

CLOUD PHYSICS - tutorial 3 Activation

Köhler and κ -Köhler curves

1. Kelvin effect

The equilibrium saturation over a pure water droplet is given by:

$$S_{eq} = \exp\left(\frac{A(T)}{r}\right) \quad \text{where} \quad A(T) = \frac{2\sigma}{\rho_l R_v T}$$

For $A(T)/r \ll 1$ the equilibrium saturation takes an approximate form:

$$S_{approx} = 1 + \frac{A(T)}{r}$$

Plot $S_{eq}(r)$ and $S_{approx}(r)$ for a given temperature (e.g. $T = 0^\circ C$). Calculate the relative error $(S_{eq} - S_{approx})/S_{ex}$. For which values of r the relative error is smaller than 1%, 0.1%?

Assume that the surface tension of water is independent of temperature.

2. Raoult/solute effect

The equilibrium saturation ratio over an aqueous solution droplet is given by:

$$S = a_w \exp\left(\frac{A(T)}{r}\right),$$

where a_w is the activity of water in solution (the Raoult effect) and is given by:

$$a_w = \frac{r^3 - r_d^3}{r^3 - r_d^3(1 - \kappa)},$$

or in approximative form as:

$$a_{w,appr} = 1 - \kappa \frac{r_d^3}{r^3}.$$

r is the radius of the droplet, r_d is the dry radius that describes the amount of solute (CCN), κ is the hygroscopicity parameter.

Plot $a_w(r)$ and $a_{w,appr}$ showing the Raoult/solute effect for NaCl ($\kappa = 1.28$) and NH_4NO_3 (ammonium nitrate, *azotan amonu*, $\kappa = 0.67$) for different values of dry radii ($r_d = 0.02, 0.05, 0.1 \mu m$).

Plot the relative error $(a_w - a_{w,appr})/a_w$ for different dry radii and different κ values.

3. Köhler curve

The κ -Köhler equation:

$$S(r, \kappa, r_d, T) = \frac{r^3 - r_d^3}{r^3 - r_d^3(1 - \kappa)} \exp\left(\frac{A(T)}{r}\right)$$

describes the saturation equilibrium pressure over a droplet containing a soluble nucleus.

Plot the Köhler curves for NaCl and NH_4NO_3 :

- $r_d = 0.01, 0.03, 0.1 \mu\text{m}$
- $T = 0, 20^\circ \text{C}$

For sufficiently big droplets (ving radius R) the equilibrium saturation converges to 100%. It means that the curvature and solute terms of Köhler equation become unimportant, Assume two scenarios of considering that those effects are unimportant: the equilibrium saturation is less then 0.1% or 0.01%) For $\kappa = 1.28$ and $\kappa = 0.2$ plot relations between dry radii and R .

4. Critical radius and critical saturation

Show how critical radii and critical supersaturations (the maximum of the Köhler curve) depend on dry radii, r_d , and hygroscopicity parameter, κ . Assume a constant value of temperature.