/\partial x_j)^2}} \right)^{1/2} = \left(\frac{U^2 \nu}{\epsilon} \right)^{1/2} \tag{4.38}$$

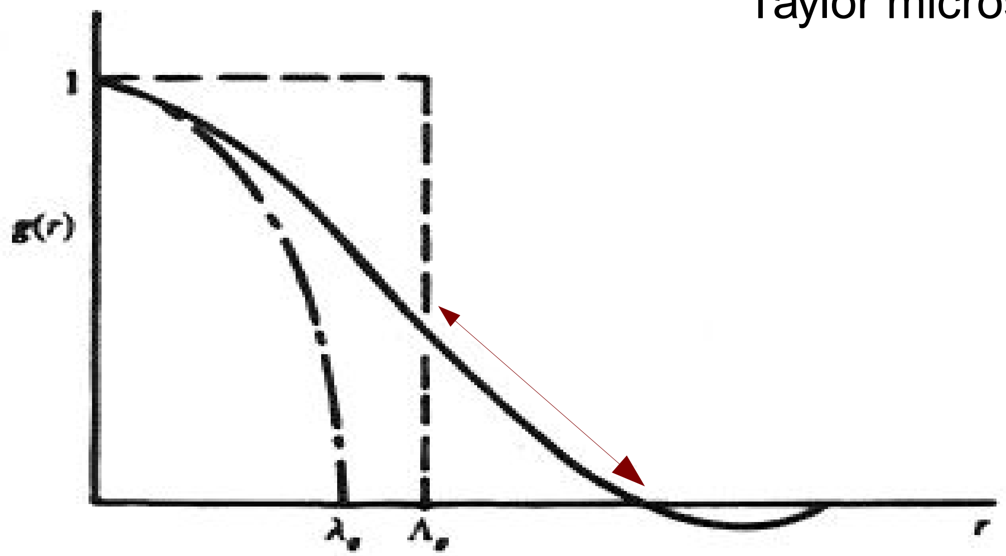
The Taylor microscale is the characteristic spatial scale of the velocity gradients. Using λ, an alternative Reynolds number can be defined:

$$Re_\lambda = \frac{U\lambda}{\nu} = \frac{U^2}{\nu^{1/2}\epsilon^{1/2}} \tag{4.39}$$

where $Re_\lambda \sim Re_{l_i}^{1/2} \sim l_i/\lambda$.

Taylor microscale Reynolds number

Taylor microscale interpretation:



Relationship of the correlation function g(r) to the integral and microscales Λ, λ.

$$g(r)=\frac{\overline{u(x)u(x+r)}}{\overline{u(x)u(x)}}$$

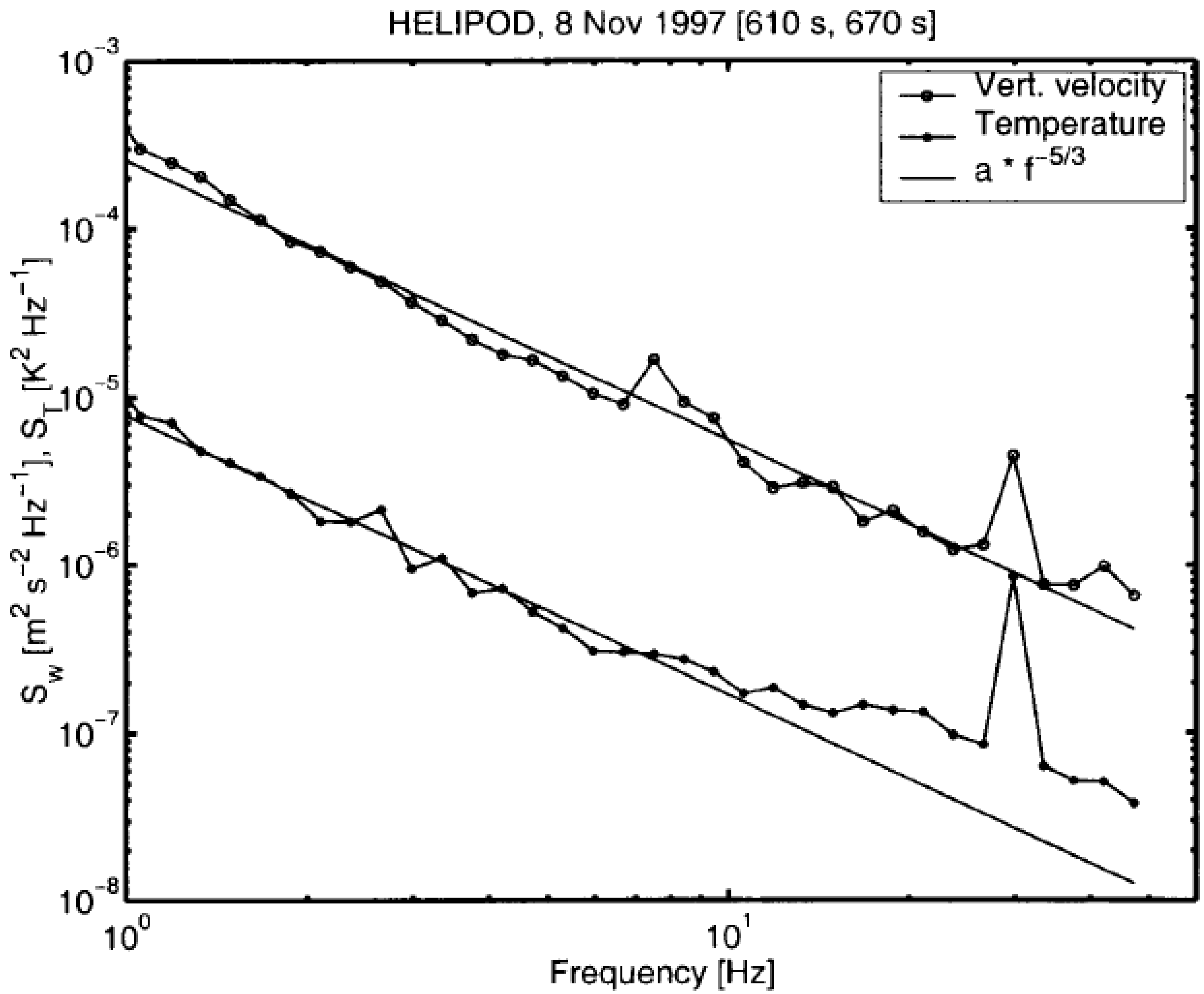


Figure 12. Frequency spectra of a 60-s-long time series of vertical wind velocity, w, and temperature, T, respectively, measured on 8 November 1997 with HELIPOD in the altitude range between 1380 m MSL and 1470 m MSL, well above the California Central Coast marine capping inversion. The noise floors are at about 1×10^{-7} m^2 s^{-2} Hz^{-1} and 3×10^{-8} K^2 Hz^{-1}, respectively, which at a Nyquist frequency of 50 Hz corresponds to uncorrelated noise standard deviations of 2.2 mm s^{-1} in w and 1.2 mK in T, respectively. The outliers at 30 Hz are due to sound waves from the helicopter rotor.

Muschinski et al., 2001

4.3 Passive tracer spectra

For a passive scalar which obeys an equation of the form

$$\frac{d\theta}{dt} = \kappa \nabla^2 \theta \tag{4.47}$$

we can write an equation for the variance $\overline{\theta'^2}$

$$\frac{d}{dt}\frac{\overline{\theta'^2}}{2} = -\kappa \overline{\nabla\theta' . \nabla\theta'} \tag{4.48}$$

and an equation for the spectrum $P(k)$ of this variance analogous to eqn 4.20:

$$2\kappa k^2 P(k) = T(k) + F(k) \tag{4.49}$$

where now $T(k)$ is the transfer of variance, and $F(k)$ is the forcing of variance. We can show that all the same properties apply as for the kinetic energy spectrum: the dissipation of variance χ must equal the total injection of variance $\int_0^\infty F(k)dk$. At wavenumbers far from the injection scale and dissipation scale, variance is fluxed at a constant rate χ (set by the injection rate). Using this information we can obtain the form of the spectrum $P(k)$. However, χ and k are not the only relevant parameters, since the tracer field is subject to the flow. The flow parameters (e.g. ϵ) also influence the tracer field.

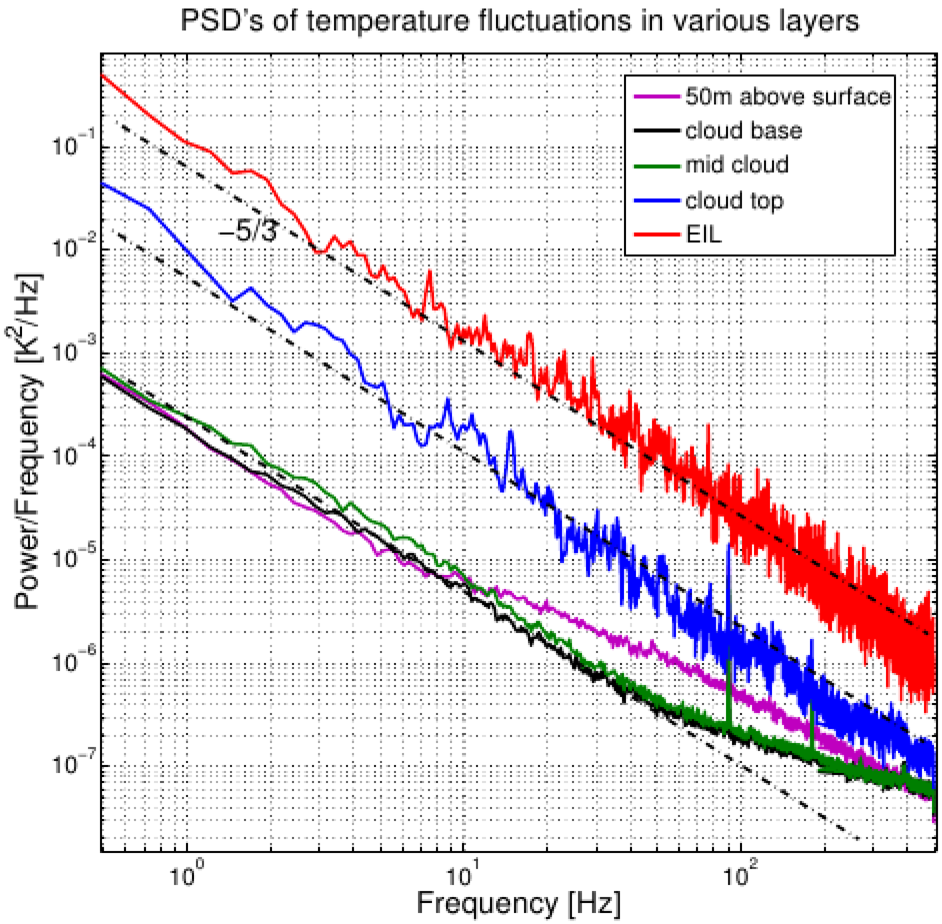


Fig. 11. Example PSDs of temperature fluctuations of a $1\,\mathrm{kS\,s^{-1}}$ signal collected at various levels of a turbulent stratocumulus topped boundary layer.

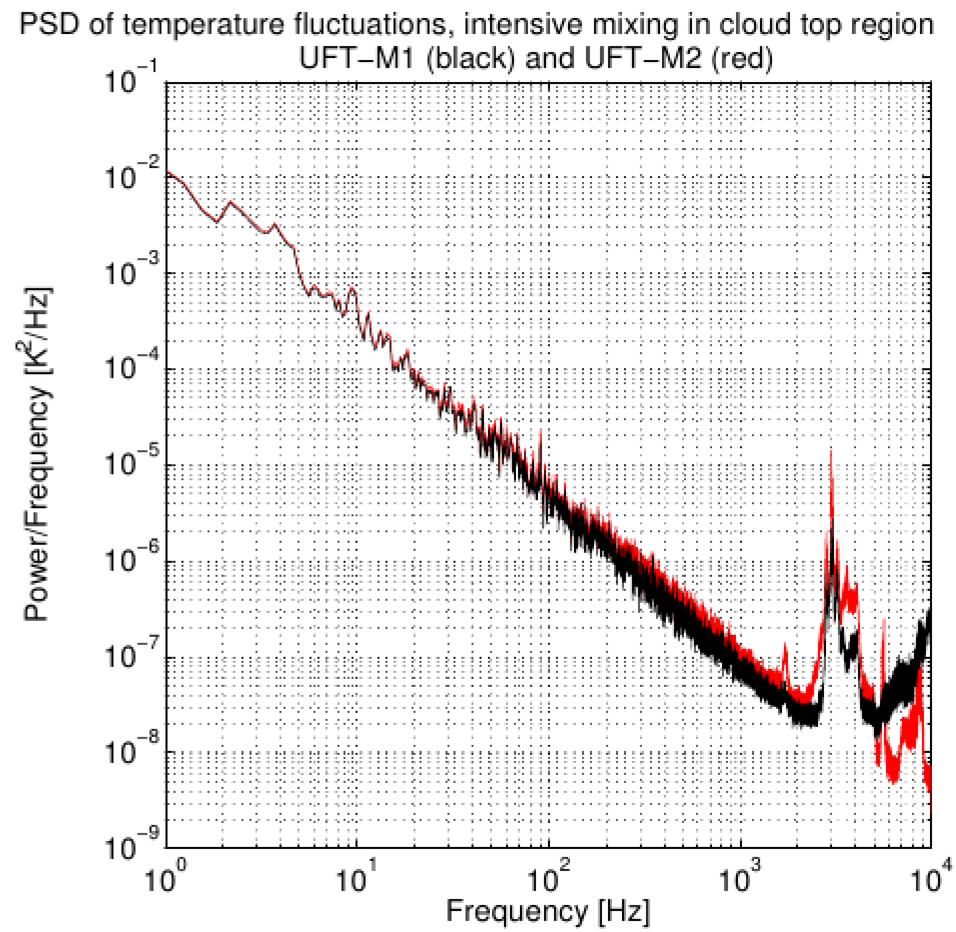


Fig. 9. Power spectra of the error corrected, unfiltered signals from two nearby sensing wires, UFT-M1 and UFT-M2, recorded in the region of intensive turbulent mixing.

Kumala et al., 2013